Bavarian Graduate Program in Economics

BGPE Discussion Paper

No. 241

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Fabian Alex

June 2025

ISSN 1863-5733

Editor: Prof. Regina T. Riphahn, Ph.D.
Friedrich-Alexander-Universität Erlangen-Nürnberg

BGPE Discussion Paper Series

Risk of Bankruptcy and the Modigliani-Miller theorem in a General Equilibrium model of Socially Responsible Investing*

Fabian Alex†

University of Regensburg

Department of Economics

93 040 Regensburg, Germany

Abstract

In the SRI-augmented version of the Arrow-Debreu-model by Arnold (2023), the restriction to no risk of bankruptcy is immaterial. Furthermore, shareholder unanimity is still valid when a firm's bond issuance (viz., its leverage) is chosen endogenously. The debt-equity-ratio of firms may not only be set arbitrarily (independent of their capital choice) with respect to shareholder value, but also to entire budget sets, implying an economy-wide Modigliani-Miller type of irrelevance given market completeness. If SRI leads individuals to constrain the set of assets they are prepared to buy and, thus, reduces their personal marketed subspace, over-indebtedness may restore it.

JEL classification: D51, D52, D53, G12, G33, M14

Key words: socially responsible investing, general equilibrium, complete markets, Modigliani-Miller, asset pricing, Arrow-Debreu

*For helpful comments I am indebted to Lutz Arnold. Declarations of interest: none.

†Phone: +49 941 943 2704; e-mail: fabian.alex@ur.de.

1 Introduction

The question whether non-financial investment criteria influence economic activity, be it via individual rates of return or through different firm behavior, has been widely discussed in the recent literature on Socially Responsible Investing (SRI). Despite some tendency towards showing reduced returns of – primarily – green, i.e., ecologically friendly investment opportunities (see the literature review of Cheong & Choi, 2020), there does not seem to be a clear consensus yet.

From a theoretical viewpoint, social responsibility criteria may lead to better payoffs for non-responsible individuals due to costly innovation by responsible firms (Oehmke & Opp, 2021). However, a varied investment attitude may also turn out neutral for the economy because that innovation would have to be carried out by firms that responsible individuals shirk (Gollier & Pouget, 2022). Lastly, the ground-breaking work of Heinkel et al. (2001) shows that a critical mass of responsible investors can shift the market in favor of more responsibility.

Empirically, the picture is much the same. One can find various studies documenting negative abnormal returns of responsible assets (see Bolton & Kacperczyk, 2020 and Flammer, 2021), equality of returns (Hamilton et al., 1993 and Andersson, 2016) or outperformance by responsible assets (Kempf & Osthoff, 2007 and Edmans, 2011).

In light of the above disagreements among different littérateurs, we go back to a very basic yet rich modeling approach, namely general equilibrium. The model is due to Arnold (2023). It is a generalization of the general equilibrium model with different states of nature in the second of two time periods, originally due to Arrow (1964) and Debreu (1959), augmented to include SRI. There, SRI turns out neutral if four conditions hold. Firstly, asset tastes and consumption have to enter utility separately. Secondly, there has to be at least one neutral (demotically, an "irresponsible") individual, drawing no utility from the sheer act of holding any asset whatsoever. Thirdly, responsible individuals must show satiation in SRI, which prevents them from financing infinite long positions of favorable firms' assets by shorting unfavorable ones. Lastly, each individual must experience a very strict version of financial market completeness (*Spanning With Assets with No Social returns*, in short *SWANS*). This is given as long as the individual can allocate consumption to any specific state of nature without using either favorable or unfavorable assets, that is, using only assets she is *indifferent towards*.

In Arnold (2023), budget constraints are built based on the assumption of no risk of bankruptcy. This is modeled by assuming that each firm offering an exogenous supply of bonds b_{jl} makes a sufficient nominal profit $p_{ls}y_{jls}$ to cover all resulting obligations in any possible state of nature $s \in S$, i.e., $p_{ls}y_{jls} \geq b_{jl}, s = 1, ..., S$. We drop this assumption and show that an adequate redefinition of budget constraints and firm values leaves the subsequent analysis unaffected.

It is apparent that, in the real world, over-indebtedness and bankruptcy are an issue. The only way of fully and inevitably circumventing it would be to oblige firms to finance all of their capital needs through equity, i.e., stocks. Hellwig (1976, p. 1) already noted this in an early version of his ground-breaking work on limited liability: "If the debt-equity ratio is large enough, there will always be a positive probability of bankruptcy". With the number of people (directly) holding stocks at all still below 20% even in industrialized countries,[1] it becomes apparent that a lot of financial planning of households must take place via debt contracts (which, in turn, are ultimately issued by some firm). So excluding the possibility of bankruptcy is simply impossible to reconcile with allowing all individuals to transfer wealth over time.

To arrive at validity of the model even with risk of bankruptcy, two (reasonable) assumptions need to be additionally employed: limited liability of shareholders as well as no transaction costs in consequence of a firm going bankrupt. Both are common in related work, comprising Stiglitz's (1969) and those following it. These assumptions are focal if one wants to arrive at the result of Modigliani &

[1]Current numbers encompass 17.6% in Germany (cf. Fey, 2024) and 15.2% in the USA (cf. Maranjian, 2023).

Miller (1958) that the capital structure of a firm is irrelevant for its value (cf. Rubinstein, 2003, p. 11), which is our declared goal.

Previous (already very early) work on this subject has shown that the above conclusion can be reached via two main strategies: using a mean-variance-framework or assuming a full set of *Arrow-Debreu securities* (ADS, cf. Stiglitz, 1969, pp. 788-792). We proceed to show that any version of financial market completeness will do, i.e., the ability to synthesize portfolios that transfer wealth into any state of nature directly is sufficient. This formulation is somewhat more general as it allows the way financial market completeness is achieved to change when comparing different scenarios of the same economy. If the Arrow-Debreu-securities were already required to be formed portfolios, the composition of these portfolios may change.

Completeness of financial markets itself is crucial in order to guarantee a Pareto-efficient allocation of ressources. In the presence of SRI, however, individuals may artificially constrain themselves in the market completeness they experience. The most prevalent instance of this state of affairs is *negative screening*, which is currently the third-most popular strategy for sustainable investments with a volume of 3,840 billion USD for the USA, Canada, Australia and Japan taken together (cf. Global Sustainable Investment Alliance, 2022, p. 13). The term refers to a refusal to buy stocks or bonds of certain firms considered irresponsible by the individual. If this kind of behavior prevents her from holding an asset that is vital to financial market completeness (i.e., if *SWANS* is violated), the latter is no longer given for her. But by over-indebting itself, a different firm she is willing to hold assets of may fundamentally change its payoff structure and thereby re-establish a fully spanned state-space.

The rest of the paper is organized as follows. Section 2 traces the model of Arnold (2023) with exogenous bond supply and over-indebtedness. Section 3 introduces the firm's decision process on capital formation and bond emissions, ultimately reaching the Modigliani-Miller theorem of corporate finance. In Section 4, we generalize this result to irrelevance of financial structures for economic activity of the entire economy in the absence of SRI. Section 5 re-introduces SRI into this discussion. Finally, Section 6 offers a discussion of the results.

2 Exogenous bond supply

The model is built around the portfolio decisions of $i = 1, ..., I$ individuals. There is a single good in $t = 0$ with a price of p_0 and $l = 1, ..., L$ goods priced at p_l, respectively, in all states $s = 1, ..., S$ in $t = 1$. After $t = 1$, the model ends. Each individual is endowed with y_{i0} units of the $t = 0$-consumption good. $t = 1$-goods are produced by J_l firms each. For their production, firms invest into physical capital k_{jl}, which can be obtained one-to-one from the $t = 0$-consumption good. The firm issues bonds b_{jl} priced at R_{jl}. Each individual has an initial share $\bar{\theta}_{ijl}$ in every firm that determines how much of their bond emission profits she is entitled to and how much of its capital she must finance. Naturally, each share must belong to someone, so $\sum_{i=1}^{I} \bar{\theta}_{ijl} = 1, j = 1, ..., J_l, l = 1, ..., L$. Capital translates into output in $t = 1$ according to state-dependent production functions $y_{jls} = f_{jls}(k_{jl})$. Furthermore, the consumer can buy bonds, where individual holdings are called a_{ijl}, change her shareholdings to θ_{ijl} at a price determined by firm values v_{jl}, and trade assets in zero net supply (securities) z_{im} with the other individuals at q_m. Taking all of this into consideration, i seeks to maximize her overall utility $U_i(u_i(\boldsymbol{c_i}, \boldsymbol{k}), \boldsymbol{\theta_i}, \boldsymbol{a_i}, \boldsymbol{k}, \boldsymbol{v})$, which depends on consumption utility $u_i(\boldsymbol{c_i}, \boldsymbol{k})$ and portfolio choice of stocks $\boldsymbol{\theta_i}$ and bonds $\boldsymbol{a_i}$ in a seperable way.[2] Dependence of u_i on physical capital formation $\boldsymbol{k}$ depicts production externalities. u_i has the usual property of decreasingly positive marginal utility of consumption in every good. In applications where SRI does not play a role, U_i and u_i coincide. As the two rationales for suffering from externalities and having (dis-)tastes for assets are allowed to coexist, one can speak of a consequentialist and a non-consequentialist

[2] Bold symbols refer to vectors of variables along the omitted index, so $\boldsymbol{\theta_i}$, e.g., stacks θ_{ijl} over $j = 1, ..., J_l$, $l = 1, ..., L$.

utility component. The former means that individuals suffer from the externality while the latter implies "warm glow" and "cold prickle" from certain asset holdings of favorable and unfavorable firms, respectively (see Dangl et al., 2024).

