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The Choice of Prices vs. Quantities under Uncertainty*

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Abstract

If duopolistic firms can choose their strategy variable, uncertainty about demand conditions and the degree of substitutability have countervailing effects on variable choice. High uncertainty favors prices, while close substitutability favors quantities. For intermediate values, a hybrid equilibrium exists.

Keywords: Competition, Strategy Variables, Demand Uncertainty

JEL Classification: D43, L13

1 Introduction

If firms are free to choose their strategy variable, namely either prices or quantities, it is well known that without uncertainty they prefer to choose quantities if their products are substitutes. The reason is that quantities soften competition and thereby guarantee higher equilibrium profits compared to prices. Singh and Vives (1984) show in a deterministic two-stage game, in which firms first choose their strategy variable and compete afterwards, that quantities are a dominant action. However, their analysis

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abstracts from the fact that firms may face uncertainty at the time the strategy variable has to be chosen. Our analysis incorporates this aspect by introducing uncertainty via shocks that affect the slope and the intercept of the demand curve.¹

In this framework the expected equilibrium profit of a price setting firm increases in the amount of uncertainty. However, expected equilibrium profits decrease with product substitutability to a greater extent for a price setting than for a quantity setting firm. As a result both firms select quantities if uncertainty is small and prices if uncertainty is high relative to the degree of substitutability. A “hybrid” equilibrium in which one firm chooses a price and the other one a quantity emerges in an intermediate range.²

Our analysis relates to Klemperer and Meyer (1986). They allow for a stochastic demand function and analyze which variable adapts more flexibly to shocks under different cost functions. They consider a one-shot game and therefore the comparative advantage of quantities is not present. Our analysis derives conditions under which either strategy variable’s comparative advantage dominates.

In Section 2, we present the model, solve for the equilibrium, and provide an intuition for our results. Section 3 concludes.

2 The Model

We consider a duopoly with differentiated products. Firms face the linear demand system

$$\begin{aligned} p_i &= \alpha - \frac{\beta}{\theta} q_i - \frac{\gamma}{\theta} q_j + \epsilon, \\ p_j &= \alpha - \frac{\beta}{\theta} q_j - \frac{\gamma}{\theta} q_i + \epsilon, \end{aligned}$$

with $\alpha > 0$, and $\beta \geq \gamma \geq 0$. When $\gamma = \beta$, products are perfect substitutes whereas with $\gamma = 0$ they are independent. θ and ϵ are two random variables. Without loss of generality we set $E[\theta] = 1$, $E[\epsilon] = 0$, and $E[1/\theta] = z$.³ We assume that the covariance between θ and ϵ is nonnegative, $\sigma_{\theta\epsilon} = \rho\sigma_\theta\sigma_\epsilon \geq 0$. So given a positive shock on the intercept ($\epsilon > 0$) the expected slope becomes

¹For an analysis of a social planner facing demand uncertainty, see Weitzman (1974).

²This seems to be in line with empirical research. For example, Aiginger (1999) asked managers of 930 manufacturing firm in Austria if they select prices or quantities as their decision variable. Roughly, 2/3 charge prices and 1/3 set quantities.

³Since $E[\theta] = 1$ we have by Jensen’s inequality that $z > 1$ if $\sigma_\theta > 0$.

flatter ($E[\theta | \epsilon > 0] > 1$).⁴ To avoid unnecessary complications we require the support of the shocks to be sufficiently small such that no equilibria emerge in which a price setting firm sells a negative quantity or a quantity setting firm receives a negative price. Further we assume that firms have zero costs.⁵

Competition between firms takes the form of a two-stage game. In stage 1 firms simultaneously choose their strategy variables. Each firm observes the other firm's choice and competes in stage 2 contingent on the chosen strategy variables. Afterwards shocks realize, markets clear and profits accrue. So after the first stage firms are committed to their strategy variable and cannot change it thereafter. We solve for the subgame perfect equilibrium.

