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"Taste heterogeneity, labor mobility and economic geography" - A critical reconsideration and correction*

(Tabuchi and Thisse 2002, JDevelEcon)

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Abstract

This article reconsiders the linear new economic geography model under heterogeneous agents developed by Tabuchi and Thisse (2002) by applying an analytical technique introduced by Ludema and Wooton (1999). Two problematic aspects are identified: first, the bifurcation pattern for countries which differ in amenities is incorrect. I show that the degree of agglomeration is highest when trade costs are high. Besides this minor problem, the second critical issue concerns the welfare analysis. It is shown in this note that this model exhibits a latent tendency for overagglomeration when trade costs are high and underagglomeration when trade costs are low, bringing it in line with other welfare analyses of new economic geography models.

JEL classification: R13; O15; O18
Keywords: Agglomeration; Migration; Welfare; New economic geography

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1 Introduction

Tabuchi and Thisse (2002) enrich the linear new economic geography model developed by Ottaviano et al. (2002) with the assumption that the migration costs differ between individuals. These heterogeneous migration costs are captured by a stochastic term which is part of their utility function. Indirect utilities are derived from quadratic quasi-linear utility functions. They are modeled such that people exhibit love of variety and, hence, all offered products will be consumed. There are two types of goods which enter the utility function: a homogeneous good which is produced under constant returns to scale and with immobile labor being the only factor of production. And there are a number of differentiated products which are produced under increasing returns to scale using a fixed amount ϕ of mobile workers. There are two countries in this model which are called home (H) and Foreign (F). By assumption, the homogeneous good does not incur trade costs when it is shipped from one country to the other. The differentiated product, instead, incurs positive trade costs of τ units of the homogeneous good per unit to be shipped. To guarantee that there is international trade, τ has to be less than a critical value τ_{trade} which depends on exogenous model parameters[1].

People migrate whenever the interregional difference of indirect utilities exceeds individual migration costs. Under these assumptions, a spatial equilibrium is reached whenever the following condition is satisfied (see Tabuchi and Thisse, 2002, p. 163):

$$\begin{aligned} J(\lambda,\tau) &\equiv \Delta V(\lambda,\tau) + d - \mu \ln \frac{\lambda}{1-\lambda} \\ &\equiv C^{*}\tau(\tau^{*}-\tau)(\lambda - 1/2) + d - \mu \ln \frac{\lambda}{1-\lambda} \stackrel{!}{=} 0 \end{aligned} \tag{1}$$

where $\Delta V(\lambda) \equiv V_H(\lambda) - V_F(\lambda)$ is the difference of indirect utilities as derived in Ottaviano et al. (2002), λ $(1-\lambda)$ stands for the proportion of people living and working in home (foreign) and C^* and τ^* are positive parameters consisting of exogenous parameters[2]. In order to describe a complete agglomeration process, it is assumed that $\tau^* < \tau_{trade}$ (see Tabuchi and Thisse, 2002, p. 163). $d \equiv d_H - d_F$ is a constant which measures the difference of country-specific amenities. If $d > 0$, home has more amenities than foreign making the first more attractive. The logarithmic expression captures heterogeneity within the population. Its functional form originates in the assumption that the stochastic part of each individual's utility function follows an identical and independent double exponential (*i.e.*, Gumbel) distribution. Here, μ is an exogenous parameter which directly measures the strength of heterogeneity (*i.e.*, the variance of the stochastic term). A spatial equilibrium is stable, whenever the slope of curve $J(\lambda,\tau)$ evaluated at the equilibrium is smaller than zero (see Tabuchi and Thisse, 2002, p. 163).

[1]See appendix A for a definition.
[2]See appendix A for a definition.

In what follows I address two critical aspects concerning the results derived by Tabuchi and Thisse (2002): first of all, the bifurcation pattern for asymmetric countries ($d > 0$) is shown to be incorrect. To prove this claim, section 2 develops a technique which differs from the one used by Tabuchi and Thisse. Secondly, in section 3 this analytical approach is used to show that the welfare analysis for asymmetric countries derived by the two authors is not correct. Furthermore, this technique sheds light on the welfare properties of this linear model when countries are symmetric ($d = 0$).