Let the number of corporate bonds b_{jl} issued by firm (j,l) be given exogenously, where no restriction on the range of values it may take on is imposed. Each of these bonds promises, as in Arnold (2023), to pay one unit of income in the subsequent period. However, this promise may turn out to be hollow: Given that in some state(s) of nature the issuing corporate goes bankrupt, the payoff of one such bond will turn out strictly less than one. If this can be the case, the price of such a bond from a firm with positive risk of bankruptcy obviously should be below that of a risk-free counterpart. Taking the resulting fair pricing of firms' bonds into account, budget constraints from the basal paper can simply be rewritten. Proceeding in this fashion, the period $t = 0$-budget constraint reads

$$p_0(c_{i0} - y_{i0}) + \sum_{l=1}^{L} \sum_{j=1}^{J_l} [\bar{\theta}_{ijl}(p_0 k_{jl} - R_{jl} b_{jl}) + (\theta_{ijl} - \bar{\theta}_{ijl}) v_{jl} + R_{jl} a_{ijl}] + \sum_{m=1}^{M} q_m z_{im} \leq 0, \tag{1}$$

where the reduced value of some (or, potentially, all) firm-issued assets is taken into account by newly introduced bond prices $R_{jl} \leq R$. We can be assured that those bond prices will coincide with the price of a safe asset R if and only if firm (j,l) does not go bankrupt in any state of nature s. Bond prices obey

$$R_{jl} = \begin{cases} = R & \text{if } p_{ls} y_{jls} \geq b_{jl},\ s = 1, ..., S \\ < R & \text{if } \exists s \in S : p_{ls} y_{jls} < b_{jl} \end{cases},$$

where we slightly abuse notation in defining the set $S = \{1, ..., S\}$. From this we can infer that the introduction of bankruptcy risk will, ceteris paribus, relax the first ($t = 0$) budget constraint somewhat for investors of a firm while it tightens it for initial shareholders. Turning to the $t = 1$-budget constraint, one now has to consider additional nonlinearity. Namely, for each firm (j,l) that i invests into, the possibility that her shares of that firm become worthless needs to be taken into account. To do so, we introduce the concept of limited liability of shareholders into the model: If $p_{ls}y_{jls} < b_{jl}$ holds for firm (j, l) of which i holds positive shares $\theta_{ijl} > 0$, she is not obliged to make up for this firm's over-indebtedness with her private wealth. Such assumptions are quite common in financial economics (see, for example, Magill & Quinzii, 1996, pp. 401-407). Rather, shares in this corporate become worthless while creditors obtain an adequate fraction of its bankruptcy estate that is proportional to the number of bonds they bought from this very firm relative to its total issuance. Considering the above, the budget constraint for date $t = 1$ can be written as

$$\sum_{l=1}^{L} p_{ls}c_{ils} - \sum_{l=1}^{L}\sum_{j=1}^{J_l}[\theta_{ijl}\max(p_{ls}y_{jls} - b_{jl}, 0) + \alpha_{ijls}] - \sum_{m=1}^{M} x_{ms}z_{im} \leq 0, \;\; s = 1, ..., S \tag{2}$$

where the bond-payoff

$$\alpha_{ijls} = \begin{cases} a_{ijl} & \text{if } p_{ls}y_{jls} \geq b_{jl} \\ \frac{a_{ijl}}{b_{jl}}p_{ls}y_{jls} & \text{if } p_{ls}y_{jls} < b_{jl} \end{cases}, \;\; s = 1, ..., S$$

expresses the seminal assumption described above that losses in bond value are borne proportionately by all bond holders (see also Sabarwal, 2000). In order to be able to apply the analysis in Arnold (2023), what remains to be altered are the equations in his Lemma 1 (cf. p. 72) determining state prices r_s. The rationale for this is that, with risk of bankruptcy, we can no longer define firm values as "state price-weighted profits minus debt" because this difference may become negative while actually any firm can be nothing worse than worthless. To correct this, one can either employ the maximum operator once again or (as done here) define a separate set of states without bankruptcy per firm.[3] We do so by specifying the sets S'_{jl} as the collection of those states s in which $p_{ls}y_{jls} \geq b_{jl}$ is satisfied for

[3]What makes this approach more attractive is the flexibility and re-usability of the so-defined set which will be heavily made use of later.

the specific firm (j, l) under consideration.[4] Using this definition, we obtain firm values obeying

$$v_{jl} = \sum_{s \in S'_{jl}} r_s(p_{ls} y_{jls} - b_{jl}), \;\; j = 1, ..., J_l, \;\; l = 1, ..., L. \tag{3}$$

The next equation necessary for pinning down state prices gives the cost of purchasing a safe asset. We can keep the original definition which reads

$$R = \sum_{s=1}^{S} r_s, \tag{4}$$

but we must note that such an asset may not exist: If every firm goes bankrupt in some state, neither of them issues a safe bond. A sufficient condition to ensure existence of such an asset would be to assume at least one firm (j, l) that satisfies $p_{ls} y_{jls} \geq b_{jl}$ in every state $s = 1, ..., S$, or put differently: a firm with enough profits to service all of its debt in each possible state of nature, $\exists (j, l) : S'_{jl} = S$. This is, however, not a necessary condition, as the nonexistence of a safe corporate bond need not impinge on market completeness. Payoff profiles may still be synthesized by means of security trade[5] or purchase of stocks. A proficient combination of bonds from various firms is also possible. That is, while we may not obtain a single safe asset, the construction of a safe portfolio can still be unhindered. Ultimately, there is nothing that prevents individuals from creating a safe asset themselves, i.e., one of the zero net-supply assets in $\boldsymbol{z}$ could be safe as well.

In fact, bankruptcy here may serve as a form of financial engineering. More precisely, a "securities innovation" can happen. The term is borrowed from Finnerty (1988), who describes it as the creation of new financial instruments. In our

[4]Naturally, there is one such set for each firm (j, l), i.e., $\sum_{l=1}^{L} J_l$ sets of this kind in total. Some of these may constitute the empty set or be identical to each other. All of them are subsets of S.

[5]Alternatively, there could be a safe financial asset in zero net supply, that is, one of the M securities could constitute the safe asset.

context, this means that new payoff profiles can be synthesized.[6] To see how a structure of payoffs unavailable before can be generated, imagine two firms, (j, l) and (j', l') with equal revenues $p_{ls}y_{jls} = p_{l's}y_{j'l's}, s = 1, ..., S$, perhaps because they produce the same good $l = l'$ with identical production structure $f_{jls}(k_{jl}) = f_{j'l's}(k_{j'l'}), s = 1, ..., S$ and will thus endogenously choose the same capital input $k_{jl} = k_{j'l'}$. Obviously, both stocks and bonds issued by one of these firms are perfect substitutes for financial products of the other if bankruptcy cannot happen. If, however, one of them instead raises so much outside capital that it cannot fully service its external debt in some states s, payoffs from both its stocks and its bonds will change, demarcating it somewhat from the more conservatively financed corporation. So the subset of the state-space spanned by the two firms under consideration alone has increased by over-indebting one of them, meaning that the risk of bankruptcy can indeed (but need not necessarily) help in achieving completeness of financial markets. In Sabarwal's (2000, cf. p. 2) words, the equilibrium allocation of an economy with bankruptcy may exert Pareto-dominance over an analogous one where bankruptcy is ruled out from the beginning. Note, however, that this may also affect state prices r_s, so the described positive effect could be mitigated or even reversed for some individuals. We can compute the prices of each single firm's corporate bond as

$$R_{jl} = \sum_{s \in S'_{jl}} r_s + \sum_{s \notin S'_{jl}} \frac{p_{ls}y_{jls}}{b_{jl}} r_s, \ \ j = 1, ..., J_l, \ \ l = 1, ..., L, \tag{5}$$

where (4) is obtained as a special case for all firms that never go bankrupt.[7] Therefore, the possibility of bankruptcy in some (that is, a fixed subset of) states can serve as a means to make a single corporation's bonds unique as already seen

[6] Finnerty (1988) distinguishes this and two further kinds of financial engineering. The other two encompass reduced transaction costs as well as more creative management strategies for debt or cash. Both are incompatible with our modeling framework where transaction costs are absent and any way of financing is available to everyone.

[7] An alternative way of writing this is $R_{jl} = \sum_{s=1}^{S} r_s \frac{\alpha_{ijls}}{a_{ijl}}, \ j = 1, ..., J_l, \ l = 1, ..., L$, where dependence on i cancels out.

in the example above. In exchange for this, however, the property of all bonds being perfect substitutes is lost. In a similar vein as in Stiglitz's work on the Modigliani-Miller theorem, this implies higher interest rates being paid (via their bonds) by firms with higher risk of bankruptcy (cf. Stiglitz, 1969, p. 788). Finally, unchanged compared to the original setup, security prices are still obtained by weighting payoffs with according state prices, i.e.,

$$q_m = \sum_{s=1}^{S} r_s x_{ms}, \; m = 1, ..., M. \tag{6}$$

Each of the subsequent analyses conducted by Arnold (2023) remains valid. To illustrate this point, we relate equilibria with different debt levels in Section 4. Before doing so, it seems adequate to discuss an endogenous optimum choice of leverage.

3 Endogenous bond supply

Within the very plannable setting of the GE model, it may seem unreasonable to assume states with bankruptcy: Nothing prevents firms from planning farsightedly enough to avoid bankruptcies given the states $s = 1, ..., S$. So some motivation for allowing bankruptcy to happen is in order. Simply obliging firms to raise outside capital to an extent which leads them to be over-indebted in some of the possible states of nature is somewhat artificial.[8] A good intuition as to why financial distress is usually avoided is given by Gilson (1989). He argues that managers of an insolvent firm bear large personal costs such as a loss in reputation and more difficult odds in the labor market, possibly mitigated somewhat by a fondness for leisure. Clearly, such long-term effects cannot be incorporated properly into

[8]The counterargument that the firm generates a free revenue in $t = 0$ in return has no bite: Note that they gain less income via bond emission at a price of R_{jl} instead of R whereas the repayment cost associated with it in all non-bankruptcy states remains the same. One could also employ some risk aversion of firms to argue for not allowing any bankruptcy to happen – after all, the firms are still owned by consumers.

the two-period framework considered here. We could merely ascribe a disutility of bankruptcy to the largest stock holder of each (j, l). If i has the absolute majority in that firm ($\theta_{ijl} > 0.5$) and if this translated into equivalent voting power, bankruptcy would be unconditionally avoided. If shares are spread out too far ($\theta_{ijl} \leq 0.5$), on the other hand, preventing bankruptcy is partly out of her control. If the latter happens, she ends up with a lower utility. Other than that, nothing changes.

We model an endogenous choice of debt b_{jl} by each firm (j, l) in order to analyze whether insolvency in some states may actually be the result of rationally optimal behavior. Our goal is to arrive at a version of the result due to Modigliani & Miller (1958) that a firm's capital structure is irrelevant for its value.[9] Note that, due to shareholders' obligations to partake in the costs of capital and their claim to the proceeds from bond emissions, what needs to be invariant is indeed not the firm value v_{jl} per se, but rather firm value less the aforementioned costs. This is precisely the shareholder value (SV):

$$SV_{jl} = v_{jl} - (p_0 k_{jl} - R_{jl} b_{jl}). \tag{7}$$

3.1 Shareholder unanimity

Before moving on to this analysis, however, we must first establish whether SV maximization is indeed the goal of shareholders. Establishing that the budget sets of all initial shareholders in a firm (j, l), i.e., all i with $\bar{\theta}_{ijl} > 0$, expands as SV increases, is sufficient for the proof of shareholder unanimity – even in the presence of SRI – given by Arnold (2023, pp. 74-75) to go through without any modification.