As spelled out before, it is a dominant strategy for firms to set quantities in the first stage of the deterministic game ($\sigma_\theta = \sigma_\epsilon = 0$) since they induce a lower degree of competition. Now turn to the case of uncertainty. We get the following result:

Proposition The subgame perfect equilibrium outcome of the two stage game is the following:

Both firms select a quantity in the first stage if

$$\max[0, \sigma_{\theta\epsilon}^*] > \sigma_{\theta\epsilon} \geq 0.$$

One firm selects a price and the other firm a quantity in the first stage if

$$\max[0, \sigma_{\theta\epsilon}^{**}] > \sigma_{\theta\epsilon} \geq \max[0, \sigma_{\theta\epsilon}^*].$$

Both firms select a price in the first stage if

$$\sigma_{\theta\epsilon} \geq \max[0, \sigma_{\theta\epsilon}^{**}] \text{ and } \gamma < \gamma^+.$$

If $\gamma > \gamma^+$ there exists no equilibrium in which both firms select a price.

The values for $\sigma_{\theta\epsilon}^*$, $\sigma_{\theta\epsilon}^{**}$, and γ^+ are defined in (6), (8), and (9).

Proof We solve the game by backward induction. First, suppose that both firms set prices as their strategy variable. Then in the second stage

⁴We restrict the covariance to be nonnegative for two reasons. Firstly, it seems natural that if demand conditions are good both the intercept and the slope are hit by a positive shock and vice versa if demand conditions are bad. Second, the analysis in case of negative correlation would be very similar and is therefore omitted.

⁵As Singh and Vives (1984) show, the analysis would not change if firms faced positive constant marginal costs c because this would only lower the effective intercept from α to $a = \alpha - c$.

each firm maximizes its expected profit by choosing

$$\max_{p_i} E \left[p_i \left(\frac{\theta((\alpha + \epsilon)(\beta - \gamma) - \beta p_i + \gamma p_j)}{\beta^2 - \gamma^2} \right) \right].$$

Since $E[\theta] = 1$, $E[\epsilon] = 0$, and $E[\theta\epsilon] = \sigma_{\theta\epsilon}$ this is equivalent to

$$\max_{p_i} p_i \left(\frac{(\alpha + \sigma_{\theta\epsilon})(\beta - \gamma) - \beta p_i + \gamma p_j}{\beta^2 - \gamma^2} \right). \quad (1)$$

Solving (1) for both firms yields equilibrium prices of $p_i = p_j = p^* = \frac{(\beta - \gamma)(\alpha + \sigma_{\theta\epsilon})}{2\beta - \gamma}$. Therefore each firm's expected profit is equal to

$$\Pi_i^{p^* p^*} = \frac{(\alpha + \sigma_{\theta\epsilon})^2 (\beta - \gamma) \beta}{(\beta + \gamma)(2\beta - \gamma)^2}.$$

Next, suppose that both firms set quantities as their strategy variable. In the second stage each firm maximizes its expected profit by choosing

$$\max_{q_i} E \left[q_i \left(\epsilon + \alpha - \frac{\beta q_i + \gamma q_j}{\theta} \right) \right]. \quad (2)$$

Since $E[\frac{1}{\theta}] = z$ the maximization problem in (2) is equivalent to

$$\max_{q_i} q_i (\alpha - z(\beta q_i + \gamma q_j)). \quad (3)$$

Solving (3) for both firms yields equilibrium quantities of $q_i = q_j = q^* = \frac{\alpha}{z(2\beta + \gamma)}$ and an expected profit of

$$\Pi_i^{q^* q^*} = \frac{\alpha^2 \beta}{z(2\beta + \gamma)^2} \quad (4)$$

for each firm.

Lastly, if firm i chooses a price while firm j sets a quantity the equilibrium price and quantity are $p^\# = \frac{(\beta - \gamma)(2z(\beta + \gamma)(\alpha + \sigma_{\theta\epsilon}) - \alpha\gamma)}{4z(\beta^2 - \gamma^2) + \gamma^2}$ and $q^\# = \frac{\alpha(2\beta - \gamma) + \gamma\sigma_{\theta\epsilon}}{4z(\beta^2 - \gamma^2) + \gamma^2}$. The expected profit of the price setting firm is

$$\Pi_i^{p^\# q^\#} = \frac{(\beta - \gamma)^2 (2z(\beta + \gamma)(\alpha + \sigma_{\theta\epsilon}) - \alpha\gamma)^2}{(4z(\beta^2 - \gamma^2) + \gamma^2)^2 \beta},$$

while the expected profit of the quantity setting firm is

$$\Pi_j^{p^\# q^\#} = \frac{z(\beta^2 - \gamma^2)(\alpha(2\beta - \gamma) + \gamma\sigma_{\theta\epsilon})^2}{(4z(\beta^2 - \gamma^2) + \gamma^2)^2 \beta}.$$

