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Endogenous Incumbency in Repeated Contests*

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Abstract

We consider a model of infinitely repeated lottery contests in which the winner of the prior contest (incumbent) additionally gains the opportunity to bias the subsequent contest by exerting early effort in an intermediate stage. An effort-maximizing contest designer strategically chooses the cost advantage of incumbency. We show that the contest designer prefers to set the cost advantage such that the incumbent only partially discourages the contender, i.e. the contender exerts less, but still positive, effort than in an unbiased contest. In this way, rent extraction is higher than under independent lottery contests with no intermediate stage, because (i) players compete fiercer to become the incumbent and (ii) the increase in early effort outweighs the decrease in effort in the biased contest. Therefore, we provide some rationale for incumbency advantages, for example in repeated procurement settings.

Keywords: repeated contests, lottery contest, incumbent, discouragement effect
JEL classification: C72, C73, D72

1 Introduction

Organizers of repeated competitive situations often warrant incumbency advantages as additional incentives. In procurement, for example, firms compete not only for the immediate reward of the current mandate, but also for an incumbency status that allows the winning firm to generate an advantage in bidding for future mandates. Specifically, when the winner of a mandate implements the project he applied for, the organizer can

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reward a high-quality implementation with an advantage in competition for the next mandate. Moreover, as the quality of implementation will usually determine the size of the future advantage, the incumbent can control this advantage by the choice of effort in implementation.

The literature on contest design gives ambiguous advice on how large an effort-maximizing contest designer should set the incumbency advantage in a repeated setting. Clearly, an incumbency advantage acts as an additional reward for winning a contest, which boosts participants' efforts. However, an incumbency advantage also introduces asymmetry into future rounds. A contender may be discouraged from fighting against a favored opponent, leading him to reduce his effort. In this article, we employ an infinitely repeated contest model between two ex-ante symmetric players to analyze how an effort-maximizing contest designer moderates the trade-off between these two effects.

We consider an infinitely repeated lottery contest with two players. In particular, we investigate a structure in which today's winner (in the following called *incumbent*) can endogenously bias tomorrow's contest in his favor. Each contest is partitioned into an investment stage and a competition stage. In the investment stage, the incumbent can exert early effort to gain a head start over his opponent (in the following called *contender*). The contest designer may encourage such behavior by (partially) compensating early effort and thus giving the incumbent a cost advantage over the contender. In the competition stage, both contestants - the incumbent as well as the contender - observe early effort and exert effort to win the contest.

We show that the extent to which the incumbent exerts early effort depends on the degree of compensation for early effort. If compensation is very high such that early effort is cheap, the incumbent fully discourages the contender from competition. If compensation is moderate, full discouragement is no longer optimal. Instead, the incumbent partially discourages the contender who still participates, but exerts less effort than he would in a symmetric contest. If compensation is zero (or even negative), the incumbent does not bias the subsequent contest at all. Consequently, the contests remain independent of each other.

Anticipating equilibrium behavior, the contest designer chooses marginal compensation of early effort to maximize rent extraction, i.e., players' total effort net of partial compensation in the investment stages, relative to the total prize sum. This moderates two opposing effects. First, compensation of early effort increases the incumbency advantage and incentivizes both players to fight hard in order to become tomorrow's incumbent, and hence increases rent extraction (*incentive effect*). Second and conversely, the incumbency advantage partially or fully discourages the contender from competing (*discouragement effect*), which reduces rent extraction.

We demonstrate that endogenous incumbency can increase rent extraction compared to a game with repeated independent contests. In particular, we find that the contest designer prefers to induce partial discouragement: he sets the incumbency advantage such that the incumbent chooses to bias tomorrow's contest in his favor, but the contender remains active in tomorrow's contest, albeit with reduced effort compared to an unbiased contest. Naturally, the optimal choice also depends on discounting. Being the incumbent today improves the chances of remaining the incumbent in the future. For patient players, these spillover effects to future periods are more important, which increases the incentive effect to exert effort today.

We also consider that players may cooperate by means of grim trigger strategies to the contest designer's detriment. For a sufficiently high discount factor, conditional cooperation equilibria can exist where both players exert zero effort, as long as no player has deviated from cooperation yet. We show that the contest designer can utilize endogenous incumbency to restrict cooperative incentives. By setting a high level of compensation, the contest designer can eliminate cooperative equilibria as deviation from cooperation becomes more profitable.

In addition, we cover two extensions of the model. First, we investigate the situation in which the incumbent's early effort is not observable for the contender. In this case, the incumbent loses the commitment opportunity, but still enjoys a cost advantage as early effort remains (partially) compensated. We show that compared to the case with observable early effort, the contest designer prefers to increase the incumbency advantage even further. However, observable early effort yields higher rent extraction than unobservable early effort, given that the contest designer acts optimally in both cases. Therefore, if the contender cannot observe early effort, the contest designer should publicly announce it.

Second, we consider situations where the contest designer only values winner's effort and not loser's effort. In this case, the contest designer prefers to further increase compensation until the incumbent finds it optimal to fully discourage the contender, that is, the contender exerts no effort. Still, competition remains intense because the incumbent must remain active preemptively to defend his position in every contest.

The remainder of the paper is structured as follows. Section 2 discusses our contribution to the related literature. In Section 3, we introduce the baseline model and derive players' equilibrium behavior. In Section 4, we show that a contest designer profits by the use of endogenous incumbency to maximize rent extraction and to avoid cooperation. Section 5 covers the scenarios where early effort is unobservable or the contest designer is only interested in winner's effort. Section 6 concludes.

2 Related Literature

Incumbency advantages and asymmetric players are a prevalent topic in several lines of literature. In studies dealing with auctions and procurement processes, incumbents may benefit from existing or emerging market barriers in the form of entry costs or switching costs (e.g. Greenstein (1993), Arozamena et al. (2014), Premik (2023)). In lobbying contests, one firm might represent the status quo and the other might challenge it (Polborn (2006)). In the contest literature, two forms of incumbency advantages are present. First, incumbents are often directly advantaged in the contest. Incumbents are either modeled to be more efficient, that is, the incumbent's effort is cheaper or more impactful compared to rivals' efforts (Fu (2006), Epstein et al. (2011), Franke et al. (2013), Franke et al. (2014)), or incumbents have a head start over their rivals, or potentially both (Fu and Wu (2020)). Second, incumbents can have commitment opportunities. In this way, the incumbency advantage is the favored leader position in sequential Stackelberg frameworks (Dixit (1987), Linster (1993), Morgan (2003), Serena (2017), Hinnosaar (2024), Gao et al. (2025)).

Our model combines these two advantages. An incumbent gains access to an exclusive investment stage. Investments (early effort) yield an observable head start for the next competition stage. Consequently, the incumbent can behave as a Stackelberg-leader. Whether the incumbent prefers to act as a Stackelberg leader or to play the simultaneous contest depends on the marginal cost of early effort, which is set by the contest designer. If early effort is cheaper than regular effort, the incumbent prefers to act as a Stackelberg leader, and additionally exerts effort more efficiently. Dixit (1987) and Linster (1993) show that a commitment opportunity in the absence of any other heterogeneity has no value in a logit contest. Conversely, if early effort is more productive than regular effort, then a mover advantage arises, as in this case it becomes optimal to invest in a head start in order to discourage the contender.

Several studies consider that an incumbency status arises as an additional reward for winning a contest (Möller (2012), Clark and Nilssen (2018), Clark et al. (2020), Häfner and Nöldeke (2022)). In this way, players can influence their chances of becoming the incumbent, but cannot influence the size of their incumbency advantage. Still, some authors discuss repeated contests in which previous behavior may indirectly influence future incumbency advantages. For example, Beviá and Corchón (2013) propose a framework where the relative share of efforts in a prior contest determines players' abilities in a subsequent contest. In Deck et al. (2024), a share of the incumbent's effort carries over into the next contest.

In contrast, the incumbency advantage is neither exogenously fixed nor is its size determined by past behavior in our model. Instead, the incumbent decides on the level of his advantage himself in a separate investment stage. In this way, we also contribute to the literature on investments prior to contests. For instance, players may invest to reduce their cost of effort (Münster (2007), Fu and Lu (2009)), or to increase their abilities (Schaller and Skaperdas (2020)).

A substantial body of work discusses how contest designers influence heterogeneity among players to maximize their objective function. Typically, the designer's goal is to maximize overall contest intensity. In ex-ante asymmetric contests, the optimal bias often counteracts existing asymmetries which levels the playing field (e.g. Lazear and Rosen (1981), Schotter and Weigelt (1992), Kirkegaard (2013)). The rationale is that, in one-shot lottery contests, equal strength maximizes aggregate effort. Under some circumstances, it is also optimal for the contest designer to bias a contest in favor of the already advantaged contestant (Meyer (1991), Meyer (1992), Epstein et al. (2011)).

In comparison, optimal biases in ex-ante symmetric contests are studied less often. Drugov and Ryvkin (2017) discuss a general class of biased contest success functions that - depending on the nature of the contest and the bias - may be optimal even when dealing with symmetric players. Barbieri and Serena (2022) show that in a best-of-three Tullock contest, it is optimal to incorporate biases, since intuitively this increases the probability for all contests to be played. When the contest designer prefers to have a winner with high ability, but is not able to ex-ante observe abilities, it can also be optimal to bias the contest (Kawamura and de Barreda (2014); Pérez-Castrillo and Wettstein (2016)). Our model discusses optimal biases in ex-ante symmetric settings as well but differs from the aforementioned papers in several aspects. First, the contest designer can only indirectly intervene in our model. In particular, the contest designer nudges contestants to bias future contests by offering a (partial) compensation of early effort. Thus, early effort, i.e., the head start, is ultimately a decision variable of the incumbent. Second, the incumbency advantage is neither an arbitrary bonus to some player nor a reflection of the incumbent's higher ability. Instead, the incumbency advantage is a reward and thus can be both earned and lost.

Finally, we contribute to the literature on infinitely repeated contests in which cooperative strategies allow for multiple equilibria. In Leininger and Yang (1994), tit-for-tat reasoning can emerge in a sequential one-shot contest where players can infinitely often add to their effort. In infinitely repeated independent Tullock contests, cooperation may be sustainable if the discount rate is sufficiently high (Linster (1994)). Brookins et al. (2021) find experimental support for this result. In our model, endogenous incumbency implies that contests are dynamically connected, as winning today provides an incumbency status tomorrow. In principle, this increases the incentive to deviate from cooperation. Indeed, we show that if incumbency provides a large advantage, then, in order to sustain cooperation, the discount rate must be higher than in Linster (1994) for unrelated contests.

3 The model

We consider two players i, j who play an infinite number of contests in succession. Players are ex-ante symmetric, i.e., their prize valuations and their productiveness coincides, except for one characteristic: one player (henceforth referred to as player i, the incumbent) has won the previous contest whereas the other (henceforth referred to as player j, the contender) has not.[1] In each contest k, players can win a non-divisible prize which is normalized to 1. Each contest is organized as a (potentially biased) lottery contest such that players i, j win contest k with probabilities given by

$$p_{i,k} = \begin{cases} \frac{1}{2} & \text{if } x_{i,k} = x_{j,k} = d_{i,k} = 0, \\ \frac{x_{i,k}+d_{i,k}}{x_{i,k}+x_{j,k}+d_{i,k}} & \text{otherwise,} \end{cases} \tag{1}$$

and

$$p_{j,k} = \begin{cases} \frac{1}{2} & \text{if } x_{i,k} = x_{j,k} = d_{i,k} = 0, \\ \frac{x_{j,k}}{x_{i,k}+x_{j,k}+d_{i,k}} & \text{otherwise.} \end{cases} \tag{2}$$

where $d_{i,k} \geq 0$ is the head start in favor of player i (henceforth called *early effort*), and $x_{i,k}, x_{j,k} \geq 0$ are players' efforts. Players take into account that player i's *effective input* into the contest success function is given by $x_{i,k} + d_{i,k}$. Players discount future periods by discount factor $\delta \in (0, 1)$ which is constant over time.[2]

A contest is divided into an investment stage and a subsequent competition stage. In the investment stage I_k of contest k, the incumbent can exert early effort $d_{i,k}$ at marginal cost of early effort $b \geq 0$ to maximize his expected payoff

$$\pi^I_{i,k} = \pi^C_{i,k}(d_{i,k}) - b \cdot d_{i,k}, \tag{3}$$

where $\pi^C_{i,k}$ is the expected payoff of the subsequent competition stage, which depends on early effort.