In order to assert anything about the budget set of i, we need to bring all she can

[9] As Rubinstein (2003, p. 7) notes, this central result in financial market theory is actually due to John Burr Williams who, in his book *The Theory of Investment Value* (1938, p. 73), described the underlying state of affairs under the name of the "Law of the Conservation of Investment Value".

afford into comparable terms. Therefore, we start with the BC of $t = 1$ which, for each state, is weighted using the adequate state price. Summing over all states afterwards delivers

$$\sum_{s=1}^{S} r_s \sum_{l=1}^{L} p_{ls} c_{ils} - \sum_{s=1}^{S} r_s \sum_{l=1}^{L} \sum_{j=1}^{J_l} [\theta_{ijl} \max(p_{ls} y_{jls} - b_{jl}, 0) + \alpha_{ijls}] - \sum_{s=1}^{S} r_s \sum_{m=1}^{M} x_{ms} z_{im} \leq 0.$$

Minor rearrangements yield

$$\sum_{l=1}^{L} \sum_{j=1}^{J_l} \left[\theta_{ijl} \sum_{s=1}^{S} r_s \max(p_{ls} y_{jls} - b_{jl}, 0) \right] \geq \sum_{s=1}^{S} r_s \sum_{l=1}^{L} p_{ls} c_{ils} - \sum_{l=1}^{L} \sum_{j=1}^{J_l} \sum_{s=1}^{S} r_s \alpha_{ijls} - \sum_{m=1}^{M} z_{im} \sum_{s=1}^{S} r_s x_{ms},$$

where the asset pricing equations (3) through (6) are applicable. To see how, note that in the above equation we have

$$\sum_{s=1}^{S} r_s \max(p_{ls} y_{jls} - b_{jl}, 0) = \sum_{s \in S'_{jl}} r_s (p_{ls} y_{jls} - b_{jl}) = v_{jl}$$

according to (3),

$$\sum_{s=1}^{S} r_s \alpha_{ijls} = \sum_{s \in S'_{jl}} r_s a_{ijl} + \sum_{s \notin S'_{jl}} r_s \frac{a_{ijl}}{b_{jl}} p_{ls} y_{jls} = a_{ijl} R_{jl}$$

due to (5) and

$$\sum_{s=1}^{S} r_s x_{ms} = q_m$$

immediately by (6). Thus, we know that

$$\sum_{l=1}^{L} \sum_{j=1}^{J_l} \theta_{ijl} v_{jl} \geq \sum_{s=1}^{S} r_s \sum_{l=1}^{L} p_{ls} c_{ils} - \sum_{l=1}^{L} \sum_{j=1}^{J_l} a_{ijl} R_{jl} - \sum_{m=1}^{M} q_m z_{im} \tag{8}$$

has to hold. To further work with the BC for $t = 0$, rewrite (1) as

$$p_0(c_{i0} - y_{i0}) + \sum_{l=1}^{L} \sum_{j=1}^{J_l} \left\{ -\bar{\theta}_{ijl} \left[v_{jl} - (p_0 k_{jl} - R_{jl} b_{jl}) \right] + R_{jl} a_{ijl} \right\} + \sum_{m=1}^{M} q_m z_{im} + \sum_{l=1}^{L} \sum_{j=1}^{J_l} \theta_{ijl} v_{jl} \leq 0, \tag{9}$$

where SV according to (7) is explicitly singled out and the left-hand value of (8) appears. As the latter is known to be no less than the RHS of (8), we can replace it with just that without violating the weak inequality in (9). After doing so, the inequality gravely simplifies to

$$p_0 c_{i0} + \sum_{s=1}^{S} r_s \sum_{l=1}^{L} p_{ls} c_{ils} \leq p_0 y_{i0} + \sum_{l=1}^{L} \sum_{j=1}^{J_l} \bar{\theta}_{ijl} SV_{jl}.$$

It thus becomes apparent that the upper limit of affordable “consumption value” increases with shareholder value SV_{jl} as long as $\bar{\theta}_{ijl} > 0$. Hence, in terms of consumption utility, each initial shareholder benefits from an increase in SV, which is in turn necessary to obtain shareholder unanimity with the goal of SV maximization. Without social responsibility concerns, it actually follows immediately. As can be seen in Arnold (2023, p. 74), it can also be guaranteed for utility functions displaying preferences that fit classification-based SRI while being independent of bond holdings. That is, we can assume for convenience that if firm (j, l) influences the utility of i, it does so via its market value v_{jl} scaled by her shareholdings θ_{ijl}. This implies a restriction toward non-consequentialist preferences. Alternatively, one can postulate that more than half of the shares of every firm are held by individuals that obtain no disutility from that k_{jl} and the resulting v_{jl} to ensure SV maximization. In the latter case, however, shareholder unanimity does not necessarily prevail.

The channel of investor voice is, thus, effectively shut down in the model at hand. While Broccardo et al. (2022) identify it as an effective way to make SRI become impactful when social responsibility is widespread, we continue to focus on the role portfolio formation can play.

Knowing that shareholders truly aim at maximizing SV is an important interim result. In what follows, this will serve as a cornerstone to analyze the effect of changing debt levels of the corporations both for themselves and the economy as a whole.

3.2 Optimizing shareholder value

Having shown that SV should indeed be maximized, the question arises of how exactly this is to be achieved. In other words, the vector of optimum capital inputs $\boldsymbol{k}^*$ needs to be determined. The SV of firm (j, l) in the no-bankruptcy-model is given by (cf. Arnold, 2023, p. 73)

$$v_{jl} - (p_0 k_{jl} - R b_{jl}), \tag{10}$$

where

$$v_{jl} = \sum_{s=1}^{S} r_s (p_{ls} y_{jls} - b_{jl}), j = 1, ..., J_l, l = 1, ..., L.$$

In this benchmark-version of the model, bankruptcy is not an issue. The SV-maximizing choice of k_{jl} obeys

$$\frac{d[v_{jl} - (p_0 k_{jl} - R b_{jl})]}{d k_{jl}} \stackrel{!}{=} 0.$$

As firms' outputs are determined by a (state-dependent) production function with capital as the only input, $y_{jls} = f_{jls}(k_{jl})$, we denote the derivative of output with respect to capital, i.e., the marginal product of capital, as $f'_{jls}(k_{jl})$. We immediately obtain the first-order condition (FOC) for SV-maximization as

$$\sum_{s=1}^{S} r_s p_{ls} f'_{jls}(k_{jl}) = p_0. \tag{11}$$

Moving on to the more general case, we cannot simply solve $\partial[v_{jl} - (p_0 k_{jl} - R_{jl} b_{jl})]/\partial k_{jl} \stackrel{!}{=} 0$ with v_{jl} given by (3) because the expression cannot be differentiated at those s where $b_{jl} = p_{ls} y_{jls}$. However, it is easy to show that (11) holds even in the presence of arbitrary risks of bankruptcy once we express SV as a function of economic fundamentals. In the general setup, SV is given by (7). Inserting the definitions of firm value v_{jl} from (3) and bond prices R_{jl} from (5), we get

$$SV_{jl} = \sum_{s \in S'_{jl}} r_s \left(p_{ls} y_{jls} - b_{jl}\right) - \left(p_0 k_{jl} - \sum_{s \in S'_{jl}} r_s b_{jl} + \sum_{s \notin S'_{jl}} r_s p_{ls} y_{jls}\right),$$

which boils down to shareholder value as a function of economic fundamentals rather than the capital structure:

$$SV_{jl} = \sum_{s=1}^{S} r_s p_{ls} y_{jls} - p_0 k_{jl}. \tag{12}$$

Differentiating (12) with respect to k_{jl} and solving for zero again delivers (11). The fact that we obtain identical FOCs with and without risk of bankruptcy has two important implications:

- From the fact that the FOC when bankruptcy is possible does not depend on the states in which it actually occurs we can infer that this condition indeed looks the same for any level of debt. Therefore, we need not concern ourselves with the more tedious analysis of discrete, that is, non-marginal differences followed in Subsection 4.2.

- If we encounter the same prices for goods and states as well as production technologies in two economies where bankruptcy happens in one and does not in the other, we can infer that those two economies will also have firms choose identical capital stocks $k^*_{jl}, j = 1, ..., J_l, l = 1, ..., L$.

These implications will serve as an important lemma in showing that allowing for bankruptcy does not matter at all for real firm decisions and individual consumption allocations.

Lemma 1: If state prices $r_s, s = 1, ..., S$, goods prices $p_0, p_{ls}, l = 1, ..., L, s = 1, ..., S$ and production technologies $f_{jls}(k_{jl})$ are the same in two economies with arbitrarily different levels of debt $b_{jl}, j = 1, ..., J_l, l = 1, ..., L$, the optimal choice of capital k^*_{jl} will be the same as well $\forall(j, l)$.

3.3 Modigliani-Miller theorem at the corporate level

Before employing any specific value for b_{jl}, we need to determine the range of values it may take on. Given that bond supply is a choice variable for firms where these firms themselves are owned by shareholders, the one central condition that needs to be satisfied is still SV maximization, now also with respect to b_{jl} rather than k_{jl}. The simplest approach that can be taken is to express SV via (12) and derive zero influence of b_{jl}. This is essentially the Modigliani-Miller theorem (MM) at the corporate level.

In order to obtain from (12) that

$$\frac{dSV_{jl}}{db_{jl}} = 0,$$

in other words, that MM at the corporate level always holds true, we have to show that $dk_{jl}/db_{jl} = 0$. This is done in Appendix A.1.

The spectatcularity of this result rests in the fact that zero influence of b_{jl} on SV_{jl} holds for changes in bond emissions of arbitrary (that is, even any non-marginal) magnitude. One can see this as the set of solvency states S'_{jl} does not appear in (12) at all.

Lemma 2: Shareholder value is fully determined by economic fundamentals. So the corporate finance version of MM holds for every single firm.