2 Bifurcation pattern for asymmetric countries

The divergence pattern for asymmetric countries ($d > 0$) shown by Fig. 1 in Tabuchi and Thisse (2002, p. 165) is not correct. To see why, it is convenient to apply a technique first used by Ludema and Wooton (1999) and to decompose the equilibrium condition given by Eq. (1) into two separate functions. The first two terms of $J(\cdot)$ are the difference of indirect utilities including differences in amenities. Following Ludema and Wooton (1999), I will refer to this curve as labor demand (LD in short).

$$LD \equiv C^{*}\tau(\tau^{*} - \tau)(\lambda - 1/2) + d \tag{2}$$

The coefficient of the linear argument ($\lambda - 1/2$) is negatively quadratic in τ and takes on the value zero when $\tau = \tau^{*}$ and $\tau = 0$. Consequently, if trade costs are greater than τ^{*} the labor demand curve is negatively sloped, whereas the gradient coefficient takes on positive values for all values of τ between zero and τ^{*}. Standard analysis reveals that for $\tau = \tau^{*}/2$ the gradient is at its maximum. d does not influence the slope of the function but rather shifts the curve upwards.

The third term of $J(\cdot)$ shall be called labor supply (LS in short) and captures the heterogeneity of the mobile population:

$$LS \equiv \mu \ln \frac{\lambda}{1 - \lambda} \tag{3}$$

Differentiating LS with respect to λ shows that the curve slopes upwards. As $\lambda \rightarrow 1$ ($\lambda \rightarrow 0$) LS tends towards (negative) infinity.
From Eq.(1) one may conclude that the necessary condition for spatial equilibria $J(\cdot) \overset{!}{=} 0$ is satisfied by each λ at which LD and LS intersect. Such equilibria are stable if at points of intersection the slope of LD is smaller than the slope of LS. Figure 1 depicts the labor supply curve and labor demand curves for three different levels of trade costs ($\tau_{trade}, \tau^{*}/2$ and $\tau = 0$). Consequently, λ_2^{*} and λ_1^{*} in figure 1 mark stable equilibria, whereas equilibrium λ_0^{*} is instable.

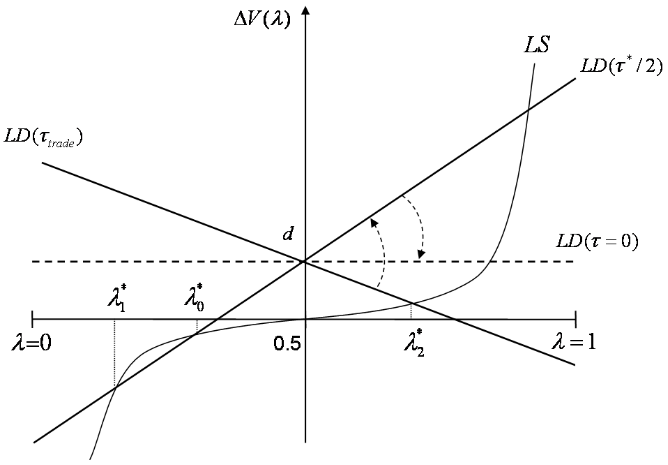

Figure 1: Labor demand and labor supply when $d > 0$

Let me concentrate on the evolution of λ_2^* with respect to falling trade costs. It becomes clear that the equilibrium share of mobile workers increases with falling trade costs as LD rotates counter-clockwise as long as trade costs are greater than $\tau^*/2$. When $\tau = \tau^*/2$, the slope of the labor demand curve is steepest and, consequently, λ_2^* takes on its greatest value. When τ continues to fall, LD rotates clockwise and, therefore, the equilibrium share λ_2^* decreases. Once trade costs have reached the lowest value possible ($\tau = 0$), the slope of LD is zero. The labor demand curve is then parallel to the horizontal axis. From this, it becomes clear that the equilibrium share of skilled industry takes on its lowest value when trade costs are close to the critical value of τ_{trade}. When trade costs are zero, the degree of agglomeration is greater than at $\tau = \tau_{trade}$. This reveals the mistake in Tabuchi and Thisse (2002, p. 165), figure 1. The corresponding divergence pattern generated by falling trade costs is shown in figure 2. Bold (dashed) lines mark stable (instable) equilibria.