In competition stage C_k, players observe the incumbent's head start $d_{i,k}$ and choose efforts $x_{i,k}$ and $x_{j,k}$ simultaneously to maximize expected payoff, i.e.,

$$\pi^C_{i,k} = p_{i,k} \cdot (1 + \delta\pi^I_{i,k+1}) + (1 - p_{i,k}) \cdot \delta\pi^C_{j,k+1} - x_{i,k}, \tag{4}$$

and

$$\pi^C_{j,k} = p_{j,k} \cdot (1 + \delta\pi^I_{i,k+1}) + (1 - p_{j,k}) \cdot \delta\pi^C_{j,k+1} - x_{j,k}, \tag{5}$$

where $\pi^I_{i,k+1}$ denotes the winner's continuation payoff (i.e. the payoff of the incumbent of the next contest), $\pi^C_{j,k+1}$ denotes the loser's continuation payoff (i.e. the payoff of the contender of the next contest) and δ is the discount factor. Since the contender j has no action in the investment stage, $\pi^I_{j,k+1} = \pi^C_{j,k+1}$, so the contender's payoff in the investment stage is simply his payoff in the subsequent competition stage. In that sense, each contest

[1] By a slight abuse of notation, we do not rename the players in the first contest, although there is no incumbent in the first contest as players are ex-ante symmetric. Notice also that depending on the outcome of the contest, the identity of player i may change, such that player i (the incumbent) in contest k is not necessarily the same player as player i in contest $k + 1$.

[2] Equivalently, δ can be interpreted as the probability with which the game continues in the next period. Then, $1 - \delta$ is the likelihood of an exogenous shock to the contest structure that causes the contest to terminate.

can be considered as a Stackelberg-variant where player i is both a Stackelberg-leader (by exerting early effort in the investment stage) and a Cournot-player (by exerting regular effort in the competition stage).[3] The first contest is an exception, as there is no investment stage because players are ex-ante symmetric.

The structure of the contest as well as the marginal costs of (early) effort are common knowledge. The timing of the game is illustrated in Figure 1.

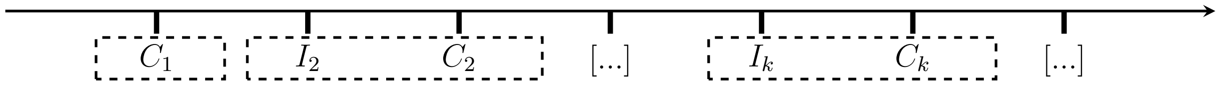

Figure 1: Timing of repeated contests (each rectangle depicts a period).

Lemma 1 (Single-period benchmark)**.** *In an unbiased one-shot lottery contest with a prize of* 1*, both players exert effort equal to* $x_i = x_j = 1/4$ *and obtain expected payoffs of* $\pi_i = \pi_j = 1/4$.

Lemma 1 summarizes the equilibrium in an unbiased one-shot lottery contest. In what follows, it will be useful to compare our findings to this single-period benchmark. Its derivation can be found in any textbook on contest theory, for example in Konrad (2009) or Beviá and Corchón (2024).

3.1 Equilibrium behavior in competition stages

In the competition stage of contest k, players observe player i's early effort $d_{i,k}$ and maximize their expected payoffs by simultaneously choosing $x_{i,k}$ and $x_{j,k}$. In every contest, players fight for a prize of value 1. Additionally, players take into account that the winner will become the incumbent and the loser will become the contender in the next contest, which is reflected in their discounted continuation payoffs. Consequently, they compete for an effective prize sum denoted by $\psi_k = 1 + \delta(\pi^I_{i,k+1} - \pi^C_{j,k+1})$. Optimal efforts and corresponding payoffs are summarized in Lemma 2.

Lemma 2. *In competition stage* C_k*, where player* i *exerted observable early effort* $d_{i,k}$ *in*

[3] In principle, player i can use both channels of effort. However, we show that player i will solely exert early (regular) effort if the net cost of early effort is lower (higher) than the cost of regular effort. If the costs of early effort and regular effort are identical, player i will be indifferent between any distribution of efforts across the two stages under the condition that $d_{i,k} + x_{i,k} = 1/4$, i.e., his effective input equals effort of the single-period benchmark (see Lemma 1).

the previous investment stage I_k, equilibrium efforts and payoffs are given by

$$x_{i,k} = \begin{cases} \frac{1}{4}\psi_k - d_{i,k} & \textit{if } d_{i,k} \leq \frac{1}{4}\psi_k \\ 0 & \textit{if } d_{i,k} > \frac{1}{4}\psi_k, \end{cases}$$

$$x_{j,k} = \begin{cases} \frac{1}{4}\psi_k & \textit{if } d_{i,k} \leq \frac{1}{4}\psi_k \\ \sqrt{d_{i,k}\psi_k} - d_{i,k} & \textit{if } \frac{1}{4}\psi_k < d_{i,k} < \psi_k \\ 0 & \textit{if } d_{i,k} \geq \psi_k, \end{cases}$$

$$\pi^C_{i,k} = \begin{cases} \frac{1}{4}\psi_k + \delta\pi^C_{j,k+1} + d_{i,k} & \textit{if } d_{i,k} \leq \frac{1}{4}\psi_k \\ \sqrt{d_{i,k}\psi_k} + \delta\pi^C_{j,k+1} & \textit{if } \frac{1}{4}\psi_k < d_{i,k} < \psi_k \\ \psi_k + \delta\pi^C_{j,k+1} & \textit{if } d_{i,k} \geq \psi_k, \end{cases}$$

$$\pi^C_{j,k} = \begin{cases} \frac{1}{4}\psi_k + \delta\pi^C_{j,k+1} & \textit{if } d_{i,k} \leq \frac{1}{4}\psi_k \\ \psi_k + \delta\pi^C_{j,k+1} + d_{i,k} - 2\sqrt{d_{i,k}\psi_k} & \textit{if } \frac{1}{4}\psi_k < d_{i,k} < \psi_k \\ \delta\pi^C_{j,k+1} & \textit{if } d_{i,k} \geq \psi_k. \end{cases}$$

The proof of Lemma 2 is in Appendix A.1. If the contest is unbiased ($d_{i,k} = 0$), players are symmetric and play $x_{i,k} = x_{j,k} = \psi_k/4$.[4] If $d_{i,k} \in (0, \psi_k/4]$, the incumbent exerts effort so that his effective input still equals $\psi_k/4$, and the contender reacts by playing $x_{j,k} = \psi_k/4$. If $\psi_k/4 < d_{i,k}$, then the incumbent does not exert additional effort ($x_{i,k} = 0$), since his effective input is already higher than $\psi_k/4$ and thus functions as a strategic substitute. This either partially discourages the contender from competing, i.e., $x_{j,k} \in (0, \psi_k/4)$ for $d_{i,k} \in (\psi_k/4, \psi_k)$ or fully discourages him, i.e., $x_{j,k} = 0$ for $d_{i,k} \geq \psi_k$. In the latter case, the competition stage degenerates and the incumbent wins with certainty.

3.2 Equilibrium behavior in investment stages

In investment stage I_k of contest k, player i who won the previous contest (the incumbent) maximizes expected payoff by choosing optimal early effort $d^*_{i,k}$. Equilibrium early effort and corresponding payoffs are summarized in Lemma 3.

Lemma 3. *In investment stage I_k, equilibrium early effort by the contender and corre-*

[4]Notice that this is not necessarily the same result as Lemma 1, as the former depends on ψ_k.

sponding payoffs are given by

$$d^*_{i,k} = \begin{cases} \psi_k & \textit{if } b \leq \frac{1}{2} \\ \frac{\psi_k}{4b^2} & \textit{if } \frac{1}{2} < b < 1 \\ 0 & \textit{if } b \geq 1, \end{cases}$$

$$\pi^I_{i,k} = \begin{cases} (1-b)\psi_k & \textit{if } b \leq \frac{1}{2} \\ \frac{\psi_k}{4b} + \delta\pi^C_{j,k+1} & \textit{if } \frac{1}{2} < b < 1 \\ \frac{1}{4}\psi_k + \delta\pi^C_{j,k+1} & \textit{if } b \geq 1, \end{cases}$$

$$\pi^I_{j,k} = \pi^C_{j,k}.$$

The proof of Lemma 3 is in Appendix A.2. Notice that ψ_k, i.e., the value of winning contest k, is not constant but a function depending on the value of incumbency that decreases in the cost of early effort b and increases in the discount factor δ.

Decreasing b moderates two opposing effects. One the one hand, any $b < 1$ makes the incumbent strong and the contender weak who suffers under a *discouragement effect*. On the other hand, any $b < 1$ implies an increased ψ_k. The higher the difference in continuation payoffs, the more valuable is the incumbency position and the fiercer players fight in contest k (*incentive effect*). If early effort is cheap ($b \leq \frac{1}{2}$), the incumbent invests in early effort to fully discourage the contender from competing. For $b \in (1/2, 1)$, early effort is moderately cheap. In this case, the incumbent exerts early effort and partially discourages the contender, who exerts $x_{j,k} < \psi_k/4$. Then, the contest's outcome is random, but the incumbent is the favorite because his input outweighs the contender's effort. Notice that the contender exerts less effort than a quarter of the effective prize. Nonetheless, depending on b and δ, $x_{j,k} \geq 1/4$ is still possible, i.e., the contender's effort under endogenous incumbency can be higher than in the single-period benchmark, if the prize ψ_k is sufficiently high such that the incentive effect dominates the discouragement effect. If early effort is costly ($b \geq 1$), then the incumbent will not invest early effort at all because - in line with Dixit (1987) and Linster (1993) - the commitment opportunity itself without a cost advantage (or even with a cost disadvantage) does not entail any value.

3.3 The infinite game

Lemma 2 and Lemma 3 characterize equilibrium behavior in any contest k, depending on b and the effective prize ψ_k. To solve for the infinite game, we apply Theorem 3.3 from Fudenberg and Levine (1983) which states that a sequence of subgame perfect Nash equilibria in the K-contest game[5] converges for $K \to \infty$ to a subgame perfect Nash equilibrium in the infinite game under certain conditions (see Appendix A.3). That is

[5]The term K-contest game refers to the finite version of the game with K periods.

why we first derive equilibrium behavior of the subgame-perfect Nash equilibrium of the finite, K-contest game by backward induction.[6] For $K \to \infty$, this equilibrium directly yields a subgame-perfect equilibrium of the infinite game, which we will call *competitive equilibrium*.[7] In the following, we will characterize the competitive equilibrium. Depending on the value of b, the incumbent prefers to fully discourage, partially discourage, or not discourage the contender. These three cases give rise to qualitatively different behavior, and we analyze them separately.

For $b \in (0, \frac{1}{2}]$, Lemma 4 characterizes equilibrium behavior in contest $k \geq 2$.[8]

Lemma 4. *For $b \in [0, \frac{1}{2}]$, then, for any contest $k \geq 2$ in which player i is the incumbent, player j is the contender and early effort $d_{i,k}$ is observable, the effective prize sum and equilibrium behavior are given by*

$$\begin{aligned} \psi_k &= \frac{1}{1-\delta(1-b)}, \\ d^*_{i,k} &= \frac{1}{1-\delta(1-b)}, \\ x_{i,k} &= x_{j,k} = 0, \end{aligned}$$

where the effective prize ψ_k and early effort $d^*_{i,k}$ are decreasing in b and increasing in δ.

The proof is in Appendix A.2. Lemma 4 is characterized full discouragement. The incumbent exerts a level of early effort equal to the effective prize sum. By doing so, the incumbent ensures that the discouragement effect for the contender is strong enough to dominate the incentive effect. The contender is fully discouraged from competing and prefers to stay out of the contest. The result of the contest is therefore not probabilistic anymore, since the incumbent wins with certainty. Consequently, the first contest's winner also wins all other contests with certainty whereas the first contest's loser does not exert any effort in any subsequent contest anymore. However, the incumbent does not win all contests for free (unless $b = 0$).

For $b \in (1/2, 1)$, Lemma 5 characterizes equilibrium behavior in contest $k \geq 2$.

Lemma 5. *If $b \in (1/2, 1)$, then, for any contest $k \geq 2$ in which player i is the incumbent*

[6] Notice that for all cases except $b = 0$ and $b = 1$, the subgame-perfect Nash equilibrium of the K-contest game is unique. For $b = 0$, there exists a continuum of optimal $d^*_{i,k} \geq \psi_k$, as early effort is costless. For $b = 1$, the incumbent is indifferent to all effort-early effort combinations under the condition $d_{i,k} + x_{i,k} = 1/4$. For simplicity, we assume that $d^*_{i,k} = 0$ for $b = 1$ and $d^*_{i,k} = \psi_k$ for $b = 0$. Additionally, we show in Section 4 that a contest designer always profits from setting $b \in (0, 1)$.

[7] The competitive equilibrium is not the unique subgame-perfect Nash equilibrium in the infinite game. We will additionally cover *cooperative equilibria* which are based on grim trigger strategies in Section 4.2.