4 General equilibrium Modigliani-Miller theorems

This Section analyzes whether MM "at large" holds in the model at hand. For this to be the case, we need to obtain not only constant SVs, which were shown in Subsection 3.3, but also unaltered consumption levels c_{i0} at $t = 0$, and c_{ils} of all goods $l = 1, ..., L$ in all states of nature $s = 1, ..., S$ at $t = 1$, for all individuals $i = 1, ..., I$. From Lemma 2, we know that firms' capital stock choice is not influenced by any alteration in leverage. Thus, we can rest assured that unchanged consumption levels are in fact sufficient to conserve consumption utility levels $u_i(\boldsymbol{c_i^*}, \boldsymbol{k^*})\forall i$ even in the presence of externalities. Of course, this does not yet imply that overall utility $U_i(u_i(\boldsymbol{c_i}, \boldsymbol{k}), \boldsymbol{\theta_i}, \boldsymbol{a_i}, \boldsymbol{k}, \boldsymbol{v})$ is indeed unaffected. Thus, although the subsequent analysis can be considered as a proof of neutrality of debt levels in the no-SRI case, the resulting modification of portfolios may point towards non-neutrality if SRI is relevant. SRI is ignored for now and will be returned to in Section 5. We introduce changes in the bond supply of firms and proceed to check if they can be neutralized. Intuitively, the analysis is similar to Stiglitz's (1974) in his seminal proof of the theorem. In his words, we show that "individuals can exactly 'undo' any financial policy undertaken by the firm" (Stiglitz, 1974, p. 859).

Clearly, from the fact that shareholder value does not change, there should not be any direct effects of changing bond supply b_{jl} in a world of complete markets. There, via their initial shareholdings $\bar{\theta}_{ijl}$ and firm values v_{jl} of the leveraging firm (j, l), what consumers experience is a shift of consumption levels between time and/or states. When confronted with such a shift, individuals can simply buy or sell other assets that restore the former order. Things become less clear-cut for the more general case where bankruptcy is allowed to happen. There, changes in b_{jl} may also influence bond prices R_{jl} as well as the return on bonds α_{ijls}. Furthermore, creating new states with bankruptcy or shifting the pending repayments in those cases might impinge upon financial market completeness. MM would then not hold due to the fact that "bankruptcy changes the opportunity set facing a given individual" (Stiglitz, 1974, p. 862).

In order to allow for the most general perspective possible, we will analyze the effect of non-infinitesimal changes in bond supply Δb_{jl}. We skip dealing with marginal changes and instead simply note that the neutrality of "small" changes follows as a corollary from the neutrality of "large" changes.

The proofs in this and the following Section comment on financial market completeness. In particular, they rely on the subset of the state-space that is spanned by all assets to remain the same when moving from one version of the economy to another. This set is known as the *marketed subspace* (see Magill & Quinzii, 1996, p. 76). If that space is not of its maximum dimension S, markets are incomplete. All remarks on unaltered spanning opportunities may be skipped by readers prepared to employ the additional assumption of the entire state-space being spanned by zero net-supply securities $\boldsymbol{z}$.

4.1 General equilibrium MM: No bankruptcy

Arguing in favor of the validity of an economic MM theorem in the baseline model is straightforward: Given that markets are complete, in a world with no bankruptcy, the forced shift via additional debt issuings of firm (j, l) can be neutralized most simply via investing all issue proceeds back in bonds: It suffices for i to change bond holdings a_{ijl} of precisely that firm which increases its issuance from b_{jl} to $b_{jl} + \Delta b_{jl}$ by an appropriate share $\bar{\theta}_{ijl}\Delta b_{jl}$. The payoff profile of those bonds remains unchanged even if more of them are given out, directly via the assumption of no bankruptcy. Given that several firms change their debt levels from one version of the economy to the other, additional freedom in the choice of which corporation's assets to buy arises for the individual. Since no new consumption possibilities can arise due to Lemma 1, the restored $\boldsymbol{c_i^*}$ is indeed optimal $\forall i$.
We can assess neutralizability of debt emissions in a similar manner as Stiglitz (1974). To do so, start with the following notion of an equilibrium:

Definition 1: $\big((\boldsymbol{c_i^*}, \boldsymbol{\theta_i^*}, \boldsymbol{a_i^*}, \boldsymbol{z_i^*})_{i=1}^I, \boldsymbol{k^*}, \boldsymbol{p^*}, \boldsymbol{v^*}, \boldsymbol{R^*}, \boldsymbol{q^*}, \boldsymbol{b}\big)$ is an equilibrium of the stock-and-debt-economy (ESDE) if, given prices $\boldsymbol{p^*}, \boldsymbol{v^*}, \boldsymbol{R^*}, \boldsymbol{q^*}$ and bond supplies $\boldsymbol{b}$ such that $p_{ls}y_{jls} \geq b_{jl}, s = 1, ..., S, j = 1, ..., J_l, l = 1, ..., L$,

- Consumption $\boldsymbol{c_i^*}$ and portfolios $(\boldsymbol{\theta_i^*}, \boldsymbol{a_i^*}, \boldsymbol{z_i^*})$ maximize consumption utility u_i for all $i = 1, ..., I$.
- Capital choices $\boldsymbol{k^*}$ maximize SV for all $j = 1, ..., J_l$, $l = 1, ..., L$.
- Markets clear.

Assume we are at an equilibrium $\big((\boldsymbol{c_i^*}, \boldsymbol{\theta_i^*}, \boldsymbol{a_i^*}, \boldsymbol{z_i^*})_{i=1}^I, \boldsymbol{k^*}, \boldsymbol{p^*}, \boldsymbol{v^*}, \boldsymbol{R^*}, \boldsymbol{q^*}, \boldsymbol{b}\big)$. Now let all firms vary their bond issuances by some Δb_{jl}, potentially taking the value 0 for some of them. The following theorem states that buying new bonds for the price of the additionally accruing issue proceeds restores optimal consumption $\boldsymbol{c_i^*} \forall i$ and, thus, equilibrium.[10]

Theorem 1: If $\left((\boldsymbol{c_i^*}, \boldsymbol{\theta_i^*}, \boldsymbol{a_i^*}, \boldsymbol{z_i^*})_{i=1}^I, \boldsymbol{k^*}, \boldsymbol{p^*}, \boldsymbol{v}^*, R^*, \boldsymbol{q^*}, \boldsymbol{b}\right)$ is an ESDE, then $\left((\boldsymbol{c_i^*}, \boldsymbol{\theta_i^*}, \boldsymbol{a_i^*} + \boldsymbol{\Delta a_i^*}, \boldsymbol{z_i^*})_{i=1}^I, \boldsymbol{k^*}, \boldsymbol{p^*}, \boldsymbol{v}^{**}, R^*, \boldsymbol{q^*}, \boldsymbol{b} + \boldsymbol{\Delta b}\right)$, where $\boldsymbol{\Delta a_i^*} = \boldsymbol{\theta_i^*} \odot \boldsymbol{\Delta b}$, is also an ESDE.

Proof: Let $\boldsymbol{p^*}$, $\boldsymbol{v^*}$, $\boldsymbol{r^*}$ and $\boldsymbol{q^*}$ be the same for both versions of the stock-and-debt-economy. Optimum capital choices are $\boldsymbol{k^*}$ in both economies due to Lemma 1. Thus, given the spanned state-space is unaltered, state prices $\boldsymbol{r^*}$ are, in fact, also the same,[11] which, of course, also extends to their sum R^*. Firm values change from $\boldsymbol{v^*}$ to $\boldsymbol{v^{**}}$ such that a constant SV is warranted.

From the fact that $\left((\boldsymbol{c_i^*}, \boldsymbol{\theta_i^*}, \boldsymbol{a_i^*}, \boldsymbol{z_i^*})_{i=1}^I, \boldsymbol{k^*}, \boldsymbol{p^*}, \boldsymbol{v}^*, R^*, \boldsymbol{q^*}, \boldsymbol{b}\right)$ is an equilibrium, we know that (1) has to hold with equality due to strictly positive marginal utility of consumption. Note that, as there is never any bankruptcy, we can replace R_{jl} with R for now. It suffices to show that the induced reallocation resulting from $\boldsymbol{\Delta b}$ can be neutralized. If we let individuals change nothing but their bond holdings, then $\Delta c_{i0} = 0$ necessitates

$$\sum_{l=1}^{L}\sum_{j=1}^{J_l} \theta_{ijl}^* R^* \Delta b_{jl} = \sum_{l=1}^{L}\sum_{j=1}^{J_l} R^* \Delta a_{ijl}.$$

So we know that total new bond purchases of i must satisfy

$$\sum_{l=1}^{L}\sum_{j=1}^{J_l} \Delta a_{ijl} = \sum_{l=1}^{L}\sum_{j=1}^{J_l} \theta_{ijl}^* \Delta b_{jl} \tag{13}$$

We are further interested in the change occurring in (2). Unsurprisingly, it turns out to be state-independent and is given by

$$\sum_{l=1}^{L}\sum_{j=1}^{J_l} \left[\theta_{ijl}^*(-\Delta b_{jl}) + \Delta a_{ijl}\right].$$

[10] Below, $\odot$ denotes the Hadamard product. The Hadamard product is defined for two vectors of the same dimension. The n-th element of this product is simply the product of the two multiplied vectors' n-th elements.

[11] Constant production decisions imply that the scarcity of consumption in all states remains unaltered. So goods prices, instruments for reflection of this scarcity, must also be identical.

It can be solved for 0 to obtain that $\Delta c_{ils} = 0$ is also possible as long as

$$\sum_{l=1}^{L}\sum_{j=1}^{J_l} \Delta a_{ijl} = \sum_{l=1}^{L}\sum_{j=1}^{J_l} \theta^*_{ijl} \Delta b_{jl},$$

which is exactly (13) again. Thus, changing bond purchases according to precisely that equation will restore the former equilibrium allocation.