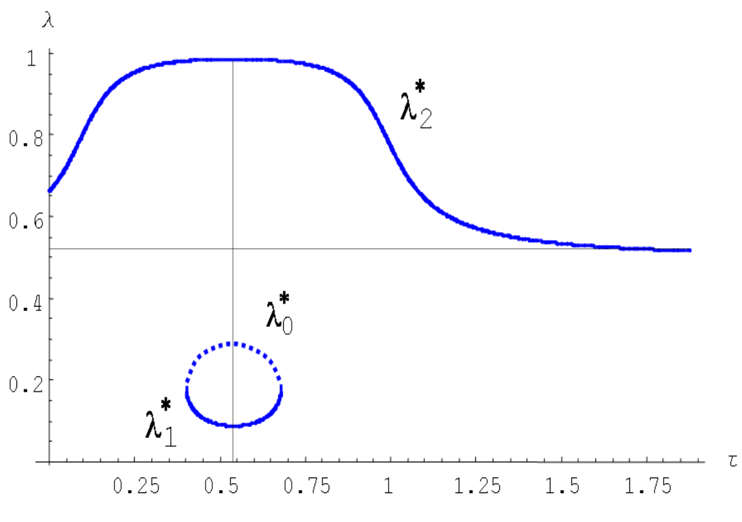

Figure 2: Divergence pattern for falling trade costs when $d > 0$[3]

3 Welfare analysis

The comparison of agglomeration induced by market forces and by a social planner presented in Tabuchi and Thisse (2002) can be shown to be incorrect for asymmetric countries ($d > 0$). Furthermore, when countries are symmetric ($d = 0$), clear patterns concerning over- and underagglomeration can be worked out.

The underlying utility functions being quasi-linear, a utilitarian welfare approach may be justified. Maximizing the overall sum of indirect utilities with respect to λ leads to the following first-order condition for extrema of the global welfare function $W(\cdot)$ (see Tabuchi and Thisse, 2002, p. 170):

$$W'(\lambda, \tau) = C^o \tau(\tau^o - \tau)(\lambda - 1/2) + d - \mu \ln \frac{\lambda}{1-\lambda} \stackrel{!}{=} 0 \tag{4}$$

Here, C^o and τ^o are positive parameters which depend on exogenous parameters[4] and it holds true that $C^o > C^*$ and $\tau^o < \tau^*$ [5]. Eq. (4) has a qualitatively identical structure to Eq. (1). Therefore, $W'(\cdot)$ can be decomposed into a linear function which embraces the first two terms of $W'(\cdot)$ and into the labor supply curve LS which is the third term of $W'(\cdot)$ and identical to the market labor supply stated in Eq. 3. The linear function shall be called "planner curve" (PC in short).

$$PC \equiv C^o \tau(\tau^o - \tau)(\lambda - 1/2) + d \tag{5}$$

Due to qualitatively identical functional structures, the evolution of the slope of PC with respect to falling trade costs is identical to LD, with τ^o instead of τ^*, indicating the critical level of trade costs at which the sign of the slope of PC turns from negative to positive.