[8] By assumption, there is no incumbent in the first contest which we characterize in Corollary 2.

and player j is the contender, the effective prize sum and equilibrium behavior are given by

$$\begin{aligned}
\psi_k &= \frac{1}{1-\delta A}, \\
d_{i,k}^* &= \frac{1}{(1-\delta A)\,4b^2}, \\
x_{i,k} &= 0, \\
x_{j,k} &= \frac{2b-1}{(1-\delta A)\,4b^2},
\end{aligned}$$

where $A = \frac{5b-1}{4b^2} - 1$. The effective prize ψ_k and early effort $d_{i,k}^*$ are decreasing in b and increasing in δ.

The proof is in AppendixA.2. Lemma 5 is characterized by partial discouragement. The incumbent fully relies on early effort. The contender exerts less effort than he would if the contest was unbiased, but still exerts positive effort. Therefore, the outcome is random, but the incumbent is the favorite. For the contender, the discouragement effect is still present, but less pronounced, such that the incentive effect now ensures a positive effort choice.

For $b \geq 1$, Lemma 6 characterizes the equilibrium in contest $k \geq 2$.

Lemma 6. *If $b \geq 1$, then, for any arbitrary contest $k \geq 2$ in which player i is the incumbent, player j is the contender and early effort $d_{i,k}$ is observable, the effective prize sum and equilibrium behavior are given by*

$$\begin{aligned}
\psi_k &= 1, \\
d_{i,k}^* &= 0, \\
x_{i,k} &= x_{j,k} = \frac{1}{4}.
\end{aligned}$$

The proof is in Appendix A.2. As discussed earlier, any $b \geq 1$ implies that the incumbent position is not valuable because it cannot be utilized in a beneficial way. All contests remain independent such that each period's contest resembles an unbiased lottery contest with two symmetric players and an effective prize sum equal to 1.

Panel (a) of Figure 2 shows the relationship between the effective prize ψ_k and δ, depending on b. Clearly, a larger continuation probability δ means that the future is more valuable, and therefore the value of becoming the incumbent is larger. Similarly, a lower b means that the value of becoming the incumbent is large, since players anticipate that if they win, they can exploit their incumbency position in the next contest at low cost.

A similar logic applies to the relationship between equilibrium early effort $d_{i,k}^*$ and the parameters b and δ, which is illustrated in Panel (b) of Figure 2. Early effort is increasing in the discount rate δ due to the incentive effect. A higher δ implies a higher effective prize given that early effort is less costly than regular effort. Then, the incumbent faces

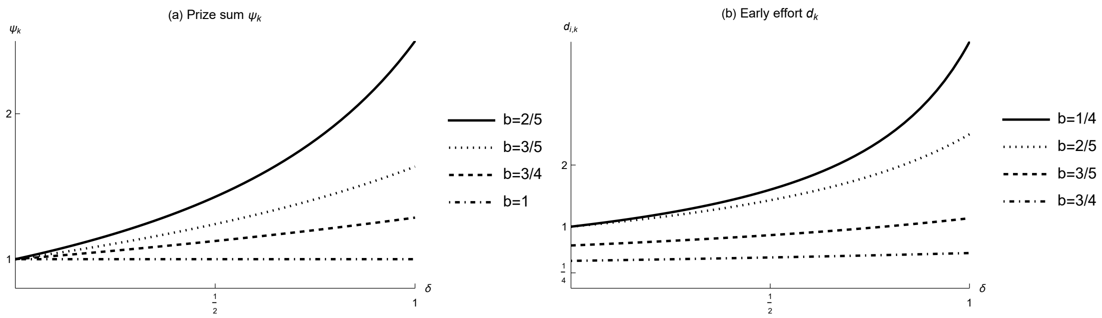

Figure 2: Effective prize sum and early effort for $b \in (0, 1)$.

stronger incentives to bias the next contest in his favor and the contender is more difficult to discourage. In contrast, the higher the cost of early effort b, the less pronounced is the incentive effect and the less severe is the asymmetry between players. Therefore, early effort is decreasing in b.

Figure 3 illustrates the interplay of the contender's effort $x_{j,k}$, discount factor δ and the cost of early effort b. A low discount factor δ implies a rather weak incentive effect. Nonetheless, player asymmetries are present. Consequently, the discouragement effect dominates, i.e., the more disadvantaged the contender, the less effort he exerts. However, a high δ implies a stronger incentive effect. Then, the contender fights fiercely even if he faces a large disadvantage in the present. If the discount factor is close to 1, the contender exerts more effort in equilibrium in case of an intermediate disadvantage ($b = 3/4$) than in case of a strong ($b = 3/5$) or a weak one ($b = 9/10$). Hence, the contender's effort is nonmonotonic in b for large δ.

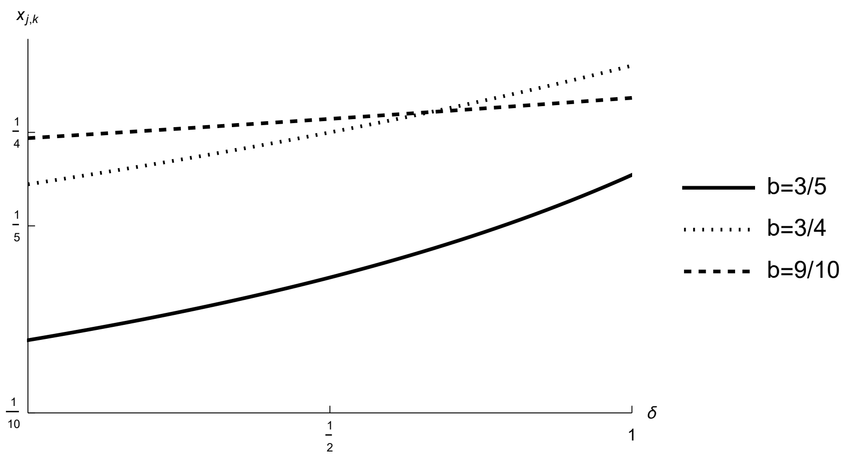

Figure 3: Effort by the contender depending on δ and b.

Figure 3 also illustrates that even in the presence of the discouragement effect, the contender can exert more effort than in the single-period benchmark. This is the case if both the discount factor and the cost of early effort are sufficiently high, such that the incentive effect dominates the discouragement effect.

Corollary 1 gives the probabilities of winning in equilibrium.

Corollary 1. *For $k \geq 2$, the probability $p_{i,k}$ with which the incumbent wins contest k is given by*

$$p_{i,k} = \begin{cases} 1 & \text{if } b \leq \frac{1}{2} \\ \frac{1}{2b} & \text{if } \frac{1}{2} < b < 1 \\ \frac{1}{2} & \text{if } b \geq 1. \end{cases}$$

In the cases $b \leq 1/2$ and $b \geq 1$, the probabilities of winning are straightforward. If $b \leq 1/2$, the contender is fully discouraged by the incumbent, who wins with certainty. If $b \geq 1$, the incumbent position is of no value and the behavior coincides with the single-period benchmark, such that $p_{i,k} = 1/2$.

In the nontrivial case $b \in (1/2, 1)$, the probabilities of winning do not depend on the discount factor but react differently to changes in the cost of early effort. Intuitively, both players share the same discount factor by assumption such that its effect cancels out. A lower b specifically benefits the incumbent and harms the contender in the current contest. Therefore, the probability of winning for the incumbent decreases in b.

Notice that Lemmas 4, 5 and 6 do not specify equilibrium effort in the first contest, in which there is no incumbent. The formulas derived for the effective prize sum ψ_k nonetheless remain valid for $k = 1$. Therefore, inserting both ψ_1 and $d_{i,1} = 0$ in Lemma 2 yields Corollary 2.

Corollary 2. *In the subgame perfect equilibrium of the infinite game, equilibrium efforts and effective prize in the first contest are given by*

$$\psi_1 = \begin{cases} 1 & \text{if } b \geq 1 \\ \frac{1}{1-\delta A} & \text{if } \frac{1}{2} < b < 1 \\ \frac{1}{1-\delta(1-b)} & \text{if } 0 < b \leq \frac{1}{2}, \end{cases}$$

$$x_{i,1} = x_{j,1} = \begin{cases} \frac{1}{4} & \text{if } b \geq 1 \\ \frac{1}{4}\frac{1}{1-\delta A} & \text{if } \frac{1}{2} < b < 1 \\ \frac{1}{4}\frac{1}{1-\delta(1-b)} & \text{if } 0 < b \leq \frac{1}{2}. \end{cases}$$

where $A = \frac{5b-1}{4b^2} - 1$.

In general, Corollary 2 shows that, if the incumbent position is valuable, players exert more effort in the first contest under endogenous incumbency compared to the single-period benchmark. Clearly, the incentive effect intensifies the initial contest, while there is no discouragement effect because players are symmetric.

4 Optimal Contest Design

4.1 Rent extraction in the competitive equilibrium

The overall aim of the contest designer is to incentivize the players to exert effort. In particular, we assume that the contest designer maximizes expected rent extraction, defined as the ratio between total effort and the total prize sum. The contest designer values effort both in the investment stages and in the competition stages. In the investment stages, the contest designer may choose to partially compensate early effort, such that if the incumbent invests $d_{i,k}$, the contest designer only receives a revenue of $b \cdot d_{i,k}$. In the competition stages, the contest designer values effort by both the incumbent and the contender.

The general range of equilibrium rent extraction is straightforward. As effort is nonnegative, rent extraction is nonnegative as well. If rent extraction were larger than unity, at least one player would face a negative expected payoff and would improve by playing zero at all stages. Therefore, rent extraction is bounded in the interval $[0, 1]$.

By choosing the cost of early effort b (equivalent to choosing the compensation of early effort $1 - b$), the contest designer maximizes rent extraction $\rho(b, \delta)$, which is defined as

$$\rho(b, \delta) = \frac{1}{\sum_{k=1}^{\infty} \delta^{k-1}} \left(\sum_{k=1}^{\infty} \delta^{k-1} (x_{i,k} + x_{j,k} + b \cdot d_{i,k})\right).$$

In what follows, we discuss how the cost of early effort influences the individual components of rent extraction. In the previous section, we showed how players' optimal effort choices in the competition stage C_k depend on b and δ. We now proceed with the remaining component of rent extraction, which is the revenue generated in the investment stage I_k.

Figure 4 illustrates the revenue from investment stage I_k depending on the cost of early effort b and the discount factor δ. Generally, $b \cdot d_{i,k} > 1/4$, so the revenue is higher than in the single-period benchmark.

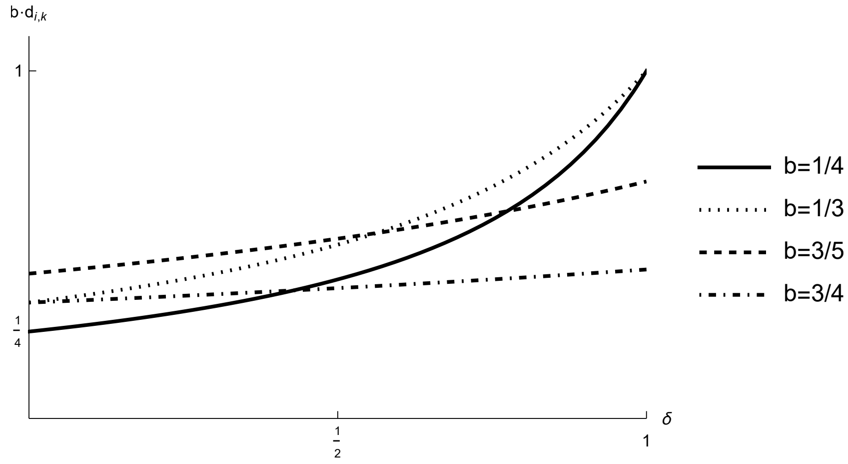

Figure 4: Revenue for the contest designer from early effort

The revenue depends on the cost of early effort b as follows. On the one hand, the incumbent exerts more early effort if early effort is less costly. On the other hand, low costs of early effort imply that the contest designer must compensate for a larger share of early effort. These two effects interact with discounting. For a low discount factor, the reward of becoming the incumbent has only little value. Then, the incentive effect is weak such that the incumbent exerts a relatively low level of early effort, and moderate compensation generates the highest revenue. In contrast, for a high discount factor, the incentive effect is strong and the players are more sensitive to changes in b because they highly value the long-term advantages of incumbency. The contest designer can use this to his benefit by setting a high level of compensation, which raises the stakes.