Furthermore, as argued above, no additional consumption possibilities arise. Therefore, no desire to change consumption can have arisen as well.[12]

It remains to show that markets still clear because else our new set of choice variables cannot be an equilibrium. To keep things simple, consider first the special case where all individuals $i = 1, ..., I$ choose $\Delta a_{ijl} = \theta^*_{ijl} \Delta b_{jl}, j = 1, ..., J_l, l = 1, ..., L$. Stacking over j and l delivers $\boldsymbol{\Delta a_i^*} = \boldsymbol{\theta_i^*} \odot \boldsymbol{\Delta b}$, which is exactly the formulation in the above theorem. With it, we can easily see that total demand for bonds of each firm (j, l) equals

$$\sum_{i=1}^{I} a_{ijl} + \Delta a_{ijl} = \sum_{i=1}^{I} a_{ijl} + \sum_{i=1}^{I} \Delta a_{ijl} = b_{jl} + \Delta b_{jl} \sum_{i=1}^{I} \theta^*_{ijl},$$

which, due to stock market clearing $\sum_{i=1}^{I} \theta^*_{ijl} = 1, j = 1, ..., J_l, l = 1, ..., L$ of our starting point equilibrium, is just total bond supply $b_{jl} + \Delta b_{jl}$ of this very firm. Thus, markets clear. Without the specification we could so far merely conclude that additional demand and additional supply satisfy

$$\sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{j=1}^{J_l} \Delta a_{ijl} = \sum_{l=1}^{L}\sum_{j=1}^{J_l} \Delta b_{jl}.$$

Of course, this implies that market clearing is possible. The division of bond holdings that individuals have from firms simply needs to be distributed accordingly. In general (as long as $\sum_{l=1}^{L} J_l > 1$), there will be infinitely many ways of achieving

[12] This last part proves that individuals indeed choose to neutralize the arisen consumption changes. Another way of seeing this is to note that nothing prevents us from choosing $\Delta b_{jl} < 0$ or, equivalently, to start at $\big((\boldsymbol{c_i^*}, \boldsymbol{\theta_i^*}, \boldsymbol{a_i^*} + \boldsymbol{\Delta a_i^*}, \boldsymbol{z_i^*})_{i=1}^{I}, \boldsymbol{k^*}, \boldsymbol{p^*}, \boldsymbol{v^*}, \boldsymbol{R^*}, \boldsymbol{q^*}, \boldsymbol{b} + \boldsymbol{\Delta b}\big)$ and move on to $\big((\boldsymbol{c_i^*}, \boldsymbol{\theta_i^*}, \boldsymbol{a_i^*}, \boldsymbol{z_i^*})_{i=1}^{I}, \boldsymbol{k^*}, \boldsymbol{p^*}, \boldsymbol{v^*}, \boldsymbol{R^*}, \boldsymbol{q^*}, \boldsymbol{b}\big)$, thereby showing the converse of what has been established.

this. At least one way is definitely feasible, namely the above special case. We conclude that market clearing is always obtainable. $\mathcal{QED}$

One objection that could still be made is that we have so far not commented on financial market completeness. Indeed, stock payoffs from firm (j, l) change in a fundamental way (that is, additively rather than multiplicatively) when increasing b_{jl}, namely by some multiple of the unit vector.[13] Notably, we have not assumed the entire state-space to be spanned, that is, merely the spanned subset of it must remain constant. This is always the case. The intuition for that lies within the above neutralization strategy: There exists an asset which can always compensate the change in the payoff structure of any stock, namely any bond (whose payoff is just that very unit vector). In other words, although stock payoffs change by construction (a point already noted by Rubinstein, 2003, p. 11), the spanned subset of the state-space is unaltered.

4.2 General equilibrium MM: Bankruptcy in an arbitrary subset of states

While proving neutralizability in the simple case where there was no risk of bankruptcy was quite straightforward, things get cumbersome once we look at the more general case. Here, rather than showing a concise strategy with which to negate the impact of additional debt issuance, we have to confine ourselves with showing "wealth neutrality" of changes Δb_{jl}. That is, we restrict ourselves to an analysis of the resulting change in total present value of consumption $p_0 c_{i0} + \sum_{s=1}^{S} r_s p_{ls} c_{ils}, i = 1, ..., I$. Given financial market completeness, each i can finance the previous $\boldsymbol{c}_i^*$ and nothing better as long as this present value does not change for her. Start again at an equilibrium:

Definition 2: $\left((\boldsymbol{c}_i^*, \boldsymbol{\theta}_i^*, \boldsymbol{a}_i^*, \boldsymbol{z}_i^*)_{i=1}^{I}, \boldsymbol{k}^*, \boldsymbol{p}^*, \boldsymbol{v}^*, \boldsymbol{R}^*, \boldsymbol{q}^*, \boldsymbol{b}\right)$ is an equilibrium of the stock-and-debt-economy with over-indebtedness (ESDO) if, given prices

[13] Of course, this additive change may become more complex once we allow for bankruptcy to happen.

$\boldsymbol{p}^*, \boldsymbol{v}^*, \boldsymbol{R}^*, \boldsymbol{q}^*$ and bond supplies $\boldsymbol{b}$ such that $p_{ls}y_{jls} < b_{jl}$ for some $s = 1, ..., S, j = 1, ..., J_l, l = 1, ..., L$,

- Consumption $\boldsymbol{c}_i^*$ and portfolios $(\boldsymbol{\theta}_i^*, \boldsymbol{a}_i^*, \boldsymbol{z}_i^*)$ maximize consumption utility u_i for all $i = 1, ..., I$.
- Capital choices $\boldsymbol{k}^*$ maximize SV for all $j = 1, ..., J_l$, $l = 1, ..., L$.
- Markets clear.

The definition of an ESDO resembles the baseline equilibrium with no over-indebtedness save for the latter being required for some firm in some state. With complete markets, debt levels may be altered arbitrarily:

Theorem 2: If $\left((\boldsymbol{c}_i^*, \boldsymbol{\theta}_i^*, \boldsymbol{a}_i^*, \boldsymbol{z}_i^*)_{i=1}^I, \boldsymbol{k}^*, \boldsymbol{p}^*, \boldsymbol{v}^*, \boldsymbol{R}^*, \boldsymbol{q}^*, \boldsymbol{b}\right)$ is an ESDO, then there exists some $\left((\boldsymbol{c}_i^*, \boldsymbol{\theta}_i^{**}, \boldsymbol{a}_i^{**}, \boldsymbol{z}_i^{**})_{i=1}^I, \boldsymbol{k}^*, \boldsymbol{p}^*, \boldsymbol{v}^{**}, \boldsymbol{R}^{**}, \boldsymbol{q}^*, \boldsymbol{b} + \boldsymbol{\Delta b}\right)$ that is also an ESDO, where $\boldsymbol{\Delta b} > \boldsymbol{0}$ must not change the marketed subspace.

Proof: Let $\boldsymbol{p}^*$, $\boldsymbol{v}^*$, $\boldsymbol{r}^*$ and $\boldsymbol{q}^*$ be the same for both versions of the economy with over-indebtedness. Optimum capital choices $\boldsymbol{k}^*$ are the same in both economies due to Lemma 1. Thus, state prices $\boldsymbol{r}^*$ are, in fact, also the same. Bond prices change from $\boldsymbol{R}^*$ to $\boldsymbol{R}^{**}$ due to the variation in debt levels, but result from these unaltered state prices. The change in firm values from $\boldsymbol{v}^*$ to $\boldsymbol{v}^{**}$ ensures constant shareholder values.

Note that we implicitly assume unaltered spanning opportunities as well, that is, the marketed subspace has to stay the same. Otherwise, Rubinstein (2003, pp. 9-10) notes that state prices may change, thus reflecting not only the scarcity of consumption in certain states, but also the impossibility of some targeted consumption relocations.

To compare optimum levels of choice variables, we calculate differences in budget constraints, starting at $t = 0$ as taken from (9). A proper derivation is outsourced to Appendix A.2. We obtain

$$\sum_{l=1}^{L}\sum_{j=1}^{J_l}\left\{-\theta_{ijl}\left[\sum_{s\in S''_{jl}} r_s\Delta b_{jl} + \sum_{s\in S'_{jl}\setminus S''_{jl}} r_s(p_{ls}y_{jls}-b_{jl})\right] + \bar{\theta}_{ijl}\Delta(SV_{jl}) + \right.$$
$$\left. a_{ijl}\left[-\sum_{s\in S'_{jl}\setminus S''_{jl}} r_s + \sum_{s\notin S''_{jl}} r_s\frac{p_{ls}y_{jls}}{b_{jl}+\Delta b_{jl}} - \sum_{s\notin S'_{jl}} r_s\frac{p_{ls}y_{jls}}{b_{jl}}\right]\right\}, \qquad (14)$$

where we distinguish states without bankruptcy before debt has risen, S'_{jl}, and afterwards, S''_{jl}.[14] $\Delta(SV_{jl})$ refers to the change in shareholder value which can be taken from Lemma 2 to be zero. The remainder of (14) may be unequal to zero, plainly reflecting the fact that different levels of corporate debt force reallocations of consumption for those individuals that are in some way invested into this firm.[15] What remains to be established is how this reallocation is compensated. To do so, write down changes in the $t=1$-BC as given by (2) as

$$-\sum_{l=1}^{L}\sum_{j=1}^{J_l}\left\{\theta_{ijl}[\max(p_{ls}y_{jls}-b_{jl}-\Delta b_{jl},0)-\max(p_{ls}y_{jls}-b_{jl},0)] + \Delta\alpha_{ijls}\right\},$$
$$s=1,...,S, \qquad (15)$$

where the expressions for maximum operators and, correspondingly, their differences, as well as $\Delta\alpha_{ijls}$ depend on the exact kind of state we are looking at. A more detailed description of the following considerations is delegated to Appendix A.3.

It is sufficient to subdivide the set of all states into three proper subsets per

[14] If we wish to compare different versions of the economy in terms of outstanding debt of all firms, we have to be careful in notation. To avoid any potential confusion, firm-specific variables that refer to the case of increased debt will always be referred to using a double-prime indexation (i.e., we distinguish sets of solvency S''_{jl} versus S'_{jl}, but also, e.g., firm values v''_{jl} versus v_{jl}).

[15] Note that, since in an equilibrium all asset markets must clear, there is always at least one individual who is actually being influenced by the new debt level of (j,l) since the latter's shares and other assets must all belong to someone.

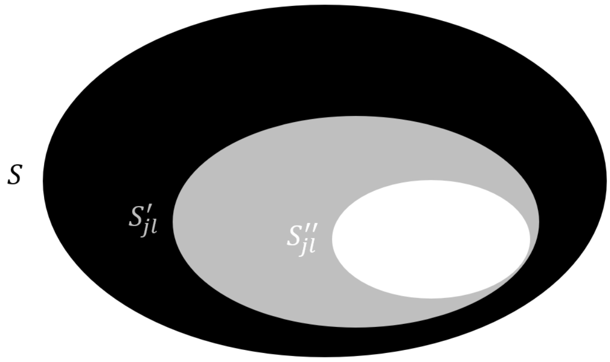

Figure 1: Interrelations of the sets of states with solvency

firm.[16] Firstly, we begin with those states that display financial solvency of this (j, l) both before and after raising additional outside capital. Consider the two alternative solvency sets. We know that $S^{'}_{jl}$ is the set of states where (j, l) does not end up bankrupt with some level of debt b_{jl}. All the states in $S^{''}_{jl}$, on the other hand, show solvency of this very firm with debt raised by Δb_{jl}, which may be fewer in number and can never be more under our assumption of $\Delta b_{jl} > 0 \forall (j, l)$. Thus, we clearly obtain $S^{''}_{jl} \subseteq S^{'}_{jl}$. Figure 1 illustrates this point.