We now proceed to the first stage. First look at the case in which firm j sets a quantity. Firm i is indifferent between setting a price or a quantity if

$$\frac{(\beta - \gamma)^2(2z(\beta + \gamma)(\alpha + \sigma_{\theta\epsilon}) - \alpha\gamma)^2}{(4z(\beta^2 - \gamma^2) + \gamma^2)^2\beta} - \frac{\alpha^2\beta}{z(2\beta + \gamma)^2} = 0. \quad (5)$$

From (5) we derive the threshold covariance $\sigma_{\theta\epsilon}^*$, above which firm i prefers to charge a price. It is implicitly defined by

$$\sigma_{\theta\epsilon}^* \equiv \alpha \left(z \frac{\gamma}{2(\beta - \gamma)} - z^2 + \frac{\sqrt{z}\beta(4z(\beta^2 - \gamma^2) + \gamma^2)}{2z^2(\beta^2 + \gamma^2)(2\beta + \gamma)} \right). \quad (6)$$

Note that an increase in σ_{θ} causes the left hand side of (6) to increase whereas the right hand side decreases due to the positive relation between σ_{θ} and z . Therefore $\sigma_{\theta\epsilon}^*$ is the relevant benchmark for covariances involving the same σ_{θ} . The threshold covariance is positive if $\gamma \geq \gamma'$, where γ' is implicitly defined by

$$0 = f(\gamma') \equiv (\beta + \gamma')(2\beta + \gamma')(z\gamma' - 2z^2(\beta - \gamma')) + \sqrt{z}\beta(4z(\beta^2 - (\gamma')^2) + (\gamma')^2).$$

Since $f(\gamma)$ is strictly convex in γ with $f(0) < 0$ and $f(\beta) > 0$, it follows that $\gamma' \in (0, \beta)$ is unique. Thus for $\sigma_{\theta\epsilon} \geq \sigma_{\theta\epsilon}^*$ firm i prefers to set a price.

Now suppose firm j sets a price. Then firm i is indifferent between choosing a price or a quantity if

$$\frac{(\alpha + \sigma_{\theta\epsilon})^2(\beta - \gamma)\beta}{(\beta + \gamma)(2\beta - \gamma)^2} - \frac{(\beta^2 - \gamma^2)z(\alpha(2\beta - \gamma) + \gamma\sigma_{\theta\epsilon})^2}{(4z(\beta^2 - \gamma^2) + \gamma^2)^2\beta} = 0. \quad (7)$$

From (7) we derive the threshold covariance $\sigma_{\theta\epsilon}^{**}$.⁷ It is implicitly defined by

$$\begin{aligned} \sigma_{\theta\epsilon}^{**} \equiv & \frac{\alpha \left(\gamma(\beta + \gamma)(2\beta^2(4\beta^2 - 6\beta\gamma + \gamma^2) + \gamma^3(5\beta - \gamma))z - 16\beta^2(\beta^2 - \gamma^2)^2z^2 - \beta^2\gamma^4 \right)}{16\beta^2(\beta^2 - \gamma^2)^2z^2 + \gamma^2(\beta + \gamma)(4\beta^3 - 8\beta^2\gamma + 3\beta\gamma^2 - \gamma^3)z + \beta^2\gamma^4} \\ & + \frac{2\alpha\sqrt{z}\beta(2\beta - \gamma)(\beta^2 - \gamma^2)(4z(\beta^2 - \gamma^2) + \gamma^2)}{16\beta^2(\beta^2 - \gamma^2)^2z^2 + \gamma^2(\beta + \gamma)(4\beta^3 - 8\beta^2\gamma + 3\beta\gamma^2 - \gamma^3)z + \beta^2\gamma^4}. \end{aligned} \quad (8)$$

⁶Since (5) is quadratic in $\sigma_{\theta\epsilon}$ there exists a second threshold covariance which is strictly negative and thus can be neglected.