Corollary 3. *Define aggregate effort in contest $k \geq 2$ as effort in competition stage C_k and revenue in investment stage I_k, that is $E_k = x_{i,k} + x_{j,k} + b \cdot d_{i,k}$. Then, aggregate effort in the competitive equilibrium of contest k is given by*

$$E_k = \begin{cases} \frac{1}{2} & \text{if } b \geq 1 \\ \frac{3b-1}{4b^2}\psi_k & \text{if } \frac{1}{2} < b < 1 \\ b\psi_k & \text{if } 0 \leq b \leq \frac{1}{2}. \end{cases}$$

Corollary 3 summarizes the aggregate effort that is generated in both the investment stage I_k and the competition stage C_k of contest $k \geq 2$ in the competitive equilibrium.[9] The cost of early effort determines the qualitative behavior in the competitive equilibrium, i.e., whether the incumbent does not, partially, or fully discourage the contender. Directly, decreasing b increases marginal compensation at the cost of the contest designer. Indirectly, decreasing b reinforces both the incentive effect and the discouragement effect.

Figure 5 illustrates the effect of the cost of early effort and discounting on aggregate

[9]The aggregate effort generated in contest 1 is characterized in Corollary 2.

effort. Obviously, aggregate effort coincides with total effort in the single-period benchmark if the incumbency advantage is worthless ($b \geq 1$).

For $b \in (1/2, 1)$, the cost of early effort is moderate and the incumbent partially discourages the contender such that the contender exerts less than $\psi_k/4$. The higher revenue generated by inducing early effort outweighs the negative effect on the contender's effort such that the competitive equilibrium outperforms the single-period benchmark in every contest.[10] Consequently, a contest designer always has an incentive to facilitate an incumbent with some advantage, because any $b \in (1/2, 1)$ dominates $b = 1$.

If cost of early effort is cheap ($b \leq 1/2$), the incumbent fully discourages the contender from competition by exerting $E_k = b\psi_k$ with $\psi_k > 1$. For $b \in (0, 1/2]$, the aggregate effort is increasing in the cost of early effort, because the negative effect of the higher compensation exceeds the positive effect of increased early effort.[11]

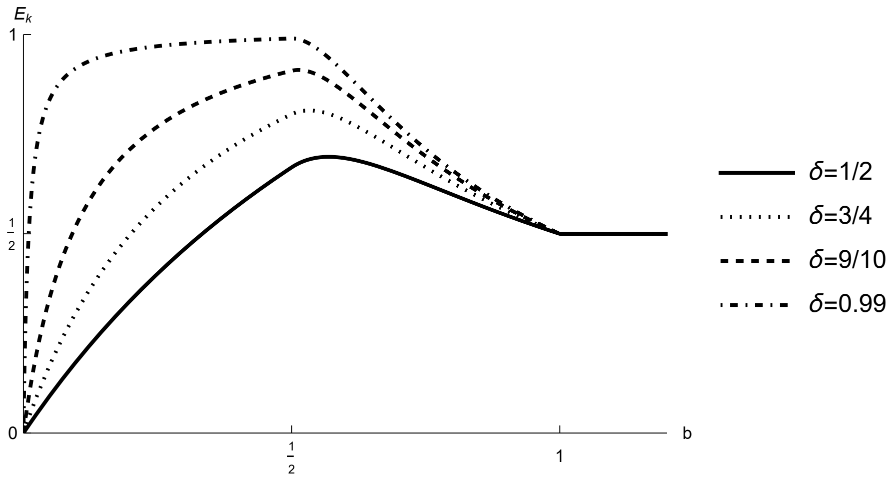

Figure 5: Aggregate effort in contest $k \geq 2$.

Corollary 3 does not cover the first contest, in which there is no incumbent. Aggregate effort in the first contest is given in Corollary 2. These two results allow us to characterize equilibrium rent extraction.

Proposition 1. *In the competitive equilibrium, rent extraction is given by*

$$\rho(b, \delta) = \begin{cases} \frac{1}{2} & \textit{if } b \geq 1 \\ \frac{1}{2} + \frac{\delta b(-8b+11)-3\delta}{8b^2+\delta(8b^2-10b+2)} & \textit{if } \frac{1}{2} < b < 1 \\ \frac{1}{2} + \frac{\delta b}{2(1-\delta+\delta b)} & \textit{if } 0 < b \leq \frac{1}{2}. \end{cases}$$

In particular, the contest designer optimally sets $b^* = \frac{\delta+\sqrt{9-5\delta}+3}{\delta+11}$.

[10] Straightforwardly, for $b \in (1/2, 1)$, $\frac{3b-1}{4b^2} > \frac{1}{2}$, $\psi_k \geq 1$ and, by Corollary 2, $E_1 > \frac{1}{2}$.

[11] In the extreme case of full compensation ($b = 0$), no effort is exerted in any contest except the first one. In fact, $\rho(b, \delta) = 1/2$ for $b = 0$. Therefore, the optimal b is necessarily positive.

The proof is in Appendix A.4. Figure 6 illustrates the results of Proposition 1. For every $\delta \in (0,1)$, the contest designer benefits from endogenous incumbency. In particular, the contest designer always prefers to set the cost of early effort such that the incumbent always partially discourages the contender. The optimal cost of early effort b^* decreases in the discount factor. The higher the discount factor, the more weight the contest designer puts on the incentive effect and the more aggressive players react to an increase of future rewards. Consequently, the contest designer decreases the cost of early effort further to increase the incumbent's early effort. This comes to the detriment of the contender, whose chances of becoming the incumbent decrease. In the limit, as $\delta \to 1$, the contender wins with probability zero, while the contest designer achieves full rent extraction.

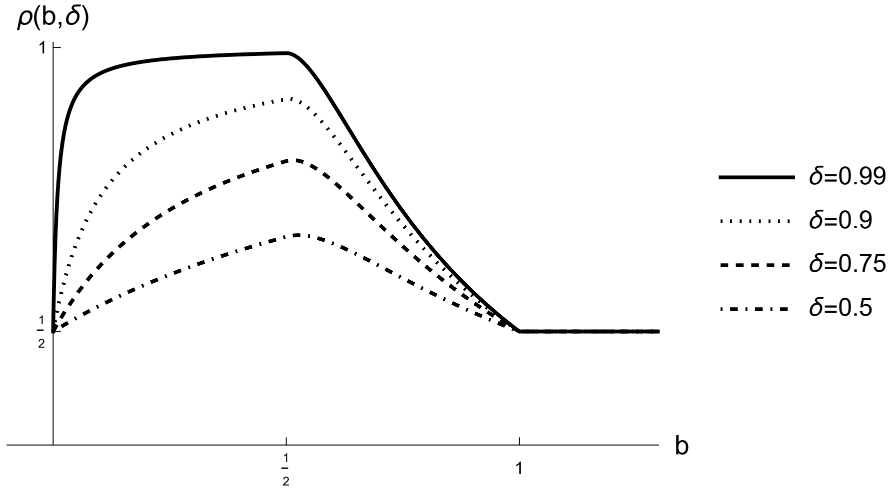

Figure 6: Rent extraction depending on b and δ.

4.2 Cooperation among players

The competitive equilibrium is not the only subgame-perfect equilibrium of the infinite game. For a sufficiently large discount factor, the infinite game allows for subgame-perfect equilibria, in which players conditionally cooperate but punish in case of a deviation. In this way, cooperation can be sustained, which benefits players through higher payoffs but reduces rent extraction to the detriment of the contest designer.

In this section, we will show that a contest designer who is concerned about cooperation among players can use endogenous incumbency to eliminate cooperative equilibria. We focus on grim trigger strategies in which players are cooperative and exert no effort as long as the opponent has not yet deviated from cooperation.[12] Notice that if both players exert no effort in all stages, the incumbent is chosen at random in every contest, but does not capitalize on the incumbency status. If a player deviates, the other player punishes by

[12] In principle, many more Nash equilibria that rely on cooperation and punishment exist, such as in tit-for-tat strategies. We focus on grim trigger strategies as a representative of this class, because an exhaustive characterization of all cooperative strategies is beyond the scope of this paper.

falling back on competitive behavior as defined in the competitive equilibrium for the rest of the game. A subgame perfect equilibrium with cooperation through mutual grim trigger strategies, henceforth called *cooperative equilibrium*, exists if deviation is not profitable in any subgame.

Formally, the trigger-strategy is denoted by

$$T(b,\delta) = \{(d_{i,k}, x_{i,k}, x_{j,k})_{k=1}^{\infty}\}$$

with

$$x_{i,k} = \begin{cases} 0 & \text{if } x_{i,n} = x_{j,n} = 0 \text{ for all } 1 \leq n \leq k-1 \text{ and } d_{i,n} = 0 \text{ for all } 1 \leq n \leq k, \\ x_{i,k}^{COMP} & \text{otherwise,} \end{cases}$$

$$x_{j,k} = \begin{cases} 0 & \text{if } x_{i,n} = x_{j,n} = 0 \text{ for all } 1 \leq n \leq k-1 \text{ and } d_{i,n} = 0 \text{ for all } 1 \leq n \leq k, \\ x_{j,k}^{COMP} & \text{otherwise,} \end{cases}$$

$$d_{i,k} = \begin{cases} 0 & \text{if } x_{i,n} = x_{j,n} = d_{i,n} = 0 \text{ for all } 1 \leq n \leq k-1 \\ d_{i,k}^{COMP} & \text{otherwise,} \end{cases}$$

where $x_{i,k}^{COMP}, x_{j,k}^{COMP}$ and $d_{i,k}^{COMP}$ denote the equilibrium levels of effort and early effort in the competitive equilibrium. Notice that players optimally deviate in the competition stage rather than the investment stage. Suppose that at least one contest has been played and no deviation has occurred yet. If the (randomly assigned) incumbent deviated in the investment stage, the contender would observe the deviation and would already be able to punish in the same contest. If the incumbent optimally deviates in the competition stage, the contender can only punish in the subsequent contests, as effort choices in the competition stage are simultaneous. Therefore, deviation in the competition stage yields higher payoffs than in the investment stage.

Proposition 2. *A cooperative equilibrium in which both players follow grim trigger strategies $T(b,\delta)$ exists*

(i) for $b \in (0, 1/2)$, if and only if $\delta \geq \frac{1}{1+b}$,

(ii) for $b \in [1/2, 1]$, if and only if $\delta \geq \frac{4b^2}{4b^2+3b-1}$, and

(iii) for $b \in (1, \infty)$, if and only if $\delta \geq 2/3$.

The proof is in Appendix A.5. Proposition 2 shows that for positive b, the cooperative equilibrium only exists if the discount factor δ is sufficiently high. Deviation from cooperation improves payoffs in the present contest, but the subsequent punishment reduces payoffs in all future periods. If players are sufficiently patient, the threat of punishment outweighs the immediate benefits from deviation. Otherwise, today's reward for deviation exceeds the cost of future punishment, and cooperation is not sustainable.[13] If the contest designer sets b optimally according to Proposition 1, the cooperative equilibrium exists for $\delta > \delta^* \approx 0.659$.

Figure 7 illustrates the existence conditions of the cooperative equilibrium. By changing the cost of early effort b, the contest designer can use endogenous incumbency to eliminate the possibility of cooperation. In the absence of endogenous incumbency (which is equivalent to $b \geq 1$), all contests are independent and the cooperative equilibrium exists for $\delta > 2/3$. By introducing endogenous incumbency, the contest designer can change the range of discount factors δ for which the cooperative equilibrium does not exist. Therefore, for any δ, the contest designer can set a sufficiently low b such that cooperation cannot be sustained. The players' expected payoffs in the competitive equilibrium are rather high, and thus the punishment for deviation is less severe.

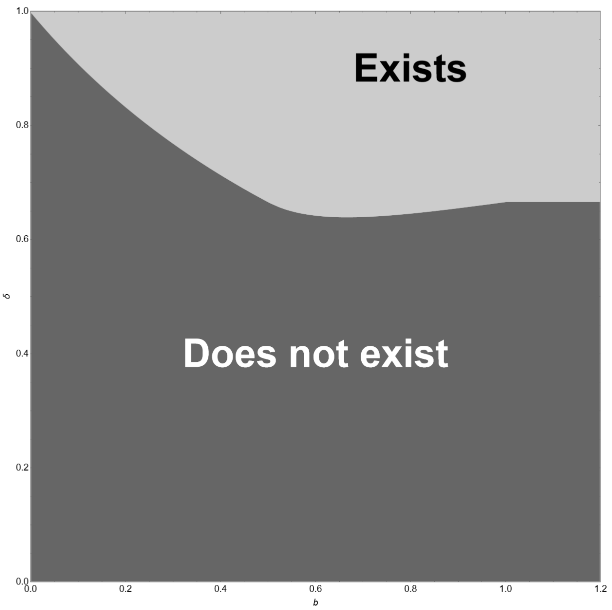

Figure 7: Existence of the cooperative equilibrium

In general, two effects characterize the shape of the boundary. On the one hand, decreasing the cost of early effort b implies a higher effective prize sum, and thus increases the incentive effect. Then, intensified competition increases the payoff difference between the competitive equilibrium and the cooperative equilibrium, which becomes more attractive. On the other hand, the deviator from cooperation will be the advantaged incumbent in the subsequent contest. Therefore, decreasing b implies a less severe punishment. If the cost of early effort is very low, the latter effect dominates the former effect, whereas the opposite holds for a cost of early effort close to $b = 1$.