Analogous to the previous section, the first-order condition of welfare extrema is satisfied at the point(s) of intersection of PC and LS. A local maximum obtains if the second derivative of $W(\cdot)$ evaluated at this point is less than zero. Therefore, the slopes of PC and LS reveal the kind of extremum: Whenever the slope of PC is smaller than the slope of LS at the point(s) of intersection, the global welfare function will exhibit a local maximum. Figure 3 shows the decomposition of $W'(\cdot)$ and the evolution of welfare extrema. λ_1^o and λ_2^o mark local welfare maxima, whereas λ_0^o is a local minimum. When countries are asymmetric in amenities, the global welfare level at λ_1^o is inferior to the welfare at λ_2^o (see Tabuchi and Thisse, 2002, p. 170). Therefore, when the market outcome and socially

[3] The bifurcation is shown for parameter values $a = 2, b = 1, A = 65, L = 1, c = 0.25, \phi = 2, \mu = 0.06, d = 0.04$.

[4] See appendix A for definitions.

[5] See appendix A.

optimal agglomeration are compared, λ_1^o can be ruled out making it completely sufficient to concentrate on λ_2^o.

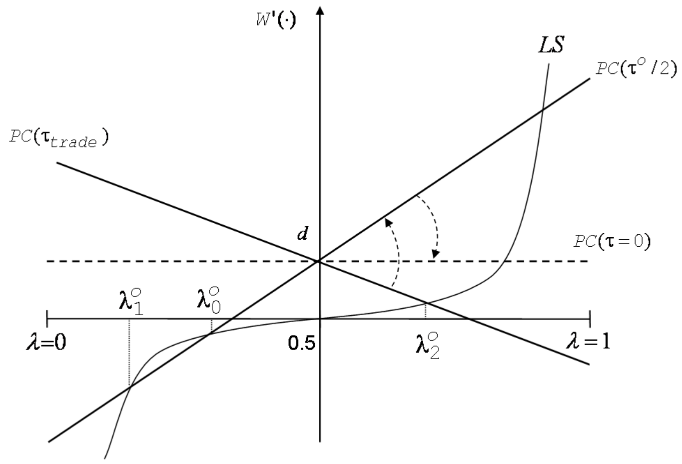

Figure 3: Planner curve and labor supply when $d > 0$

It is straightforward that a social planner will prefer more (less) agglomeration than the market outcome if the level of partial agglomeration at which PC and LS intersect is greater (smaller) than the degree of agglomeration at which LD and LS intersect. As PC and LD are linear curves, this issue can be reduced to a comparison of slopes - whenever the slope of PC is greater (less) than the slope of LD, the market will exhibit underagglomeration (overagglomeration). Subtracting the slope of PC from the slope of LD reveals that the slope of PC is greater (less) if trade costs are lower (greater) than the critical value τ^c given by Eq. (6) [6]:

$$\tau^c \equiv \frac{C^o\tau^o - C^*\tau^*}{C^o - C^*} > 0 \tag{6}$$

Subtracting τ^c from τ^o reveals that $\text{sgn}(\tau^o - \tau^c) = \text{sgn}(\tau^* - \tau^o) > 0$. As the latter holds true (see before), τ^c is always smaller than τ^o and τ^*. Furthermore, τ^* is assumed to be smaller than τ_{trade} so that there is no such case where τ^c can be greater than τ_{trade}. This is in contrast to what is claimed in Tabuchi and Thisse (2002, p. 171). Figure 4 summarizes the results and shows the evolution of both slopes with respect to the level of trade costs[7].

When countries are symmetric ($d = 0$) the position of τ^c relative to $\tau^o/2$ and $\tau^*/2$ (*i.e.*, the levels of trade costs which maximize the slope of PC and LD) as well as the degree of heterogeneity influence the shape of bifurcation patterns which include *both* the degree of partial agglomeration by market forces *and* the social planner. It is important to observe

[6] It can be shown that $C^o\tau^o - C^*\tau^* > 0$ and $C^o - C^* > 0$; see appendix A.

[7] This is in the spirit of Pflüger and Südekum (2007). In analogy to their analysis, the slopes indicate the private net agglomeration force for the market and the social net agglomeration force for the social planner, respectively. The vertical difference between these slopes can then be interpreted as the net pecuniary externality associated with the mobiliy of agents.