⁷As in the previous case, there exists a second threshold covariance, that is strictly negative and is therefore not relevant. Again an increase in σ_{θ} causes the left hand side of (8) to increase and its right hand side to decrease.

Now we determine the range in which $\sigma_{\theta\epsilon}^{**} \geq 0$. Let γ'' and γ^+ be implicitly defined by

$$0 = g(\gamma'') \equiv \gamma''(\gamma'' + \beta)(\beta^2(8\beta^2 + 2(\gamma'')^2 - 12\gamma''\beta) + (\gamma'')^3(5\beta - \gamma''))z - \beta^2(\gamma'')^4 \\ - 16\beta^2(\beta^2 - (\gamma'')^2)^2z^2 + 2\sqrt{z}\beta(2\beta - \gamma'')(\beta^2 - (\gamma'')^2)(4z(\beta^2 - (\gamma'')^2) + (\gamma'')^2),$$

and

$$0 = h(\gamma^+) \equiv 16\beta^2(\beta^2 - (\gamma^+)^2)^2z^2 \\ + (\gamma^+)^2(\beta + \gamma^+)(4\beta^3 - 8\beta^2\gamma^+ - 3\beta(\gamma^+)^2 - (\gamma^+)^3)z + (\gamma^+)^4\beta^2, \quad (9)$$

where $g(\gamma)$ is the numerator of (8) divided by α and $h(\gamma)$ is the denominator of (8).

First, we show, that $\gamma'', \gamma^+ \in (0, \beta)$ are unique. Since $g(\gamma)$ decreases in γ for γ smaller than some $\gamma^* \in (0, \beta)$ and strictly increases thereafter with $g(0) < 0$ and $g(\beta) > 0$ it follows that $\gamma'' \in (0, \beta)$ is unique. Similarly, $h(\gamma)$ increases in γ for γ smaller than some $\gamma^{**} \in (0, \beta)$ and strictly decreases thereafter with $h(0) > 0$ and $g(\beta) < 0$. Therefore $\gamma^+ \in (0, \beta)$ is unique. Next we show that $\gamma'' < \gamma^+$. Since $\sigma_{\theta\epsilon}^{**}$ is strictly convex in γ for $\gamma \in [0, \gamma^+)$ with $\sigma_{\theta\epsilon}^{**} < 0$ if $\gamma = 0$ and $\lim_{\gamma \rightarrow \gamma^+} \sigma_{\theta\epsilon}^{**} \rightarrow \infty$ it follows that $\gamma'' < \gamma^+$. Thus, the numerator and the denominator of $\sigma_{\theta\epsilon}^{**}$ have the same sign if and only if $\gamma \in (\gamma'', \gamma^+)$. Therefore $\sigma_{\theta\epsilon}^{**} \geq 0$ if $\gamma \in [\gamma'', \gamma^+)$.

Now we show that for $\gamma'' \leq \gamma < \gamma^+$ firm i prefers to set a quantity contingent on firm j choosing a price if $0 \leq \sigma_{\theta\epsilon} < \sigma_{\theta\epsilon}^{**}$. Firm i sets a quantity if the difference in expected profits, given by the left hand side of (7), is negative. Differentiating the left hand side of (7) twice with respect to $\sigma_{\theta\epsilon}$ yields that it is convex if

$$16\beta^2(\beta^2 - \gamma^2)^2z^2 + \gamma^2(\gamma + \beta)(4\beta^3 - 8\beta^2\gamma + 3\beta\gamma^2 - \gamma^3)z + \gamma^4\beta^2 = h(\gamma) > 0,$$

which is the case if $\gamma < \gamma^+$. Thus, if $\gamma'' \leq \gamma < \gamma^+$, then for $\sigma_{\theta\epsilon} < \sigma_{\theta\epsilon}^{**}$ firm i prefers to set a quantity, while for $\sigma_{\theta\epsilon} \geq \sigma_{\theta\epsilon}^{**}$ it prefers a price.