[13] In the extreme case $b = 0$, a cooperative equilibrium cannot be sustained, because deviations cannot be punished. Unilateral deviation makes the deviator the incumbent in the subsequent contest. Since early effort is costless, the incumbent can fully discourage the contender at no cost. However, for $b = 0$, the contest designer only ensures a rent extraction of $\rho = 1/2$. Proposition 2 shows that the contest designer can, for any δ, optimally choose a level of compensation that yields higher rent extraction.

5 Extensions

5.1 Unobservable early effort

The Stackelberg structure implicitly assumes that early effort $d_{i,k}$ is observable by the contender before he decides upon $x^C_{j,k}$. If early effort is not observable prior to the contender's decision, the incumbent loses his first-mover advantage as there is no visible commitment. Therefore, players play as if the decision were simultaneous. The game is then equivalent to a simultaneous move game. For ease of notation, we will analyze the game in the simultaneous version.

Even if early effort is unobservable, it can still be more cost effective than regular effort. In contrast, if early effort is more costly than regular effort, the incumbent prefers to exert regular effort instead of early effort. In this case, equilibrium behavior coincides with Lemma 6. In this section, we focus on the cases where the incumbent enjoys a cost advantage ($b \leq 1$).

Consider contest k where player i is the incumbent and player j is the contender. Then, both players' maximization problems are given by

$$\max_{x_{i,k}} \pi_{i,k} = \frac{x_{i,k}}{x_{i,k}+x_{j,k}}(1+\delta\pi_{i,k+1}) + (1-\frac{x_{i,k}}{x_{i,k}+x_{j,k}})\delta\pi_{j,k+1} - bx_{i,1}$$

and

$$\max_{x_{j,k}} \pi_{j,k} = \frac{x_{j,k}}{x_{i,k}+x_{j,k}}(1+\delta\pi_{i,k+1}) + (1-\frac{x_{j,k}}{x_{i,k}+x_{j,k}})\delta\pi_{j,k+1} - x_{j,1}.$$

The maximization problems are symmetric except for the incumbent's marginal cost of effort. To solve the game, we follow the same method as in previous sections. First, we characterize equilibrium behavior of the finite K-contest game by backward induction. Then, we apply Fudenberg and Levine (1983) to characterize the competitive equilibrium of the infinite game. Detailed calculations are relegated to the Appendix.

In case of unobservable effort, rent extraction is given by

$$\rho(b,\delta) = \frac{1}{\sum_{k=1}^{\infty}\delta^{k-1}}\sum_{k=1}^{\infty}\delta^{k-1}(b\cdot x^*_{i,k} + x^*_{j,k}).$$

Proposition 3. *In the competitive equilibrium with unobservable early effort, rent extraction is given by*

$$\rho(b,\delta) = \frac{1}{2} + \frac{b\delta(1-b)}{(1+b)(1+b-\delta+\delta b)}.$$

The contest designer who maximizes rent extraction optimally sets

$$b^* = \frac{\delta + 2\sqrt{1-\delta} - 1}{\delta + 3}.$$

The proof is in Appendix A.7. Figure 8 illustrates the results from Proposition 3. We find that rent extraction is increasing in the discount factor δ. Intuitively, a higher δ implies a stronger incentive effect, which increases rent extraction. Again, the interplay between the discouragement effect and the incentive effect shapes players' effort choices. On the one hand, decreasing the incumbent's cost of effort b strengthens the incentive effect and positively influences players' efforts in equilibrium. On the other hand, the decreasing b also strengthens the discouragement effect which reduces the contender's effort in equilibrium. In contrast to the case of observable early effort, the incumbent never chooses to fully discourage the contender.

The higher the discount factor δ, the lower the optimal level of b that maximizes rent extraction. Clearly, the more the contest designer (and the players) value the future, the more emphasis is put on the incentive effect and the less on the discouragement effect. Consequently, contests become more intense in that case, and the level of optimal rent extraction increases as well.

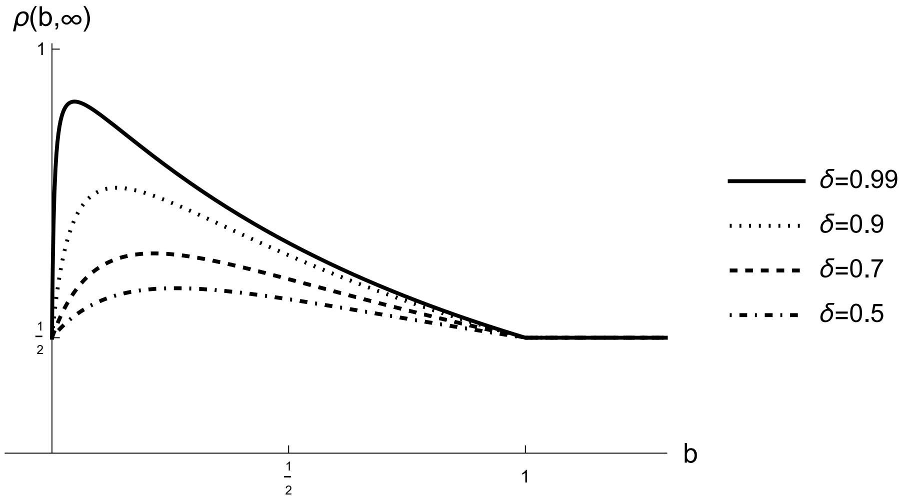

Figure 8: Rent extraction in the infinite game depending on δ

Corollary 4. *For all $\delta \in (0, 1)$, the competitive equilibrium yields a higher level of rent extraction if early effort is observable than if it were unobservable, given that the contest designer sets the cost of early effort b optimally.*

Corollary 4 compares the competitive equilibrium of the case with observable early effort with the one with unobservable early effort. It highlights that the contest designer prefers early effort to be observable for any discount factor δ. Therefore, he has an incentive to always publicly announce the level of the incumbent's total effort because it restores the incumbent's commitment opportunity.[14] This increases both the discouragement effect and the incentive effect, but the incentive effect remains the stronger force for an optimally chosen level of compensation.

[14] We assume such these announcements must be truthful. Allowing for the possibility that the contest designer misreports effort levels and analyzing strategic consequences is left for future research.

5.2 Winner's effort

Next, consider a contest designer who is only interested in winner's effort. In other words, a player's effort is only valuable if the respective player also wins the contest. In a repeated contest for mandates, the contest designer prefers to give the mandate to a strong winner, with whom he enters into a contract. The loser is not contracted. Thus, the contest designer cannot utilize the contender's effort.

However, the contest for a mandate remains a noisy process (depending on b). With some probability, a player can win the contest even if he exerts a relatively low level of effort. In this case, the incumbent's high effort is wasted from the contest designer's perspective. By contrast, we assume that the contest designer always benefits from the incumbent's early effort irrespective of whether the incumbent wins the contest. This reflects the interpretation of early effort as effort in implementation during a current project. Formally, the contest designer sets the cost of early effort b to maximize a variant of rent extraction ρ_W, which is given by

$$\max_{b} \quad \rho_W(b,\delta) = \frac{1}{\sum_{k=1}^{\infty}\delta^{k-1}}\left(\sum_{k=1}^{\infty}\delta^{k-1}(p_{i,k}x_{i,k} + (1-p_{i,k})x_{j,k} + b\cdot d_{i,k})\right).$$

The players observe b and optimally choose their levels of effort and early effort. The players' behavior is given by Lemmas 4, 5 and 6.

Then, rent extraction ρ_W is summarized in Proposition 4.

Proposition 4. *Consider a contest designer that is interested in maximizing winner's effort. Then, rent extraction is*

$$\rho_W(b,\infty) = \begin{cases} \frac{1}{4} & \text{if } b \geq 1, \\ \frac{1}{4} + \frac{\delta(2-b(1-b)(9-8b))}{4b(4b^2(1+\delta)-5b\delta+\delta)} & \text{if } \frac{1}{2} < b < 1, \\ \frac{1}{4} + \frac{3b\delta}{4(1-\delta(1-b))} & \text{if } b \leq \frac{1}{2}. \end{cases}$$

A contest designer who maximizes rent extraction optimally sets $b = 1/2$.

The proof is in Appendix A.8. Figure 9 illustrates the results of Proposition 4. When the contest designer maximizes winner's effort instead of total effort, rent extraction changes depending on the value of b. In the area of full discouragement ($b < 1/2$), winner's effort is very close to total effort, because the incumbent wins all contests after the first one with certainty. Only in the first contest, both players have a positive chance of winning, such that also the loser has exerted positive effort, which the contest designer does not value. In the area of partial discouragement ($b \in (1/2, 1)$), the contender exerts some positive effort in every contest. Therefore, in every contest, the loser exerts positive effort that is not valuable to the contest designer. The incumbent, who exerts more effort, also wins with higher probability.[15] In the area of no discouragement ($b > 1$), there is no incumbency advantage and the incumbent exerts exactly as much effort as the contender. Therefore, in every contest, the contest designer does not value half of players' total effort.

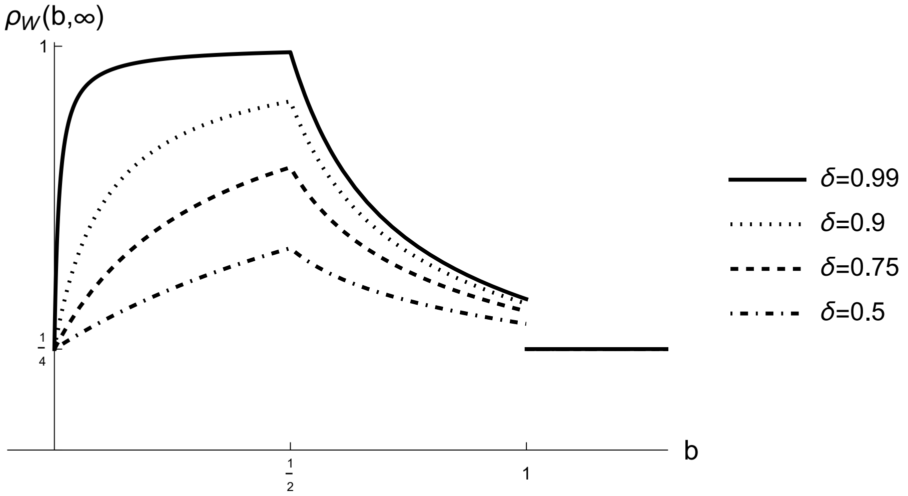

Figure 9: Rent extraction depending on δ, where the contest designer maximizes winner's effort.

Proposition 4 shows that a winner's effort maximizing contest designer prefers to always set $b = 1/2$, irrespective of δ. In this case, the incumbent always fully discourages the contender who exerts no effort, and the incumbent wins all contests after the first with certainty in equilibrium. This result differs from the optimal cost of early effort when the contest designer values both players' efforts (Proposition 1). In this case, the contender always exerts positive effort, which gives him a positive chance of winning each contest. When concentrating on winner's effort, this structure cannot be optimal because it implies that the contest designer induces some positive level of effort that he does not value. Consequently, he imposes a structure in which all effort in the contest comes from the winner.

6 Conclusion

This paper introduces endogenous incumbency as a tool a designer of repeated contests can use to incentivize players to increase intensity. The contest designer provides the incumbent with the opportunity to be active in the contest before the contender enters. This opportunity may (i) yield a mover advantage and (ii) may also be more cost effective for the incumbent. Therefore, in each contest, the reward of winning is not only the

[15]Note that there is a discontinuity in rent extraction for $b = 1$. At this point, the incumbent shifts his effort from the investment stage to the competition stage. In the investment stage, the contest designer always values the incumbent's effort irrespective of whether he wins. In the competition stage, the contest designer only values the incumbent's effort if he wins, which happens with probability $1/2$ in equilibrium.

present period's prize, but also the opportunity to become the incumbent in the next contest, which implicitly raises the stakes in each contest. At the same time, this comes at the cost of a permanent asymmetry between the incumbent and the contender.

We find that an incumbent only chooses to exert early effort if it is cheaper than regular effort. Depending on the cost advantage's size, he either partially or fully discourages the contender from competition. However, this is not to the detriment of the contest designer, who enjoys increased rent extraction compared to independent lottery contests without endogenous incumbency. Therefore, we show that the contest designer prefers to introduce endogenous asymmetry into an ex-ante symmetric game and provides a cost advantage as well as a mover advantage.

By partially compensating early effort, the contest designer balances the incentive effect and the discouragement effect. We find that a moderate incumbency advantage is effort-maximizing, so that the incumbent partially discourages the contender, whose level of effort remains positive in equilibrium. Consequently, the outcome of every contest is uncertain, and the incumbency status is at stake in every period.