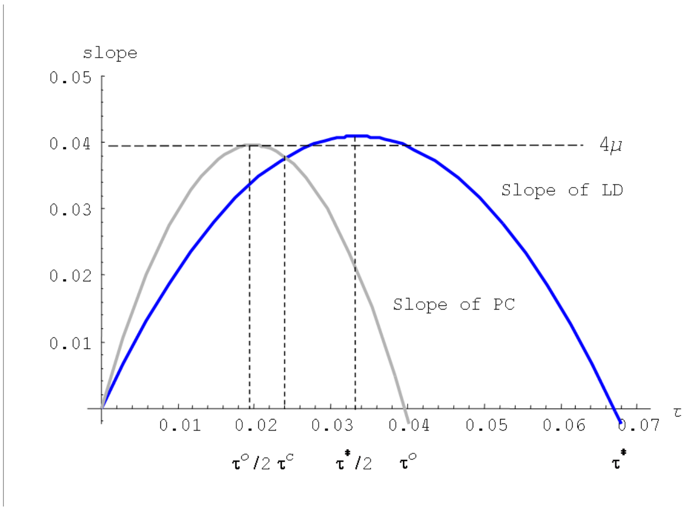

Figure 4: The slope of the labor demand and the planner curve[8]

that the exact position of τ^c relative to $\tau^*/2$ and $\tau^o/2$ is parameter dependent. If returns to scale (proportionally measured by ϕ) are sufficiently high and the ease of substitution (proportionally measured by c) is sufficiently low, then τ^c will be greater than $\tau^*/2$ [9]. Whenever this is the case, the maximum slope of PC will be greater than the maximum slope of LD[10]. Such a situation is shown in figure 5. If returns to scale are sufficiently low and the ease of substitution is sufficiently high, then τ^c will be smaller than $\tau^o/2$ [11]. Figure 6 depicts this situation. The parameters may also be such that τ^c lies between $\tau^o/2$ and $\tau^*/2$ as shown by figure 4.

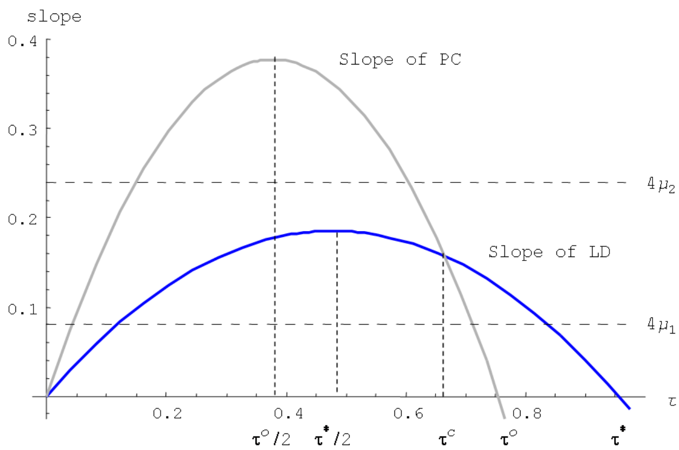

Figure 5: Slopes when ϕ is large and c is small[12]

[8]The slopes are depicted for parameter values $a = 1, b = 1, A = 65, L = 1, c = 1.5, \phi = 1$; see appendix B.1 for further details.

[9]See appendix A.

[10]It holds true that if $\tau^c - \tau^*/2 > 0 \Rightarrow C^o\tau^{o2} - C^*\tau^{*2} > 0$; see appendix A.

[11]See appendix A.

[12]The slopes are depicted setting $a = 1, b = 1, A = 65, L = 1, c = 0.1, \phi = 2$, see appendix B.2 for further details.