The sets of solvency states before and after debt was raised can consequently be summarized as $S^{''}_{jl}$ and for them, the budget restriction (2) or, equivalently, the possible consumption expenditures $\sum_{l=1}^{L} p_{ls} c_{ils}$, change by

$$\sum_{l=1}^{L} \sum_{j=1}^{J_l} \theta_{ijl} \Delta b_{jl}.$$

Next, consider states for (j, l) where solvency was not even given before debt was raised. For these $s \in S \setminus S^{'}_{jl}$ or, equivalently, $s \notin S^{'}_{jl}$, (15) takes the form

$$-\sum_{l=1}^{L} \sum_{j=1}^{J_l} p_{ls} y_{jls} \left(\frac{a_{ijl}}{b_{jl} + \Delta b_{jl}} - \frac{a_{ijl}}{b_{jl}} \right).$$

[16]The resulting subsets are always proper subsets of S given the firm goes bankrupt at least in some state given the lower debt level b_{jl}. Either of the following subsets may constitute the empty set.

Finally, we have to deal with states where the surge in debt was causal for over-indebtedness. This set is given by $S'_{jl} \setminus S''_{jl}$ and its concrete form of (15) is

$$-\sum_{l=1}^{L}\sum_{j=1}^{J_l}(-p_{ls}y_{jls} + b_{jl}) + \frac{a_{ijl}p_{ls}y_{jls}}{b_{jl} + \Delta b_{jl}} - a_{ijl}.$$

It can be readily seen that multiplying the above three types of changes by r_s and afterwards summing them over the corresponding sets of states per firm yields exactly the negative equivalent of (14). Hence, the net present value of possible new consumption expenditures is indeed equal to zero. Thus, no wealth effects arise, and as long as financial market completeness is not an issue, consumption profiles are permitted to remain the same once again.

The proof still lacks a rigorous derivation of market clearing. Without specifying the exact new portfolios $(\boldsymbol{\theta_i^{**}}, \boldsymbol{a_i^{**}}, \boldsymbol{z_i^{**}})_{i=1}^{I}$, this canoot be done. Hence, we resort to an example. The intuition for why markets must still clear is obvious:

In the new equilibrium, individuals buy the same consumption profiles $(\boldsymbol{c_i^*})_{i=1}^{I}$ as in the old one. Claims to firms' proceeds $p_{ls}y_{jls}$ are sliced differently among the various kinds of stakeholders, but all those claims still have to add up to the very same proceeds. Asset prices, as given by firm values v_{jl} for stocks and by R_{jl} for bonds, may change accordingly, still following suit after state prices r_s. Individuals may need to buy more assets than before, but save enough on all of their asset purchases such that financing the same consumption vectors is exactly feasible.

The following considerations sketch the final part of the proof for the special case where a safe bond is not required for complete financial markets:

Assume first an economy with no bonds, $b_{jl} = 0, j = 1, ..., J_L, l = 1, ..., L$. Let the allocation and prices from $\left((\boldsymbol{c_i^*}, \boldsymbol{\theta_i^*}, \boldsymbol{z_i^*})_{i=1}^{I}, \boldsymbol{k^*}, \boldsymbol{p^*}, \boldsymbol{v^*}, \boldsymbol{q^*}\right)$ constitute an equilibrium of the stock market economy (ESME, defined analogously to Definition 1 with $b_{jl} = 0, j = 1, ..., J_l, l = 1, ..., L$) with complete markets.[17] Then the in-

[17]Here, one may also assume that financial markets were incomplete in the ESME and continue to be incomplete in the same dimensions in the ESDE.

troduction of bonds leads to an ESDE $\big((\boldsymbol{c_i^*}, \boldsymbol{\theta_i^*}, \boldsymbol{a_i^*}, \boldsymbol{z_i^*})_{i=1}^I, \boldsymbol{k^*}, \boldsymbol{p^*}, \boldsymbol{v^*}, R^*, \boldsymbol{q^*}, \boldsymbol{b}\big)$ where all bond issuance effects are neutralized by investors, that is, $a_{ijl} = \theta_{ijl} b_{jl}$.[18] A notable implication of the starting point featuring no bonds is that the safe bond must not be required for financial market completeness. Now increase bond issuances to an even higher level such that the ESDE turns into an ESDO. This equilibrium, $\big((\boldsymbol{c_i^*}, \boldsymbol{\theta_i^*}, \boldsymbol{a_i^*} + \boldsymbol{\Delta a_i^*}, \boldsymbol{z_i^*})_{i=1}^I, \boldsymbol{k^*}, \boldsymbol{p^*}, \boldsymbol{v^*}, \boldsymbol{R^*}, \boldsymbol{q^*}, \boldsymbol{b} + \boldsymbol{\Delta b}\big)$, has the same bond purchase differentials as the one in Theorem 1, $\boldsymbol{\Delta a_i^*} = \boldsymbol{\theta_i^*} \odot \boldsymbol{\Delta b}$. Since $\theta_{ijl}^*(b_{jl} + \Delta b_{jl}) = a_{ijl}^* + \Delta a_{ijl}^*$ follows immediately from $\theta_{ijl}^* b_{jl} = a_{ijl}^*$, $i = 1, ..., I, j = 1, ..., J_l, l = 1, ..., L$, we obtain both identical consumption levels $\boldsymbol{c_i^*}$ and unimpeded market clearing. The fact that Δb may be set arbitrarily relates any ESDO to the ESDE above and, hence, every conceivable ESDO to one another, which completes the proof. $\mathcal{QED}$

5 MM and SRI

What has been widely left out so far is the role of SRI in the context of varied debt. Solid conclusions are near impossible to reach here as we can assume virtually anything on how utility functions take account of the different valuations of assets they depend on. An intuitive way to proceed here would be to assume that i gains utility from the value of financial claims she holds towards any firm (j, l), but not from the number or kind of assets purchased from it. It then becomes apparent that the same claims towards all firms remain affordable without additional costs or benefits, such that a naïve neutralization strategy ensures neutrality of SRI.

Generally speaking, note that, provided that satiation in SRI holds for our reference equilibrium, individual utility improvements are impossible to be made. This

[18]See Stiglitz (1974) on how any change in the debt-equity ratio can be neutralized by investors given complete markets with no risk of bankruptcy. It is easy to verify that the BCs (1) and (2) without bankruptcy collapse to versions entirely without bonds, that is, $a_{ijl} = b_{jl} = 0, i = 1, ..., I, j = 1, ..., J_l, l = 1, ..., L$ if we set $a_{ijl} = \theta_{ijl} b_{jl}$.

is due to the fact that, while (j, l) can increase the number of bonds they issue, the same holds true for any individual via short sales. Since the latter do not compromise the number of solvency states S'_{jl} for that firm, they are the superior means of increasing bond supply and will in fact have already been performed to the optimum extent.

5.1 Neutrality of SRI with over-indebtedness

Carrying forth the logic from the final step in the proof of Theorem 2, we wish to relate an equilibrium with over-indebtedness and SRI (ESRIO) to an ESDO.

Definition 3: $\big((\boldsymbol{c_i^*}, \boldsymbol{\theta_i^*}, \boldsymbol{a_i^*}, \boldsymbol{z_i^*})_{i=1}^I, \boldsymbol{k^*}, \boldsymbol{p^*}, \boldsymbol{v^*}, \boldsymbol{R^*}, \boldsymbol{q^*}, \boldsymbol{b}\big)$ is an equilibrium of the stock-and-debt-economy with socially responsible investing and over-indebtedness (ESRIO) if, given prices $\boldsymbol{p^*}, \boldsymbol{v^*}, \boldsymbol{R^*}, \boldsymbol{q^*}$ and bond supplies $\boldsymbol{b}$ such that $p_{ls}y_{jls} < b_{jl}$ for some $s = 1, ..., S, j = 1, ..., J_l, l = 1, ..., L$,

- Consumption $\boldsymbol{c_i^*}$ and portfolios $(\boldsymbol{\theta_i^*}, \boldsymbol{a_i^*}, \boldsymbol{z_i^*})$ maximize overall utility U_i for all $i = 1, ..., I$.
- Capital choices $\boldsymbol{k^*}$ maximize SV for all $j = 1, ..., J_l$, $l = 1, ..., L$.
- Markets clear.

More precisely, we want to show that every ESRIO is an ESDO and that for every ESDO, there exists an equivalent ESRIO with the same consumption choices by all individuals. This is the same step carried out by Arnold (2023), where an equilibrium with SRI (ESRI) is related to an ESDE.

Theorem 3: Let $\big((\boldsymbol{c_i^*}, \boldsymbol{\theta_i^*}, \boldsymbol{a_i^*}, \boldsymbol{z_i^*})_{i=1}^I, \boldsymbol{k^*}, \boldsymbol{p^*}, \boldsymbol{v^*}, \boldsymbol{R^*}, \boldsymbol{q^*}, \boldsymbol{b}\big)$ be an ESRIO, where $\theta_{ijl}^* b_{jl} = a_{ijl}^*$, $i = 1, ..., I, j = 1, ..., J_l, l = 1, ..., L$. Then it is also an ESDO.

Proof: The proof follows that from Arnold's (2023) Theorem 1 (p. 90) verbatim when replacing R^* with $\boldsymbol{R^*}$. $\mathcal{QED}$

Note that Theorem 3 requires $\theta_{ijl}^* b_{jl} = a_{ijl}^*$ to be utility-maximizing for every

$i = 1, ..., I$. This assumption is not innocuous as it restricts the permitted set of utility functions. Utility has to be either dependent on financial claims rather than asset numbers, as sketched in the introductory part of this Section, or independent of bond holdings altogether. The latter may seem like a stark restriction. However, it was, in fact, precisely the requirements one has to employ in order to arrive at shareholder unanimity in the presence of SRI (see also the discussion at the end of Subsection 3.1 on the assumptions made in Arnold, 2023, p. 74). Hence, we can conclude that the assumptions on utility necessary to obtain neutrality of the abolishment of SRI are no stronger with over-indebtedness than they are without.