In addition, we consider the case of cooperative equilibria, where players follow grim-trigger strategies. Interestingly, for any discount factor, a contest designer can employ endogenous incumbency with a sufficiently low cost of early effort to preclude such cooperative behavior from being sustainable.

Given that the contest designer sets the cost of early effort optimally, rent extraction is always higher if early effort is observable than if early effort is unobservable. The latter case misses the commitment opportunity and decreases the value of becoming the incumbent. Therefore, the contest designer always has an incentive to publicly announce the level of the incumbent's early effort. We also account for a contest designer who solely values the winner's effort. Then, the contest designer compensates the incumbent's early effort just enough such that the incumbent fully discourages the contender.

In procurement, suppliers fight for mandates issued by the same firm again and again. Many firms utilize preferred supplier programs that give some preferential treatment to suppliers who, among other criteria, shine in current projects for which they have already been contracted. Our findings suggest that those preferred supplier programs may be profitable even in the absence of synergy effects, if the preferred supplier position is conditional on performance in a current project. Although such a preferred partner program creates an asymmetry between suppliers by design, it motivates them to work harder to attain this incumbency position.

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A Appendix

A.1 Proof of Lemma 2

In competition stage C_k, player i's maximization problem is

$$\pi_{i,k}^C = \frac{x_{i,k}+d_{i,k}}{x_{i,k}+x_{j,k}+d_{i,k}}(1+\delta\pi_{i,k+1}^I) + \left(1-\frac{x_{i,k}+d_{i,k}}{x_{i,k}+x_{j,k}+d_{i,k}}\right)\delta\pi_{j,k+1}^C - x_{i,k},$$

and player j's maximization problem is

$$\pi_{j,k}^C = \frac{x_{j,k}}{x_{i,k}+x_{j,k}+d_{i,k}}(1+\delta\pi_{i,k+1}^I) + \left(1-\frac{x_{j,k}}{x_{i,k}+x_{j,k}+d_{i,k}}\right)\delta\pi_{j,k+1}^C - x_{j,k}.$$

Solving the two maximization problems and using $\psi_k = 1 + \delta(\pi^I_{k+1} - \pi^C_{k+1})$ yields

$$x_{i,k} = \frac{\psi_k}{4} - d_{i,k},$$
$$x_{j,k} = \frac{\psi_k}{4}.$$

Since efforts are nonnegative, the above solution is only valid for $d_{i,k} \leq \psi_k/4$. Otherwise, player i exerts zero effort in optimum. In this case, effort of player j is given by player j's best response to $x_{i,k} = 0$, which is

$$x_{j,k} = \sqrt{d_{i,k}\psi_k} - d_{i,k}.$$

Also, player j's effort must be nonnegative. Therefore, the above solution is only valid for $d_{i,k} \in [\psi_k/4, \psi_k]$. If instead $d_{i,k} > \psi_k$, player j exerts zero effort in optimum.

Together, the equilibrium levels conditional on the value of $d_{i,k}$ are

$$x^*_{i,k} = \begin{cases} \frac{\psi_k}{4} - d_{i,k} & \text{if } d_{i,k} \leq \frac{\psi_k}{4}, \\ 0 & \text{if } d_{i,k} > \frac{\psi_k}{4}, \end{cases}$$
$$x^*_{j,k} = \begin{cases} \frac{\psi_k}{4} & \text{if } d_{i,k} \leq \frac{\psi_k}{4}, \\ \sqrt{d_{i,k}\psi_k} - d_{i,k} & \text{if } \frac{\psi_k}{4} < d_{i,k} < \psi_k, \\ 0 & \text{if } d_{i,k} \geq \psi_k, \end{cases}$$

Expected payoffs in competition stage C_k are then given by

$$\pi^C_{i,k}(x^*_{i,k}, x^*_{j,k}) = \begin{cases} \frac{1}{4}\psi_k + \delta\pi^C_{j,k+1} + d_{i,k} & \text{if } d_{i,k} \leq \frac{\psi_k}{4}, \\ \sqrt{d_{i,k}\psi_k} + \delta\pi^C_{j,k+1} & \text{if } \frac{\psi_k}{4} < d_{i,k} < \psi_k, \\ \psi_k + \delta\pi^C_{j,k+1} & \text{if } d_{i,k} \geq \psi_k, \end{cases}$$
$$\pi^C_{j,k}(x^*_{i,k}, x^*_{j,k}) = \begin{cases} \frac{1}{4}\psi_k + \delta\pi^C_{j,k+1} & \text{if } d_{i,k} \leq \frac{\psi_k}{4}, \\ \psi_k + \delta\pi^C_{j,k+1} + d_{i,k} - 2\sqrt{d_{i,k}\psi_k} & \text{if } \frac{\psi_k}{4} < d_{i,k} < \psi_k, \\ \delta\pi^C_{j,k+1} & \text{if } d_{i,k} \geq \psi_k. \end{cases}$$

which proves Lemma 1.

A.2 Proof of Lemma 3

Using the expected payoffs given by Lemma 2, player i's maximization problem is given by

$$\underset{d_{i,k}}{max} \quad \pi^I_{i,k} = \begin{cases} \frac{1}{4}\psi_k + \delta\pi^C_{j,k+1} + d_{i,k} - bd_{i,k} & \text{if } d_{i,k} \leq \frac{1}{4}\psi_k, \\ \sqrt{d_{i,k}\psi_k} + \delta\pi^C_{j,k+1} - bd_{i,k} & \text{if } \frac{1}{4}\psi_k < d_{i,k} < \psi_k, \\ \psi_k + \delta\pi^C_{j,k+1} - bd_{i,k} & \text{if } d_{i,k} \geq \psi_k. \end{cases}$$

For each case, we solve the maximization problem separately, Afterwards, we compare the payoffs of the three cases for a given b to determine player i's reaction function $d^*_{i,k}(b)$.

Case 1: $d_{i,k} \leq \frac{1}{4}\psi_k$

Obviously, $\frac{\partial\pi^I_{i,k}}{\partial d_{i,k}} = 1 - b$ is positive for $b < 1$ and negative for $b > 1$. Then, $d_{i,k}$ is given by

$$d_{i,k} = \begin{cases} \frac{1}{4}\psi_k & \text{if } b < 1, \\ [0, \frac{1}{4}\psi_k] & \text{if } b = 1, \\ 0 & \text{if } b > 1. \end{cases}$$

and the incumbent's corresponding payoff is

$$\pi^I_{i,k} = \begin{cases} \frac{(2-b)\psi_k}{4} + \delta\pi^C_{j,k+1} & \text{if } b < 1, \\ \frac{\psi_k}{4} + \delta\pi^C_{j,k+1} & \text{if } b \geq 1. \end{cases}$$

Case 2: $d_{i,k} \in (\psi_k/4, \psi_k)$

Maximization yields $d_{i,k+1} = (\psi_k)/(4b^2)$. However, the solution is only valid for $(\psi_k)/(4b^2) \in ((\psi_k)/4, \psi_k)$. If $b \geq 1$, then $(\psi_k)/(4b^2) \leq (\psi_k)/4$, so the lower bound of the interval is optimal. If $b < 1/2$, then $(\psi_k)/(4b^2) \geq \psi_k$, so the upper bound of the interval is optimal. Therefore,

$$d_{i,k} = \begin{cases} \psi_k & \text{if } b \leq \frac{1}{2}, \\ \frac{\psi_k}{4b^2} & \text{if } \frac{1}{2} < b < 1, \\ \frac{1}{4}\psi_k & \text{if } b \geq 1, \end{cases}$$

$$\pi_{i,k} = \begin{cases} \psi_k(1-b) + \delta\pi^C_{j,k+1} & \text{if } b \leq \frac{1}{2}, \\ \frac{\psi_k}{4b} + \delta\pi^C_{j,k+1} & \text{if } \frac{1}{2} < b < 1, \\ \frac{(2-b)\psi_k}{4} + \delta\pi^C_{j,k+1} & \text{if } b \geq 1. \end{cases}$$

Case 3: $d_{i,k} \geq \psi_k$

Obviously, $\frac{\partial \pi_{i,k}}{\partial d_{i,k}} = -b$ is negative for all $b > 0$. Therefore,

$$
\begin{aligned}
d_{i,k} &= \psi_k, \\
\pi_{i,k} &= \psi_k(1-b) + \delta\pi^C_{j,k+1}.
\end{aligned}
$$

Given the the case-dependent equilibrium payoffs for all possible values of b, we can now determine the optimal $d^*_{i,k}(b)$. We identify three intervals, i.e., $b \leq 1/2$, $b \in (1/2, 1)$ and $b \geq 1$. Notice that fixing b implies a fixed ψ_k as well in equilibrium. For these three ranges of b, we now determine the optimal $d^*_{i,k}(b)$.

If $b \leq \frac{1}{2}$, then

$$\psi_k(1-b) + \delta\pi^C_{j,k+1} > \frac{(2-b)\psi_k}{4} + \delta\pi^C_{j,k+1} \quad \Leftrightarrow \quad b < \frac{2}{3},$$

which is always true if $b < 1/2$. Therefore,

$$d^*_{i,k} = \psi_k \quad \text{if } b \leq \frac{1}{2}.$$

If $b \in (1/2, 1)$, then

$$\frac{\psi_k}{4b} + \delta\pi^C_{j,k+1} > \psi_k + \delta\pi^C_{j,k+1} - b\psi_k \quad \Leftrightarrow \quad (2b-1)^2 > 0,$$

and

$$\frac{\psi_k}{4b} + \delta\pi^C_{j,k+1} > \frac{\psi_k(2-b)}{4} + \delta\pi^C_{j,k+1} \quad \Leftrightarrow \quad (b-1)^2 > 0,$$

are both always true. Therefore,

$$d^*_{i,k} = \frac{\psi_k}{4b^2} \quad \text{if } \frac{1}{2} < b < 1.$$

If $b \geq 1$, then

$$\frac{\psi_k}{4} + \delta\pi^C_{i,k+1} \geq \frac{\psi_k(2-b)}{4} + \delta\pi^C_{j,k+1} \quad \Leftrightarrow \quad b \geq 1,$$

and

$$\frac{\psi_k}{4} + \delta\pi^C_{j,k+1} > \psi_k + \delta\pi^C_{j,k+1} - b\psi_k \quad \Leftrightarrow \quad b \geq \frac{3}{4},$$

which are both always true for $b \geq 1$.

Therefore, we can combine the three cases and obtain

$$d^*_{i,k} = \begin{cases} \psi_k & \text{if } b \leq \frac{1}{2}, \\ \frac{\psi_k}{4b^2} & \text{if } \frac{1}{2} < b < 1, \\ 0 & \text{if } b \geq 1, \end{cases}$$

which proves Lemma 3.

A.3 Proof of Lemmas 4, 5, 6 and Corollary 2

To prove the three lemmas, we first solve the finite, K-contest game by backward induction. Then, we use Theorem 3.3 from Fudenberg and Levine (1983) to obtain the competitive equilibrium of the infinite game.

For $b \geq 1$ in the finite game, Lemma 3 yields $d^*_{i,k} = 0 \, \forall k \in \{2, ..., K\}$. Therefore, Lemma 2 and Lemma 3 immediately reveal $\pi^I_{i,k} = \pi^C_{j,k}$ for all k, so $\psi_k = 1$ and $x_{i,k} = x_{j,k} = 1/4$.

Now assume $b \leq 1/2$ in the finite game. By Lemma 2 and Lemma 3, the contender exerts $x_{j,k} = 0$, and the incumbent exerts some positive level of early effort. Therefore, the contender wins with probability zero in equilibrium in every contest and obtains an equilibrium payoff of $\pi^C_{j,k} = 0$ for all k. Given that $\psi_k = 1 + \delta(\pi^I_{i,k+1} - \pi^C_{j,k+1})$, we can insert the values of $\pi^I_{i,k+1}$ and $\pi^C_{j,k+1}$ from Lemma 2 and Lemma 3 and obtain

$$\psi_k = 1 + \delta(1-b)\psi_{k+1}.$$

From that we can immediately see that ψ_k is increasing in δ and decreasing in b. We now show that

$$\psi_k = \frac{1 - (\delta(1-b))^{K-k+1}}{1 - \delta(1-b)} \tag{6}$$

for all $k \in \{1, ..., K\}$ by induction, where the base case covers period K and the induction step is from $k+1$ to k.