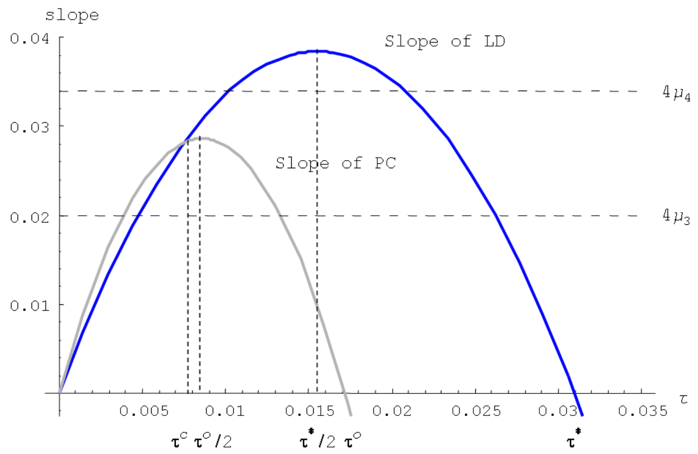

Figure 6: Slopes when ϕ is small and c is large[13]

Next, consider the impact of heterogeneity: the market (the social planner) will partially agglomerate whenever the slope of LD (PC) is greater than the slope of LS evaluated at $\lambda = 0.5$ (*i.e.*, 4μ) (see Tabuchi and Thisse, 2002, p. 166). Agglomeration forces are then stronger than the dispersion force originating in heterogeneity. Three different cases can be distinguished: *First of all*, when there is no heterogeneity (*i.e.*, $\mu = 0$), the model leads to "bang-bang" solutions known from core-periphery models and only overagglomeration becomes visible for trade costs between τ^* and τ^o. For trade costs greater than τ^* or smaller than τ^o market and socially optimal concentration coincide. *Secondly*, whenever the degree of heterogeneity is low such that 4μ is smaller than the slope of LD or PC evaluated at τ^c, bifurcation diagrams will show overagglomeration for levels of trade costs greater than τ^c, whereas there is underagglomeration for trade costs lower than τ^c (see figure 5, $4\mu_1$ and figure 6, $4\mu_3$). And *thirdly*, for values of 4μ greater than this critical threshold, the shape of bifurcation patterns depends on the relative position of τ^c. If τ^c is greater than $\tau^*/2$, bifurcation diagrams exhibit only underagglomeration (see figure 5, $4\mu_2$). If τ^c is smaller than $\tau^o/2$, only overagglomeration becomes visible (compare figure 6, $4\mu_4$). If τ^c lies between these levels, there are three characteristic regions: first, overagglomeration will be found for high levels of trade costs (the social planner will keep people dispersed). This is followed by an interval of τ where both the market and the social planner will exhibit dispersion. And lastly, when trade costs continue to fall, there might be underagglomeration as market forces will lead to dispersion whereas the social planner might prefer some level of partial agglomeration. The latter depends on whether there is an interval of trade costs for which the slope of PC is greater than the dispersion force 4μ (see figure 4, 4μ).

Consequently, there are clear patterns which determine the shape of the bifurcation dia-

[13]The slopes are shown for parameter values $a = 1, b = 1, A = 65, L = 1, c = 2.1, \phi = 0.6$; see appendix B.3 for further details.

gram of the equilibrium and the optimum, but there is no robust result - whether there is over- or underagglomeration depends on the actual values of model parameters.

However, when countries are asymmetric due to amenity differences, there is an unambiguous and robust pattern for bifurcation diagrams of the equilibrium and the optimum. With interregional differences in "first nature", partial agglomeration in home obtains by necessity at *any* level of trade costs smaller than τ_{trade}, both in the equilibrium and in the optimum (see figure 1 for the market). The greater the slope of PC (LD), the greater is the degree of partial agglomeration. Fig. 4 shows that as long as $\tau > \tau^c$, the slope of LD will be greater than the slope of PC. Consequently, market forces lead to more agglomeration than a social planner would prefer, which is equivalent to overagglomeration. When trade costs are lower than τ^c, the slope of PC exceeds the slope of LD leading to market underagglomeration. Figure 7 shows the bifurcation pattern of market forces and a social planner for $d > 0$. Bold blue lines mark agglomeration by market forces, light gray lines show the degree of socially optimal agglomeration. The position of τ^c relative to the levels of trade costs which maximize the slope of LD and PC (*i.e.*, $\tau^*/2$ and $\tau^o/2$) does not *qualitatively* influence the result. Consequently, if $d > 0$, there is no such case where market forces lead to overall overagglomeration, nor is it possible that the market exhibits excessive agglomeration for low levels of τ.