Given the specific bond holding structure, even the converse of Theorem 3, that is, neutrality of the introduction of SRI when there is over-indebtedness, goes through unmodified:

Theorem 4: Assume $\left((\boldsymbol{c}_i^*, \boldsymbol{\theta}_i^*, \boldsymbol{a}_i^*, \boldsymbol{z}_i^*)_{i=1}^I, \boldsymbol{k}^*, \boldsymbol{p}^*, \boldsymbol{v}^*, \boldsymbol{R}^*, \boldsymbol{q}^*, \boldsymbol{b}\right)$ is an ESDO, where $\theta_{ijl}^* b_{jl} = a_{ijl}^*$, $i = 1, ..., I, j = 1, ..., J_l, l = 1, ..., L$. Then there exists an ESRIO $\left((\boldsymbol{c}_i^*, \boldsymbol{\theta}_i^{**}, \boldsymbol{a}_i^{**}, \boldsymbol{z}_i^{**})_{i=1}^I, \boldsymbol{k}^*, \boldsymbol{p}^*, \boldsymbol{v}^*, \boldsymbol{R}^*, \boldsymbol{q}^*, \boldsymbol{b}\right)$.

Proof: The proof follows that from Arnold's (2023) Theorem 2 (p. 90) verbatim when replacing R^* with $\boldsymbol{R}^*$. $\mathcal{QED}$

Theorems 3 and 4 taken together state that SRI, referring to both its abolishment and its introduction, respectively, is neutral, that is, does not affect consumption or production choices in equilibrium. What remains to be commented on is the associated notion of market completeness. The criterion used by Arnold (2023) is *SWANS*. However, it can be weakened to requiring a constant marketed subspace via assets they are indifferent towards for all individuals. Indeed, as our starting point was an ESME with complete markets, safe bonds cannot have impacted financial market completeness (see also Theorem 2). That does not imply, however, that no bond contributes to market completeness in the ESDO-economy. In fact, it may well be the case that stocks and securities alone span the state-space no more after some firms over-indebt themselves. But these blemishes are

$$\begin{tikzcd}
\text{ESME} \arrow[r] & \text{ESDE} \arrow[d] & \text{ESRI} \arrow[l] \\
 & \text{ESDO} & \text{ESRIO} \arrow[l]
\end{tikzcd}$$

Figure 2: Equilibrium relations

automatically corrected via financial engineering on behalf of no longer risk-free bonds given their precise holding level.

So neutrality of SRI holds even in the presence of over-indebtedness. The set of equilibria this result applies to seems rather limited due to the upheld assumption of $\theta^*_{ijl} b_{jl} = a^*_{ijl}$. However, note that, instead of searching for an ESRIO, we could have simply shown any $\big((\boldsymbol{c}^*_{\boldsymbol{i}}, \boldsymbol{\theta}'_{\boldsymbol{i}}, \boldsymbol{a}'_{\boldsymbol{i}}, \boldsymbol{z}'_{\boldsymbol{i}})_{i=1}^I, \boldsymbol{k}^*, \boldsymbol{p}^*, \boldsymbol{v}^*, \boldsymbol{R}^*, \boldsymbol{q}^*, \boldsymbol{b}\big)$ to be another ESDO where we arbitrarily assign payoff-consistent portfolios to all i except one, where the latter is then tasked with guaranteeing market clearing and turns out to obtain the same payoffs as before, too (as is the case for the neutral i in Arnold's 2023 proof). So every ESDO relates back to the basal one and, thus, to the ESRIO. The logical chains relating equilibria are depicted in Figure 2. It can be interpreted in the following way: An equilibrium from the arrow's starting point has an equilibrium with identical claims to all firms by all individuals at the arrow's endpoint. In the counterdirection, the Figure indicates that an equilibrium with identical consumption vectors for each i exists, but portfolios possibly need to be shifted.

Notably, the allocation $(\boldsymbol{c}^*_{\boldsymbol{i}}, \boldsymbol{k}^*)$ is the same across all equilibria, indicating irrelevance of both debt levels and SRI. So in the SRI-augmented AD-GE-model, MM holds, SRI-neutrality holds and, consequently, the combination of both holds as well.

Figure 2 also reveals that an ESRI is always a special case of an ESDE, and that an

ESRIO is always a special case of an ESDO. Thus, linking the ESRI and the ESRIO to each other also necessitates that the restrictions on asset payoff structures implied by a constant marketed supspace from the respective proofs separately also hold simultaneously. This assumption is not innocuous. It says that if a stock or bond has become vital for the achieved marketed subspace when moving from an ESDE to an ESDO, every individual must be indifferent toward that asset. Else, SRI would proscribe a precise holding level of it for some individual which may run counter to the amount required to finance $\boldsymbol{c}_i^*$ in the ESRIO, but not in the ESRI. The converse holds for an asset that has become obsolete for the marketed subspace by new debt levels: it may prevent the establishment of $\boldsymbol{c}_i^*$ in an ESRI, but not in an ESRIO.[19]

5.2 Over-indebtedness as a means to market completion

After what has been established, it is clear that the only channel through which over-indebtedness could influence utility is by changing the marketed subspace. Consecutively, if financial markets were complete with some debt levels $\boldsymbol{b}$, no Pareto-improvement is possible, even if debt levels differ. The converse argument (see also the discussion in Section 2), however, may also hold: given financial markets were incomplete under $\boldsymbol{b}$, a Pareto-improvement could be achieved by altered debt levels if that enhances financial market completeness. The example provided below is an application of this idea.

Consider an economy where three firms ($J_1 = 3$) produce the single $t = 1$-consumption good, $L = 1$. Their payoffs for the $S = 3$ states at $t = 1$ are $p_1 y_{11} = p_1 y_{21} = (3, 2, 1)$ and $p_1 y_{31} = (1, 2, 1)$ where we stack states in vector notation. Firm 2 issues one bond, $b_{21} = 1$, with payoff $(1, 1, 1)$. Consequently, the payoff on all stocks of firm 2 is $p_1 y_{21} - b_{21} = (2, 1, 0)$. For now, none of

[19] A plausible special case where even (dis-)liked bonds may contribute to the marketed subspace is if utilities depend on asset holdings only in relation to the financial claims implied therein, which was already noted at the beginning of this Section.

the other firms emit debt. Without SRI, the financial market is complete as the stocks of firm 1 together with the riskless bond and the stocks of firm 3 span the state-space. That is, one can construct ADSs for each state by

- $s = 1$: buying 50% of the shares of firm 1 while shorting those of firm 3 by the same amount,
- $s = 2$: buying 100% of the shares of firm 3 while shorting the bond and
- $s = 3$: buying 2 bonds while shorting firms 1 and 3 by 50% each.

When we introduce SRI as firm 3 being negatively screened by some i, however, the picture changes. $SWANS$-completeness of financial markets then necessitates the state-space to be spanned by assets of firms 1 and 2 alone (i.e., their respective shares and bond issuances). However, the synthesization of an $s = 2$-ADS is now simply impossible. The newly created difference between firms 1 and 2, which stems from firm 2 issuing a bond, did not create any newly available payment structures. If, on the other hand, firm 1 raises its bond issuance to $b_{11} = 2$, there are now four different assets, three of which together allow i to construct that ADS. Investing directly into firm 1 delivers $p_1 y_{11} - b_{11} = (1, 0, 0)$ while its bond pays off $(1, 1, 0.5)$. So i simply has to buy the two bonds issued by firm 1 and sell both a 100%-share of firm 1 and the bond of firm 2 short. Equivalently, she could buy all firm-2-stocks and sell double of the firm-1-stocks short. As the ADS for $s = 1$ is just the firm-1-stock, and that for $s = 3$ can be obtained from two firm-2-bonds long with two firm-1-bonds short, the state-space is spanned. Hence, over-indebtedness of firm 1 has achieved financial market completeness in the stricter sense of $SWANS$, that is, with SRI.

6 Discussion and Outlook

Frequently, it is claimed that the Modigliani-Miller theorem does not hold in the presence of bankruptcy risks. In a standard general equilibrium model, this is entirely due to the fact that the marketed subspace may change. The modeling framework of Arnold (2023) augments the general equilibrium model due to Arrow (1964) and Debreu (1959) in order to allow for an integration of Socially Responsible Investing into it. We show that even in this generalized framework, allowing firms to go bankrupt does not have any real effects: shareholder value is unaltered and consumption possibilities remain the same as well. What we need to employ in order to reach this stark conclusion is a stronger notion of complete markets which must not be hampered via firms going bankrupt (at all instead of never or, generally speaking, more frequently). Notably, market completeness itself is actually not required as long as the kinds of market imperfections remain identical. Without over-indebtedness, this even is an automatism. Such an assumption can easily be extended to include the presence of SRI uncritically. However, in that process, the assumption becomes comparatively starker as SRI restricts the amount of assets that contribute to a complete market for some individuals.

Our analysis comes with some drawbacks, the most salient of which is the maintained assumption of complete markets (or, more generally, constance of the marketed subspace). As Rubinstein (2003) argues, state prices may change in response to altered debt levels if this act creates new securities or destroys existing ones, i.e., if the marketed subspace changes. If the latter happens, however, there may be benefits from over-indebtedness as it can, in fact, add marketed dimensions.

Another caveat is that we cannot make the distinction introduced by Green & Roth (2025) between *values-aligned* and *impact-aligned* investors, that is, people seeking to invest in firms based on whether these do good or can do more good with additional financing, respectively. The reason for this impossibility is that the general equilibrium structure and the inherrent MM conclusion leave no leeway for additional project financing via bonds. So even if there is *impact-alignment*, it cannot affect equilibrium.

We have continuously assumed constant prices of both goods and states. There

are good reasons to believe in those two assumptions for the model as it stands, due to unaltered production quantities and financial market completeness, respectively. A caveat applies, however, once we take social preferences beyond portfolio construction into account: a responsible individual may not just shirk a disliked firm's assets, but also its products, thus possibly affecting prices via the goods demand channel. This idea is pursued by Hakenes & Schliephake (2024), who find that the combination of SRI with Socially Responsible Consumption leads to constant prices and considerable reductions of externalities.
It becomes apparent that SRI on its own can, at best, have limited effects on real economic activity. This implies that one cannot simply unload one's conscience onto the fact that the investments undertaken stem from responsible firms. It is either consumption or the factual act of making available new financial means to sound corporations that affect the environment, not trading.

A Appendix

The appendix serves to give comprehensive, rigorous proofs and derivations that would compromise readability of the main text if left there. More precisely, it takes a close look on how shareholder value SV_{jl} as well as the budget constraints (1) and (2) change when debt b_{jl} is varied by some small db_{jl} and a non-marginal amount Δb_{jl}, respectively.