In period K, there is no next period, so continuation payoffs of winning and losing are identical, meaning $\psi_K = 1$. Indeed, from (6), we have

$$\psi_K = \frac{1 - (\delta(1-b))^{K-K+1}}{1 - \delta(1-b)} = 1.$$

For the induction step, assume the statement holds for $k+1$. Then,

$$
\begin{aligned}
&\psi_k = 1+\delta(1-b)\psi_{k+1} = 1+\delta(1+b)\frac{1-(\delta(1-b))^{K-k}}{1-\delta(1-b)}\\
&= \frac{1-\delta(1-b)+\delta(1-b)(1-(\delta(1-b))^{K-k})}{1-\delta(1-b)} = \frac{1+\delta(1-b)(1-(\delta(1-b))^{K-k}-1)}{1-\delta(1-b)}\\
&= \frac{1-(\delta(1-b))^{K-k+1}}{1-\delta(1-b)}.
\end{aligned}
$$

This concludes the proof of (6). From Lemma 2 and Lemma 3 we then immediately get

$$
d^*_{i,k} = \frac{1-(\delta(1-b))^{K-k+1}}{1-\delta(1-b)}, \qquad x_{i,k} = x_{j,k} = 0.
$$

Now assume $b \in (1/2, 1)$ in the finite game. Given that $\psi_k = 1+\delta(\pi^I_{i,k+1} - \pi^C_{j,k+1})$, inserting continuation payoffs from Lemma 2 and Lemma 3 yields

$$
\begin{aligned}
\psi_k &= 1+\delta(\frac{\psi_{k+1}}{4b} + \delta\pi^C_{j,k+2} - \psi_{k+1} - \delta\pi^C_{j,k+2} - \frac{\psi_{k+1}}{4b^2} + \frac{\psi_{k+1}}{b})\\
&= 1+\delta(-\psi_{k+1} + \frac{\psi_{k+1}}{4b} - \frac{\psi_{k+1}}{4b^2} + \frac{\psi_{k+1}}{b})\\
&= 1+\delta\psi_{k+1}(\frac{5b-1}{4b^2} - 1)
\end{aligned}
$$

We now show by induction that

$$
\psi_k = \frac{1-(\delta A)^{K-k+1}}{1-\delta A}, \tag{7}
$$

where $A := \frac{5b-1}{4b^2} - 1$.
The base case is given by

$$
\psi_K = \frac{1-(\delta A)^{K-K+1}}{1-\delta A} = 1.
$$

In the induction step, assume that the statement holds for $k+1$. Then,

$$
\begin{aligned}
\psi_k &= 1+\delta A\psi_{k+1} = 1+\delta A\frac{1-(\delta A)^{K-k}}{1-\delta A} = \frac{1-\delta A+\delta A(1-(\delta A)^{K-k})}{1-\delta A}\\
&= \frac{1-(\delta A)^{K-k+1}}{1-\delta A},
\end{aligned}
$$

which proves (7). Inserting this expression for ψ_k into the respective values for $d^*_{i,k}$ and $x_{j,k}$ yields

$$\begin{aligned}
d_{i,k}^{*} &= \frac{1-(\delta A)^{K-k+1}}{(1-\delta A)4b^2} \\
x_{i,k} &= 0 \\
x_{j,k} &= \frac{(1-(\delta A)^{K-k+1})(2b-1)}{(1-\delta A)4b^2}
\end{aligned}$$

The above expressions cover equilibrium behavior in the finite, K-contest game for all contests $k \geq 2$. In the first contest, player's behavior is different, as there is no incumbent. Nonetheless, the prize sum in the first contest ψ_1 is given by the above expressions for ψ_k above, dependent on b. Then, players exert effort equal to a quarter of the prize in the unbiased first contest:

$$x_{1,1} = x_{2,1} = \begin{cases} \frac{1}{4} & \text{if } b \geq 1 \\ \frac{1}{4}\frac{1-(\delta A)^{K-k+1}}{1-\delta A} & \text{if } \frac{1}{2} < b < 1 \\ \frac{1}{4}\frac{1-(\delta(1-b))^{K-k+1}}{1-\delta(1-b)} & \text{if } 0 < b \leq \frac{1}{2}. \end{cases}$$

This result for the first contest, together with the above results of contests $2, ..., K$ constitute the unique subgame-perfect Nash equilibrium of the finite game. Let g_K denote the subgame perfect Nash equilibrium for $K \in \mathbb{N}$. Then, Theorem 3.3 from Fudenberg and Levine (1983) states that $g^* = \lim_{K\to\infty} g_K$ is a subgame perfect Nash equilibrium of the infinite game (for $T(n) = n$ and $\epsilon_n = 0\ \forall n$, in their notation). Therefore, the expressions above give, for $K \to \infty$, a subgame perfect Nash equilibrium of the infinite game. This directly leads to the results of Lemmas 4, 5 and 6 and Corollary 2.

A.4 Proof of Proposition 1

Rent extraction is given by

$$\rho(b,\delta) = \frac{1}{\sum_{k=1}^{\infty}\delta^{k-1}}(2x_{i,1} + \sum_{k=2}^{\infty}\delta^{k-1}E_k).$$

We now insert the results from Lemmas 4, 5 and 6 and Corollaries 2 and 3 into this expression.

For $b \geq 1$ this yields

$$\rho(b,\delta) = (1-\delta)(2\cdot\frac{1}{4} + \sum_{k=2}^{\infty}\delta^{k-1}\cdot\frac{1}{2}) = \frac{1}{2}.$$

For $b \in (1/2, 1)$ this yields

$$\begin{aligned}
\rho(b,\delta) &= (1-\delta)(2 \cdot \frac{1}{4(1-\delta A)} + \sum_{k=2}^{\infty} \delta^{k-1} \frac{3b-1}{4b^2} \frac{1}{1-\delta A}) \\
&= \frac{1-\delta}{2(1-\delta A)} + \frac{3b-1}{4b^2} \cdot \frac{1-\delta}{1-\delta A} \cdot \frac{\delta}{1-\delta} = \frac{1-\delta}{2(1-\delta A)} + \frac{3b-1}{4b^2} \cdot \frac{\delta}{1-\delta A} \\
&= \frac{1}{2} + \frac{\delta b(-8b+11) - 3\delta}{8b^2 + \delta(8b^2 - 10b + 2)}.
\end{aligned}$$

For $b \in (0, 1/2)$ this yields

$$\begin{aligned}
\rho(b,\delta) &= (1-\delta)(2 \cdot \frac{1}{4(1-\delta(1-b))} + \sum_{k=2}^{\infty} \delta^{k-1} \cdot b \cdot \frac{1}{1-\delta(1-b)}) \\
&= \frac{1-\delta}{2(1-\delta(1-b))} + b \cdot \frac{1-\delta}{1-\delta(1-b)} \cdot \frac{\delta}{1-\delta} = \frac{1}{2} + \frac{\delta b}{2(1-\delta+\delta b)}.
\end{aligned}$$

This finalizes equilibrium rent extraction for all values of b. One can quickly verify that for $b \leq 1/2$, rent extraction is increasing in b. For $b \in (1/2, 1)$, $\rho(b,\delta)$ has a local maximum at

$$b^* = \frac{\delta + \sqrt{9-5\delta} + 3}{\delta + 11}.$$

which is for all δ the global maximum of $\rho(b,\delta)$.

A.5 Proof of Proposition 2

Consider the grim trigger strategy $T(b,\delta)$. In what follows, we determine the conditions for which the profile of grim trigger strategies forms a subgame perfect equilibrium of the infinite game. To do so, we fix some contest $\bar{k} \geq 1$ and consider the following two cases.

Case 1: Some deviation has already occurred before $\bar{k}$

Assume that some deviation from cooperation has already occurred in a previous contest. Then, both players play the effort levels from the competitive equilibrium in their grim trigger strategies, which is by Lemmas 4, 5 and 6 a combination of mutual best responses.

Case 2: No deviation until $\bar{k} - 1$

Assume that both players have cooperated in all previous contests. Consider a unilateral deviation by some player p whereas the other player $-p$ follows his grim trigger strategy.

Assume player p deviates from $T(b,\delta)$ in competition stage $\bar{k}$. Given that the other player follows the grim trigger strategy, the best deviation is to exert an infinitesimally small effort level $\epsilon > 0$ to win the contest with certainty. In line with the usual conventions, deviation yields a payoff equal to 1 in stage $\bar{k}$. In all subsequent contests, both players will follow behavior of the competitive equilibrium: player $-p$ has pulled the trigger and punishes forever and player p optimally reacts to the punishment which is the effort level of the competitive equilibrium. Let $\pi^C_{D,\bar{k}}$ denote the payoff stream that results from deviation in competition stage $\bar{k}$, from the perspective of period $\bar{k}$. Then, the deviating player p generates

$$\pi^C_{D,\bar{k}} = 1 + \delta \pi^I_{i,\bar{k}+1},$$

where $\pi^I_{i,\bar{k}+1}$ is the incumbent's continuation payoff in investment stage $\bar{k}+1$ that results from the competitive equilibrium.

If player p continues to follow the grim trigger strategy, his payoff stream is given by

$$\pi^C_{T,\bar{k}} = \sum_{k=0}^{\infty} \frac{1}{2}\delta^k = \frac{1}{2(1-\delta)}.$$

The deviation is profitable if $\pi^C_{D,\bar{k}} > \pi^C_{T,\bar{k}}$, that is,

$$1 + \delta \pi^I_{i,\bar{k}+1} > \frac{1}{2(1-\delta)}. \tag{8}$$

Lemma 7. *The payoff $\pi^I_{i,\bar{k}+1}$ is given by*

$$\pi^I_{i,\bar{k}+1} = \begin{cases} \frac{1}{4(1-\delta)} & \text{if} \quad b \geq 1 \\ \frac{A}{1-\delta A} + \frac{(2b-1)^2}{(1-\delta A)(1-\delta)4b^2} & \text{if} \quad \frac{1}{2} < b < 1 \\ \frac{1-b}{1-\delta(1-b)} & \text{if} \quad b \leq \frac{1}{2} \end{cases}$$

where $A = \frac{5b-1}{4b^2} - 1$.

The proof of Lemma 7 is relegated to Appendix A.6. Inserting Lemma 7 in Equation (8) yields

$$\pi^C_{D,\bar{k}} > \pi^C_{T,\bar{k}} \Leftrightarrow \begin{cases} \delta < \frac{2}{3} & \text{if} \quad b \geq 1, \\ \delta < \frac{4b^2}{4b^2+3b-1} & \text{if} \quad \frac{1}{2} < b < 1, \\ \delta < \frac{1}{1+b} & \text{if} \quad b \leq \frac{1}{2}. \end{cases} \tag{9}$$

Equation 9 gives conditions under which the profitable deviation precludes the cooperative equilibrium. Otherwise, since $\bar{k}$ is arbitrary, there does not exist a profitable deviation from the cooperative equilibrium in any competition stage. Therefore, we additionally check whether there exists a profitable deviation in an investment stage.

Assume player p deviates from $T(b,\delta)$ in investment stage $\bar{k}$. A deviation in the investment stage $I_{\bar{k}}$ implies that both players follow the competitive equilibrium from $\bar{k}$ onward. Let $\pi^I_{D,\bar{k}}$ denote the payoff stream that results from a deviation in investment stage $\bar{k}$, from the perspective of period $\bar{k}$. Then, the deviating player p generates

$$\pi^I_{D,\bar{k}} = \Pi_{\bar{k}} + \delta\Pi_{\bar{k}+1}$$

where $\Pi_{\bar{k}}$ is the immediate payoff in period $\bar{k}$, and $\Pi_{\bar{k}+1}$ is the continuation payoff in period $\bar{k}+1$. Note that

$$\Pi_{\bar{k}+1} = p_{i,k}\pi^I_{i,\bar{k}+1} + (1-p_{i,k})\pi^C_{j,\bar{k}+1}$$

where $p_{i,k}$ is the deviator's probability of winning. As $\pi^I_{i,\bar{k}+1} \geq \pi^C_{j,\bar{k}+1}$, the continuation payoff Π_{k+1} is highest for $p_{i,k} = 1$, where it is equal to $\pi^I_{i,\bar{k}+1}$. Therefore, $\Pi_{\bar{k}+1} \leq \pi^I_{i,\bar{k}+1}$. In addition, the immediate payoff in the period of deviation $\Pi_{\bar{k}}$ is strictly smaller than one. Therefore, we have

$$\Pi_{\bar{k}} + \delta\Pi_{k+1} < 1 + \delta\pi^I_{i,\bar{k}+1} \Leftrightarrow \pi^I_{\bar{k},D} < \pi^C_{\bar{k},D},$$

that is, for any $\bar{k}$ and $b > 0$, deviating in investment stage $\bar{k}$ is less profitable than deviating in competition stage $\bar{k}$. Therefore, Equation 9 yields the conditions for which the cooperative equilibrium exists, that is

$$\pi^{\bar{k},C}_D \leq \pi^{\bar{k},C}_T \Leftrightarrow \begin{cases} \delta \geq \frac{2}{3} & \text{if} \quad b \geq 1, \\ \delta \geq \frac{4b^2}{4b^2+3b-1} & \text{if} \quad \frac{1}{2} < b < 1, \\ \delta \geq \frac{1}{1+b} & \text{if} \quad b \leq \frac{1}{2}, \end{cases}$$

which proves Proposition 2.