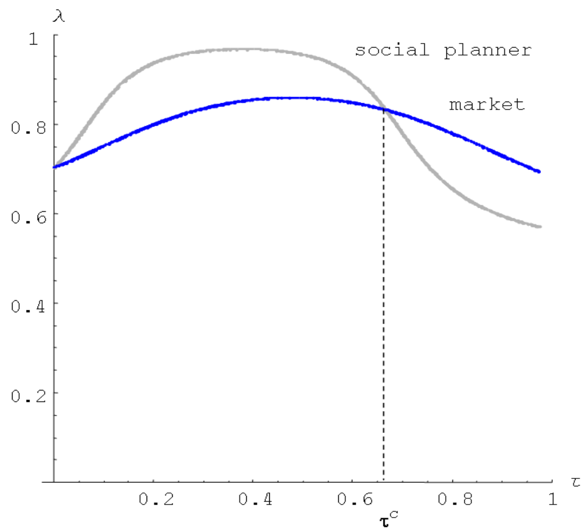

Figure 7: Socially optimal agglomeration in comparison to the market outcome[14]

4 Conclusion

This linear new economic geography model exhibits a tendency for overagglomeration when trade costs are high, whereas it has a tendency for underagglomeration when trade costs are low. This is unambiguously revealed by the presented slope diagrams. When

[14]Numerical evaluation setting $a = 1, b = 1, A = 65, L = 1, c = 0.1, \phi = 2, \mu = 0.07, d = 0.06$.

countries are symmetric in amenities, this welfare property does not necessarily become visible in bifurcation diagrams, as they map functions for only one particular set of parameters. When heterogeneity is sufficiently small, divergence patterns will show overagglomeration which converts to underagglomeration once trade costs have fallen below a critical level. If people are characterized by strong heterogeneity, divergence patterns will show either overagglomeration, underagglomeration or a combination of both, depending on model parameters. When countries are asymmetric in amenities, comparative bifurcation diagrams are qualitatively identical and robust. This is due to that fact that every level of trade costs is unambiguously translated into a specific level of partial agglomeration. Furthermore, it becomes clear that the inefficiency of market forces originates solely in the underlying market model of imperfect competition. Taste heterogeneity as well as amenity differences between countries are considered in the same way by a social planner and by market forces.

This result is qualitatively similar to what has been worked out by Pflüger and Südekum (2007). Assuming logarithmic quasi-linear utility functions, they show that there is market overagglomeration for high values of trade costs, whereas one observes market underagglomeration once trade costs have fallen below a critical threshold. Like in this article, this welfare property remains latent unless a dispersion force is introduced. Furthermore, they are also able to show that the inefficiency originates solely in the market model and not in the dispersion force. This similarity between Pflüger and Südekum (2007) and the findings presented in this article is very reassuring, as it seems that there is a robustness concerning the welfare aspects of new economic geography models with mobile labor.