Now consider the case $\gamma > \gamma^+$. This implies that $\sigma_{\theta\epsilon}^{**}$ is negative and the difference in expected profits is concave in $\sigma_{\theta\epsilon}$. Consequently, firm i prefers to set a quantity for every positive covariance.

Finally, it is easy to show that $\sigma_{\theta\epsilon}^{**} > \sigma_{\theta\epsilon}^*$ for all $\gamma'' \leq \gamma < \gamma^+$. This implies that $\gamma' > \gamma''$. So for $\gamma \geq \gamma'$ both firms select a quantity if $0 \leq \sigma_{\theta\epsilon} < \sigma_{\theta\epsilon}^*$. For $\gamma \geq \gamma''$ one firm selects a quantity and the other one a price if $\max[0, \sigma_{\theta\epsilon}^*] \leq \sigma_{\theta\epsilon} < \sigma_{\theta\epsilon}^{**}$. Both firms select prices if $\gamma < \gamma^+$ and $\sigma_{\theta\epsilon} \geq \max[0, \sigma_{\theta\epsilon}^{**}]$. ■

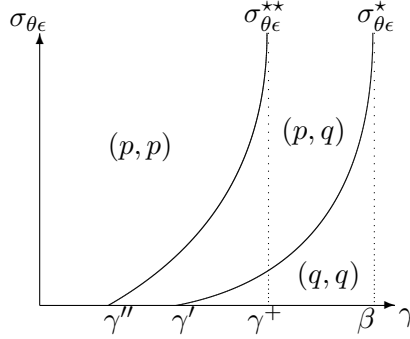


Figure 1: Outcome of the game

The outcome of the game is depicted in Figure 1. One can see that the degree of substitutability and the amount of uncertainty have offsetting effects.

The intuition behind the result is the following: the closer substitutes the two products are, the fiercer is competition but even more so under price competition than under quantity competition. Thus, the higher γ , the more firms are inclined to set quantities. If products are nearly perfect substitutes ($\gamma \rightarrow \beta$), then $\sigma_{\theta\epsilon}^* \rightarrow \infty$ and quantities are the preferred choice for every positive covariance.

By contrast, the expected profit of a price setting firm increases in every component of $\sigma_{\theta\epsilon}$ while that of a quantity setting firm decreases in σ_{θ} irrespective of the strategy variable chosen by the other firm. The underlying reason is that if demand shifts upwards it also becomes flatter in expectation, while it becomes steeper with a downward shift. Therefore with fixed prices, the quantity increase in a good demand state overcompensates in expectations the loss in a bad demand state. By contrast, if quantities are fixed the price drop in the bad state overcompensates in expectations the price increase in the good state. So if $\gamma \leq \gamma''$ we have that $\sigma_{\theta\epsilon}^{**} < 0$. Thus prices are preferred even if ρ and therefore $\sigma_{\theta\epsilon} = 0$ when demands are sufficiently independent.⁸

There also exists a hybrid equilibrium. Consider a $\gamma \in (\gamma'', \gamma^+)$. For $\sigma_{\theta\epsilon} = \sigma_{\theta\epsilon}^*$ uncertainty is sufficiently high such that firm i sets a price. Still firm j prefers to set a quantity because this softens competition. As a consequence, there always exists some intermediate range of $\sigma_{\theta\epsilon}$ in which firms play a hybrid equilibrium. Yet, in this equilibrium firm j 's profit

⁸Therefore the uncertainty effect is of particular importance in a monopolistic setting, see Reis (2006).

increases in the covariance by less than if it charged a price and so for $\sigma_{\theta\epsilon} \geq \sigma_{\theta\epsilon}^{**}$ the quantity setting firm leaves the hybrid equilibrium and ends up setting a price. For $\gamma \in [\gamma^+, \beta]$, the products are such close substitutes that even for an arbitrarily high covariance no (price, price) equilibrium emerges.

3 Conclusion

This paper shows that the superiority of quantity competition for firms might no longer hold if there is a substantial amount of uncertainty concerning demand conditions. It also provides the testable implication that if firms have some degree of choice about their strategy variable, they should tend to choose quantities in industries with relatively stable and certain demand, but choose prices if demand is fluctuating and uncertain.

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