A.6 Proof of Lemma 7

We prove the three cases separately. Suppose first $b \geq 1$. Then, for all k, $d_{i,k} = 0$ and $x_{i,k} = x_{j,k} = 1/4$. Therefore $\pi^I_{i,k} = \sum_{k=0}^{\infty} \frac{1}{4}\delta^k$ for all k. In particular, also for $k = 2$ we have

$$\pi^I_{i,2} = \frac{1}{4(1-\delta)}.$$

Suppose now $b \in (1/2, 1)$. We calculate the payoff $\pi^I_{i,2}$ of the infinite game as the limit of payoffs $\pi^I_{i,2}$ of the K-contest game and then make again use of Theorem 3.3 from Fudenberg and Levine (1983). Then, according to Lemma 2 and Lemma 3, we can express $\pi^C_{j,k}$ recursively:

$$\pi^C_{j,k} = \psi_k + \delta\pi^C_{j,k+1} + \frac{\psi_k}{4b^2} - 2\sqrt{\frac{\psi_k^2}{4b^2}} = \frac{(2b-1)^2}{4b^2}\psi_k + \delta\pi^C_{j,k+1}.$$

By induction we show that $\pi^C_{j,k}$ has the representation

$$\pi^C_{j,k} = \frac{(2b-1)^2}{4b^2} \sum_{n=k}^{K} \delta^{n-k} \psi_n.$$

The base case $k = K$ is clear from $\psi_K = 1$ and Lemma 2. Induction step from $k+1$ to k:

$$\begin{aligned}
\pi^C_{j,k} &= \frac{(2b-1)^2}{4b^2}\psi_k + \delta \pi^C_{j,k+1} = \frac{(2b-1)^2}{4b^2}\psi_k + \delta \frac{(2b-1)^2}{4b^2} \sum_{n=k+1}^{K} \delta^{n-(k+1)} \psi_n \\
&= \frac{(2b-1)^2}{4b^2}(\psi_k + \sum_{n=k+1}^{K} \delta^{n-k}\psi_n) = \frac{(2b-1)^2}{4b^2} \sum_{n=k}^{K} \delta^{n-k}\psi_n.
\end{aligned}$$

Inserting this into $\pi^I_{i,k}$ from Lemma 3, we obtain

$$\begin{aligned}
\pi^I_{i,k} &= \frac{1}{4b}\psi_k + \delta \pi^C_{j,k+1} = \frac{1}{4b}\psi_k + \delta(\frac{(2b-1)^2}{4b^2} \sum_{n=k+1}^{K} \delta^{n-(k+1)}\psi_n) \\
&= \frac{1}{4b}\psi_k + \delta^0 \frac{(2b-1)^2}{4b^2}\psi_k - \delta^0 \frac{(2b-1)^2}{4b^2}\psi_k + \delta \frac{(2b-1)^2}{4b^2} \sum_{n=k+1}^{K} \delta^{n-(k+1)}\psi_n = \\
&= \left(\frac{5b-1}{4b^2} - 1\right)\psi_k + \frac{(2b-1)^2}{4b^2} \sum_{n=k}^{K} \delta^{n-k}\psi_n.
\end{aligned}$$

From Lemma 5, we insert the value of ψ_n into this expression and obtain

$$\begin{aligned}
\pi^I_{j,k} &= A \cdot \frac{1-(\delta A)^{K-k+1}}{1-\delta A} + \frac{(2b-1)^2}{4b^2} \sum_{n=k}^{K} \delta^{n-k} \frac{1-(\delta A)^{K-n+1}}{1-\delta A} \\
&= A \cdot \frac{1-(\delta A)^{K-k+1}}{1-\delta A} + \frac{(2b-1)^2}{4b^2(1-\delta A)}(\sum_{n=k}^{K} \delta^{n-k} - \sum_{n=k}^{K} \delta^{K-k+1} A^{K-n+1}) \\
&= A \cdot \frac{1-(\delta A)^{K-k+1}}{1-\delta A} + \frac{(2b-1)^2}{4b^2(1-\delta A)}(\sum_{n=0}^{K-k} \delta^{n} - \delta^{K-k+1} \sum_{n=1}^{K-k+1} A^{n}) \\
&= A \cdot \frac{1-(\delta A)^{K-k+1}}{1-\delta A} + \frac{(2b-1)^2}{4b^2(1-\delta A)}(\frac{1-\delta^{K-k+1}}{1-\delta} - \delta^{K-k+1} \frac{A(1-A^{K-k+1})}{1-A}).
\end{aligned}$$

Setting $k = 2$ and letting $K \to \infty$ in this expression yields $\pi^I_{i,2}$ in the infinite game:

$$\lim_{K\to\infty} \pi^I_{i,2} = \frac{A}{1-\delta A} + \frac{(2b-1)^2}{(1-\delta A)(1-\delta)4b^2}.$$

Suppose now $b \in (0, 1/2)$. Then, from Lemma 4, $\pi^I_{i,k} = (1-b)\psi_k$. Inserting the value of ψ_k from Lemma 4 yields

$$\pi^I_{i,k} = (1-b)\frac{1-(\delta(1-b))^{K-k+1}}{1-\delta(1-b)}.$$

Now, again, for $k = 2$ and $K \to \infty$ we obtain

$$\pi^I_{i,2} = \frac{(1-b)}{1-\delta(1-b)},$$

which finalizes the proof of the Lemma.

A.7 Proof Proposition 3

If early effort is unobservable, the maximization of players' expected payoffs yields

$$x^*_{i,k} = \frac{1}{(1+b)^2}\psi_k, \quad x^*_{j,k} = \frac{b}{(1+b)^2}\psi_k,$$

and players' payoffs are

$$\pi_{i,k} = \frac{\psi_k}{(1+b)^2} + \delta\pi_{j,k+1}, \quad \pi_{j,k} = \frac{b^2\psi_k}{(1+b)^2} + \delta\pi_{j,k+1},$$

where, again, $\psi_k = 1 + \delta\pi^*_{i,k+1} - \delta\pi^*_{j,k+1}$ denotes the effective prize sum of contest k.

To characterize the competitive equilibrium with unobservable early effort, we first solve the finite, K-contest game by backward induction. Then, we use Theorem 3.3 from Fudenberg and Levine (1983) to obtain the equilibrium of the infinite game.

For the finite game with unobservable early effort, we show by induction that the effective prize in period $k \geq 2$ is given by

$$\psi_k = \frac{1-(\delta B)^{K-k+1}}{1-\delta B}, \qquad B = \frac{1-b}{1+b}.$$

The base case $k = K$ is clear since $\psi_K = 1$. Induction step $k+1 \to K$:

$$\begin{aligned}
\psi_k &= 1 + \delta(\pi_{i,k+1} - \pi_{j,k+1}) = 1 + \delta(\frac{\psi_{k+1}}{(1+b)^2} - \frac{b^2\psi_{k+1}}{(1+b)^2}) \\
&= 1 + \frac{\delta(1-b)}{1+b}\psi_{k+1} = 1 + \frac{\delta B(1-(\delta B)^{K-k})}{1-\delta B} \\
&= \frac{1-\delta B + \delta B(1-(\delta B)^{K-k})}{1-\delta B} = \frac{1-(\delta B)^{K-k+1}}{1-\delta B}.
\end{aligned}$$

Then, equilibrium efforts are

$$\begin{aligned}
x_{i,k} &= \frac{1}{(1+b)^2}\frac{1-(\delta B)^{K-k+1}}{1-\delta B}, \\
x_{j,k} &= \frac{b}{(1+b)^2}\frac{1-(\delta B)^{K-k+1}}{1-\delta B}.
\end{aligned}$$

As the expression of ψ_k is also valid for $k = 1$ and players in the first contest both exert effort of a quarter of the prize, we have

$$x_{i,1} = x_{j,1} = \frac{1 - (\delta B)^K}{4(1 - \delta B)}.$$

By Fudenberg and Levine (1983), we then have for the infinite game

$$\begin{aligned} x_{i,k} &= \frac{1}{(1 - \delta B)(1 + b)^2} \quad \text{for } k \geq 2, \\ x_{j,k} &= \frac{b}{(1 - \delta B)(1 + b)^2} \quad \text{for } k \geq 2, \\ x_{i,1} = x_{j,1} &= \frac{1}{4(1 - \delta B)}. \end{aligned}$$

Inserting these values into $\rho(b, \delta)$, we obtain

$$\begin{aligned} \rho(b, \delta) &= \frac{1}{\sum_{k=1}^{\infty} \delta^{k-1}} (2 \frac{1}{4(1 - \delta B)} + \sum_{k=2}^{\infty} \delta^{k-1} \frac{2b}{(1 - \delta B)(1 + b)^2}) \\ &= (1 - \delta)(\frac{1}{2(1 - \delta B)} + \frac{2b}{(1 - \delta B)(1 + b)^2} \sum_{k=2}^{\infty} \delta^{k-1}) \\ &= \frac{1 - \delta}{2(1 - \delta B)} + \frac{2b\delta}{(1 - \delta B)(1 + b)^2} = \frac{(1 - \delta)(1 + b)^2 + 4b\delta}{2(1 + b)(1 + b - \delta + \delta b)} \\ &= \frac{1}{2} + \frac{b\delta(1 - b)}{(1 + b)(1 + b - \delta + \delta b)}, \end{aligned}$$

which proves Proposition 3.

A.8 Proof Proposition 4

When the contest designer maximizes winner's effort, rent extraction is given by

$$\rho_W(b, \delta) = \frac{1}{\sum_{k=1}^{\infty} \delta^{k-1}} (2x_{i,1} + \sum_{k=2}^{\infty} \delta^{k-1} W_k), \tag{10}$$

where $W_k = p_{i,k} x_{i,k} + (1 - p_{i,k}) x_{j,k} + b \cdot d_{i,k})$ is the expected effort of the winner. W_k depends on the incumbent's probability of winning, which is given by Corollary 1. Therefore, expected winner's effort in the equilibrium of contest $k \geq 2$ is given by

$$W_k = \begin{cases} \frac{1}{4} & \text{if } b \geq 1 \\ \frac{6b^2 - 4b + 1}{8b^3} \psi_k & \text{if } \frac{1}{2} < b < 1 \\ b\psi_k & \text{if } 0 \leq b \leq \frac{1}{2}. \end{cases} \tag{11}$$

We can directly obtain expected winner's effort in the first contest from Corollary 2. Then, we use equation 11 and Lemmas 4, 5 and 6 to explicitly calculate $\rho_W(b,\delta)$. For $b \geq 1$, we can immediately see that

$$\rho_W(b,\delta) = (1-\delta)\sum_{k=1}^{\infty}\delta^{k-1}\frac{1}{4} = \frac{1}{4}.$$

For $b \in (1/2, 1)$, we have for $A = (5b-1)/(4b^2) - 1$:

$$\begin{aligned}
\rho_W(b,\infty) &= (1-\delta)\Big(\frac{1}{4(1-\delta A)} + \sum_{k=2}^{\infty}\delta^{k-1}\frac{6b^2-4b+1}{8b^3}\frac{1}{1-\delta A}\Big) \\
&= \frac{1-\delta}{4(1-\delta A)} + \frac{6b^2-4b+1}{8b^3}\cdot\frac{1-\delta}{1-\delta A}\sum_{k=2}^{\infty}\delta^{k-1} \\
&= \frac{1-\delta}{4(1-\delta A)} + \frac{6b^2-4b+1}{8b^3}\cdot\frac{1}{1-\delta A} \\
&= \frac{2b^3-2b\delta(2-b)(1-b)+\delta}{2b\left(4b^2(1+\delta)-5b\delta+\delta\right)} = \frac{1}{4} + \frac{\delta(2-b(1-b)(9-8b))}{4b\left(4b^2(1+\delta)-5b\delta+\delta\right)}.
\end{aligned}$$

For $b \leq 1/2$, we have

$$\begin{aligned}
\rho_W(b,\infty) &= (1-\delta)\Big(\frac{1}{4(1-\delta(1-b))} + \sum_{k=2}^{\infty}\delta^{k-1}\cdot b\cdot\frac{1}{1-\delta(1-b)}\Big) \\
&= \frac{1-\delta}{4(1-\delta(1-b))} + b\cdot\frac{1-\delta}{1-\delta(1-b)}\sum_{k=2}^{\infty}\delta^{k-1} \\
&= \frac{1-\delta}{4(1-\delta(1-b))} + \frac{b\delta}{1-\delta(1-b)} \\
&= \frac{(4b-1)\delta+1}{4(1-\delta(1-b))} = \frac{1}{4} + \frac{3b\delta}{4(1-\delta(1-b))},
\end{aligned}$$

which finalizes the proof of Proposition 4.