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A Parameter definitions and derivations

$$C^* \equiv [2b\phi(3b\phi + 3cL + cL_A) + c^2L(L_A + L)]\frac{L(b\phi + cL)}{2\phi^2(2b\phi + cL)^2} > 0 \quad (7)$$

$$C^o \equiv \frac{[2b\phi + c(L_A + L)]L}{\phi^2} > 0 \quad (8)$$

$$\tau_{trade} \equiv \frac{2a\phi}{2b\phi + cL} > 0 \quad (9)$$

$$\tau^* \equiv \frac{4a\phi(3b\phi + 2cL)}{2b\phi(3b\phi + 3cL + cL_A) + c^2L(L_A + L)} > 0 \quad (10)$$

$$\tau^o \equiv \frac{4a\phi}{2b\phi + c(L_A + L)} > 0 \quad (11)$$

$$\text{sgn}(C^o - C^*) = \text{sgn}(B_1) \qquad \text{where} \quad (12)$$

$$B_1 \equiv 10b^3\phi^3 + 12b^2\phi^2cL + 5b\phi c^2L^2 + 6b^2\phi^2cL_A + 5b\phi c^2LL_A + $$
$$+ c^3L^2L_A + c^3L^3 > 0$$

$$\text{sgn}(C^o\tau^o - C^*\tau^*) = \text{sgn}(B_2) \qquad \text{where} \quad (13)$$

$$B_2 \equiv 5b\phi + 3cL > 0$$

$$\text{sgn}(\tau^* - \tau^o) = \text{sgn}(B_3) \qquad \text{where} \quad (14)$$

$$B_3 \equiv b\phi cL + b\phi cL_A + c^2LL_A + c^2L^2 > 0$$

$$\text{sgn}(\tau^o - \tau^*/2) = \text{sgn}(B_4) \qquad \text{where} \quad (15)$$

$$B_4 \equiv 6b^2\phi^2 + 5b\phi cL + b\phi cL_A > 0$$

$$\text{sgn}(\tau^c - \tau^*/2) = \text{sgn}(2C^o\tau^o - C^*\tau^* - C^o\tau^*) = \text{sgn}(B_5) \qquad \text{where} \quad (16)$$

$$B_5 \equiv 30b^4\phi^4 + 40b^3\phi^3cL + 2b^3\phi^3cL_A + 7b^2\phi^2c^2L^2 - 5b^2\phi^2c^2LL_A -$$
$$- 7b\phi c^3L^2L_A - 7b\phi c^3L^3 - 2c^4L^3L_A - 2c^4L^4$$

$$
\begin{aligned}
\mathrm{sgn}(\tau^c & - \tau^o/2) = \mathrm{sgn}(C^o\tau^o - 2C^*\tau^* + C^*\tau^o) = \mathrm{sgn}(B_6) \qquad \text{where} \qquad (17) \\
B_6 & \equiv 10b^3\phi^3 + 4b^2\phi^2 cL_A + 10b^2\phi^2 cL + b\phi c^2 LL_A + b\phi c^2 L^2 - \\
& - c^3L^2L_A - c^3L^3
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{sgn}(C^o\tau^o\tau^o & - C^*\tau^*\tau^*) = \mathrm{sgn}(B_7) \qquad \text{where} \qquad (18) \\
B_7 & \equiv 30b^4\phi^4 + 45b^3\phi^3 cL + 7b^3\phi^3 cL_A + 3b^2\phi^2 c^2 LL_A + 15b^2\phi^2 c^2 L^2 - \\
& - 4b\phi c^3L^2L_A - 4b\phi c^3 L^3 - 2c^4L^3L_A - 2c^4L^4
\end{aligned}
$$

B Numerical analysis

B.1 Figure 4

$$
\begin{aligned}
\tau_{trade} &= 0.571429 \\
\tau^* &= 0.0669456 \\
\tau^c &= 0.0240773 \\
\tau^*/2 &= 0.0334728 \\
\tau^o/2 &= 0.019802
\end{aligned}
$$

B.2 Figure 5

$$
\begin{aligned}
\tau_{trade} &= 0.97561 \\
\tau^* &= 0.956421 \\
\tau^c &= 0.66595 \\
\tau^*/2 &= 0.478211
\end{aligned}
$$

B.3 Figure 6

$$
\begin{aligned}
\tau_{trade} &= 0.363636 \\
\tau^* &= 0.0309957 \\
\tau^c &= 0.00747957 \\
\tau^o/2 &= 0.00858369
\end{aligned}
$$