A.1 Changes in SV

To begin with, allow firms to react to $db_{jl} > 0$ with $dk_{j,l} \neq 0$, leading to $dy_{jls} \neq 0$. That is, firms use the additionally gathered debt capital in order to adjust their business plan. In doing so, they change SV taken from (12) by

$$dSV_{jl} = \sum_{s=1}^{S} r_s p_{ls} dy_{jls} - p_0 dk_{jl},$$

which is equal to zero as long as

$$p_0 = \sum_{s=1}^{S} r_s p_{ls} \frac{dy_{jls}}{dk_{jl}}. \tag{A.1}$$

Since (A.1) is just (11), we can conclude that the corporation will optimally set $dk_{jl} = 0$ if it has already acted in an SV-maximizing way before debt was raised. Given the results of Subsection 3.1, this turns out to be a completely innocuous assumption. On the contrary, allowing $dk_{jl}/db_{jl} \neq 0$ would force us to assume an inefficient management given lower debt levels.

A.2 Changes in the BC for $t = 0$

The sources of change for the first period are threefold. Firstly, of course, everything being multiplied by b_{jl} changes by Δb_{jl}, but it doesn't stop there. Rather, two arguments of the BCs react to changes in bond issuance as well, one of them being firm values v_{jl} (but not shareholder values). They change by

$$\Delta v_{jl} = \sum_{s \in S''_{jl}} r_s(p_{ls}y_{jls} - b_{jl} - \Delta b_{jl}) - \sum_{s \in S'_{jl}} r_s(p_{ls}y_{jls} - b_{jl}).$$

Here, we can truncate some terms of $r_s(p_{ls}y_{jls} - b_{jl})$ because $S''_{jl} \subseteq S'_{jl}$. This leads to the expression

$$\Delta v_{jl} = -\sum_{s \in S''_{jl}} r_s \Delta b_{jl} - \sum_{s \in S'_{jl} \setminus S''_{jl}} r_s(p_{ls}y_{jls} - b_{jl}). \tag{A.2}$$

Furthermore, bond prices R_{jl} of the corresponding corporation (j, l) are also affected. Resulting from their definition (5), they experience a shift of magnitude

$$\Delta R_{jl} = \sum_{s \in S''_{jl}} r_s + \sum_{s \notin S''_{jl}} r_s \frac{p_{ls}y_{jls}}{b_{jl} + \Delta b_{jl}} - \sum_{s \in S'_{jl}} r_s - \sum_{s \notin S'_{jl}} r_s \frac{p_{ls}y_{jls}}{b_{jl}}$$

We can make use of basic set theory and omit some summands. More precisely, it is sums over r_s that leave only those addends from $S'_{jl} \setminus S''_{jl}$. We obtain

$$\Delta R_{jl} = -\sum_{s \in S'_{jl} \setminus S''_{jl}} r_s + \sum_{s \notin S''_{jl}} r_s \frac{p_{ls}y_{jls}}{b_{jl} + \Delta b_{jl}} - \sum_{s \notin S'_{jl}} r_s \frac{p_{ls}y_{jls}}{b_{jl}}. \tag{A.3}$$

Furthermore, what appears in the BC (1) is the product $R_{jl}b_{jl}$. Following (5), it changes by

$$\Delta(R_{jl}b_{jl}) = \sum_{s\in S''_{jl}} r_s(b_{jl}+\Delta b_{jl}) + \sum_{s\notin S''_{jl}} r_s p_{ls} y_{jls} - \sum_{s\in S'_{jl}} r_s b_{jl} - \sum_{s\notin S'_{jl}} r_s p_{ls} y_{jls},$$

which, analogously to the previous differences, simplifies to

$$\Delta(R_{jl}b_{jl}) = \sum_{s\in S''_{jl}} r_s \Delta b_{jl} - \sum_{s\in S'_{jl}\backslash S''_{jl}} r_s b_{jl}. \tag{A.4}$$

The absolute differences in parameters defined above will prove helpful in determining the change in wealth as implied by the first budget constraint.

To see how the BC for $t=0$ changes, start with its formulation according to (9). Differencing it with and without raising debt by Δb_{jl} (assuming $dc_{i0}=0$) results in

$$\sum_{l=1}^{L}\sum_{j=1}^{J_l}\left\{-\bar{\theta}_{ijl}\left[\Delta v_{jl}+\Delta(R_{jl}b_{jl})\right]+\Delta R_{jl}a_{ijl}\right\}+\sum_{l=1}^{L}\sum_{j=1}^{J_l}\theta_{ijl}\Delta v_{jl}.$$

Note that $\Delta v_{jl}+\Delta(R_{jl}b_{jl})$ is just $\Delta(SV_{jl})$, meaning that the term in square brackets reduces to zero. One can easily verify this using the definitions of differences from (A.2) and (A.4):

$$\Delta v_{jl}+\Delta(R_{jl}b_{jl}) = -\sum_{s\in S''_{jl}} r_s\Delta b_{jl} - \sum_{s\in S'_{jl}\backslash S''_{jl}} r_s(p_{ls}y_{jls}-b_{jl}) + \sum_{s\in S''_{jl}} r_s\Delta b_{jl} - \sum_{s\in S'_{jl}\backslash S''_{jl}} r_s b_{jl} = 0.$$

The remaining change after applying (A.2) and (A.3) is then

$$\sum_{l=1}^{L}\sum_{j=1}^{J_l}\left[-\sum_{s\in S''_{jl}} r_s\Delta b_{jl} - \sum_{s\in S'_{jl}\backslash S''_{jl}} r_s(p_{ls}y_{jls}-b_{jl})\right]a_{ijl} + \sum_{j=1}^{J_l}\theta_{ijl}\left(-\sum_{s\in S'_{jl}\backslash S''_{jl}} r_s + \sum_{s\notin S''_{jl}} r_s\frac{p_{ls}y_{jls}}{b_{jl}+\Delta b_{jl}} - \sum_{s\notin S'_{jl}} r_s\frac{p_{ls}y_{jls}}{b_{jl}}\right),$$

which is just (14). *QED*

A.3 Changes in the BC for $t = 1$

The approach undertaken to evaluate wealth changes in $t = 1$ is somewhat different from the previous one. Here, we need to assess (15) for firm (j, l) depending on the state s that has materialized before weighting those changes with the adequate state price r_s each in order to make them directly comparable to the changes in $t = 0$ taken from (14). To do so, start with the simplest subset of states: those where bankruptcy never occurs, neither with lower nor with higher debt levels. This set reads $S^{'}_{jl} \cap S^{''}_{jl}$, which simplifies to the smaller of both, namely $S^{''}_{jl}$, due to the fact that $S^{''}_{jl} \subseteq S^{'}_{jl}$. For any such state, both maximum operators in (15) obtain their first element. Furthermore, α_{ijls} is just a_{ijl} for both debt levels, implying $d\alpha_{ijls} = 0$. Thus, we get a total change in the budget constraint of

$$\sum_{l=1}^{L} \sum_{j=1}^{J_l} \theta_{ijl} \Delta b_{jl} \tag{A.5}$$

for all $s \in S^{''}_{jl}$.

Next, look at states where bankruptcy was inevitable: Neither the increased nor the benchmark level of debt would have led to solvency. Formally, we have this case for all (j, l) such that $s \notin S^{'}_{jl}$ and $s \notin S^{''}_{jl}$. This intersection is simply the set of all $s \notin S^{'}_{jl}$ because $S^{''}_{jl} \subseteq S^{'}_{jl}$. Here, both maximum operators obtain their second entry, namely 0, while $d\alpha_{ijls} \neq 0$ leads to a change

$$-\sum_{l=1}^{L} \sum_{j=1}^{J_l} p_{ls} y_{jls} \left(\frac{a_{ijl}}{b_{jl} + \Delta b_{jl}} - \frac{a_{ijl}}{b_{jl}} \right) \tag{A.6}$$

for any $s \notin S^{'}_{jl}$.

Finally, consider those states where the additional debt is responsible for over-indebtedness. We intersect sets of solvency before debt was raised, $s \in S^{'}_{jl}$, with sets of bankruptcy after the surge in debt occurred, $s \notin S^{''}_{jl}$. This leads to $s \in S^{'}_{jl} \setminus S^{''}_{jl}$. With two different realizations of the maximum operator and α_{ijls}, we end up with

$$-\sum_{l=1}^{L} \sum_{j=1}^{J_l} (-p_{ls} y_{jls} + b_{jl}) + \frac{a_{ijl} p_{ls} y_{jls}}{b_{jl} + \Delta b_{jl}} - a_{ijl} \tag{A.7}$$

for all $s \in S'_{jl} \setminus S''_{jl}$.

Before we can move on to add up all changes, i.e., (A.5), (A.6) and (A.7), make sure that each state is neither omitted nor counted twice in this procedure. Indeed, the intersection of $s \in S''_{jl}$ and $s \notin S'_{jl}$ is just the empty set $\emptyset$, as is that of $s \notin S'_{jl}$ with $s \in S'_{jl} \setminus S''_{jl}$. The same holds for $(S'_{jl} \setminus S''_{jl}) \cap S''_{jl}$. The overall set union of those three, on the other hand, is veritably S. Thus, our partitioning of sets was indeed genuine.

To arrive at the MM-conclusion, weight all changes by corresponding state prices and add them up. As a consequence of the above calculations, we get a total $t = 1$-originated welfare change of size

$$\sum_{s \in S''_{jl}} r_s \left(\sum_{l=1}^{L} \sum_{j=1}^{J_l} \theta_{ijl} \Delta b_{jl} \right) + \sum_{s \notin S'_{jl}} r_s \left[- \sum_{l=1}^{L} \sum_{j=1}^{J_l} p_{ls} y_{jls} \left(\frac{a_{ijl}}{b_{jl} + \Delta b_{jl}} - \frac{a_{ijl}}{b_{jl}} \right) \right] + \sum_{s \in S'_{jl} \setminus S''_{jl}} r_s \left[- \sum_{l=1}^{L} \sum_{j=1}^{J_l} (-p_{ls} y_{jls} + b_{jl}) + \frac{a_{ijl} p_{ls} y_{jls}}{b_{jl} + \Delta b_{jl}} - a_{ijl} \right] \text{(A.8)}$$

The final step in our proof is to add up (14) and (A.8). This yields a present value of wealth changes which is equal to zero. Thus, changing the debt structure of firms does not alter individual consumption possibilities for all $i = 1, ..., I$ (given the marketed subspace is unchanged). Hence, MM holds. $\mathcal{QED}$

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