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Competition in an Increasing Variety Growth Model*

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Abstract

This paper introduces competitive markets in the Grossman-Helpman [1991, ch. 3] increasing variety growth model. In this standard model of endogenous growth theory, competition has a negative incentive effect. Accordingly, a larger resource base is required to sustain long run growth. In an intermediate range, however, there is path dependence. In this case, too much initial competition may ultimately stall the growth process. Moreover, by introducing asymmetry in market-power, competition gives rise to static welfare losses. In economies with a small positive growth rate, welfare losses due to varying mark-up factors may be large enough to offset the benefits of growth.

JEL Classification: O34, O41

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1 Introduction

This paper introduces competitive markets in the Grossman-Helpman [1991, ch. 3] increasing variety growth model. In this standard model of endogenous growth theory, the prospect of earning future rents is the crucial incentive for innovation. If the notion of competition is such that it implies lower mark-up factors, an increase in the degree of competition will reduce the equilibrium growth rate by lowering the incentive to innovate. We show that for a given process of knowledge diffusion, long-run positive growth can be sustained as long as the resource base in the economy is sufficiently high. If the economy is endowed with an "intermediate" resource base however, it may find itself stuck in a no-growth trap depending on the initial degree of competition. If so, too much *initial* competition will ultimately stall the growth process. Thus, we make the case for less endowed economies to take their time when it comes to engage in ambitious competition policies.

Besides lowering the incentives to innovate, allowing for product market competition also introduces asymmetry. As different sectors typically face different degrees of competition, mark-up factors throughout the economy will differ. Accordingly, competition removes the somewhat artificial absence of static welfare losses in the standard model. Due to the coexistence of markets with different degrees of competition, relative prices do no longer adequately reflect marginal cost. Thus, household decisions are biased and labor is no longer allocated efficiently. We demonstrate that the associated loss in aggregate welfare can be large enough to offset the dynamic gains resulting from a small but positive growth rate. Thus, the asymmetry introduced by competition primarily adds the often claimed trade-off between static welfare losses and dynamic gains to the standard model.

The Grossman-Helpman model is swiftly described as follows. Economic growth is captured by an increasing number of varieties. The engine of growth is intentional innovation. R&D is fueled by the chances of extracting future rents from imperfectly competitive product markets. Labor is the only resource and can either be used in production or in R&D. Resources allocated in production increase the quantity of existing varieties while resources in R&D increase the knowledge of how to build new varieties. As new products become introduced to the economy, the incentives to innovate diminish just as the present value of future profits decreases with an increasing number of existing varieties (horizontal competition). Accordingly, it takes an opposed force to sustain long-run growth. This force is found in knowledge spill-overs. It is assumed that with any newly invented blueprint, there comes along an increase in knowledge capital easing future research. The net impact of knowledge spill-overs and diminishing returns finally determines the scale of long-run growth.

Originally, the blueprints for new varieties are assumed to be protected by infinitely lasting patents. Thus, profit maximizing firms are monopolies and perpetually charge a constant mark up over marginal cost. In what follows, we allow for competition in existing varieties (vertical competition). Vertical competition has previously been considered by Barro and Sala-i-Martin [1995], Walz [1995] and Arnold [1995] who analyze the impact of product imitation. More recently, competition by means of overlapping varieties originating from a countries opening up to trade have been considered by Tang and Wälde [2001] and Impullitti [2006]. Also, Gancia and Zilibotti [2006] spend a section of their chapter on horizontal innovation in the handbook of economic growth on the erosion of monopoly power in the variety type growth model. We extend the existing analysis along three lines. Firstly, presenting a global solution, we are able to consider stability issues and reveal the possibility of a growth trap. This secondly allows us to confirm the danger of a no-growth trap found by Tang and Wälde for the complete-overlap-open-economy in a closed economy. Thirdly, by explicitly accounting for static welfare losses due to unequal mark-ups we explore the possibility that a small but positive growth rate may be welfare inferior to zero growth. Far from suggesting actual policies, we point out that from a social planers view, small resource base economies may actually find it worthwhile not to engage in growth enhancing policies. Throughout the paper we stick to the growth mechanics of the variety model and thereby put up its weaknesses like e.g. the predicted strong scale effects (Jones [2004]) or the absence of escape-competition effects (Aghion, Harris and Howitt [2001]). In addition, our exclusive focus on the rent decreasing effect of competition puts aside, e.g. improvements of knowledge spillovers due to international trade or productivity effects of imitation (Arnold [1995]) which potentially qualify our results.

The rest of the paper is structured as follows. Section 2 introduces the baseline model. After clarifying our notion of competition (section 3), we present the imitation model in section 4. An analysis of different endowed economies is considered in the following subsections. The deregulation model is derived as a special case of the imitation model in section 5. Subsequently, we explore the micro perspective and show that the gains from a small positive growth rate may be offset by welfare losses due to market power. The final section draws some conclusions.

2 Baseline model

Households

There is a continuum of mass 1 of identical households and each is assumed to inelastically supply L units of labor. Their preferences exhibit an intertemporal elasticity of substitution equal to 1 and satisfy[1]

$$U(c) = \int_0^\infty e^{-\rho t} \ln c \; dt, \quad c = x^\sigma y^{1-\sigma}. \tag{1}$$

Consumption c consists of two goods x and y and instantaneous utility is discounted to time 0 using the subjective rate of time preference, ρ. For ease of interpretation, in this one resource economy, x and y may be regarded as representing capital intensive and less intensive goods, respectively.

Following Grossman and Helpman [1991], we take advantage of the fact that the associated indirect utility function only depends upon total consumption expenditures, E, and thus consider the problem of optimal consumption in two separate decisions. That is, we firstly derive the optimal intertemporal path of total consumption and consider the division of consumption expenditures onto both final goods afterwards.

Denote by B^{-1} consumers' stock of bonds. Maximizing intertemporal utility then requires maximizing the current value Hamiltonian

$$\mathcal{H} \equiv \ln c + \lambda_H (rB^{-1} + wL - E) \tag{2}$$

subject to expenditures and baring of securities. The first order conditions, which by the strict concavity of the Hamiltonian are sufficient to maximize

[1] For notational convenience, the derivative of any variable x with respect to time is represented by $\dot{x}$ and $\hat{x}$ denotes its proportional rate of change. Also, time arguments are suppressed whenever this causes no confusion.

discounted utility (see Mangasarian [1966]), read

$$E = \lambda_H^{-1} \tag{3}$$
$$\hat{\lambda}_H = \rho - r. \tag{4}$$

Given total expenditures on consumption, the optimal division is obtained by maximizing the Lagrangian

$$\mathcal{L} \equiv \sigma \ln x + (1-\sigma)\ln y + \lambda_L\left[E - p_x c_x - p_y y\right] \tag{5}$$

with respect to x, y and λ_L. Due to the Cobb-Douglas specification in (1), the first order conditions, which again are sufficient for a maximum, state that throughout time a constant share of expenditures is devoted to both goods.

$$p_x x = \lambda_L \sigma \tag{6}$$
$$p_y y = \lambda_L (1-\sigma) \tag{7}$$
$$E = p_x x + p_y y. \tag{8}$$

Using total expenditures E as numéraire, (3) requires constancy of λ_H such that, by (4), individuals rate of time preference has to equal the interest rate, $\rho = r(t)$. Moreover, by (6), (7) and (8), $E_y \equiv p_y y = 1 - \sigma$ and $E_x \equiv p_x x = \sigma$. However, although helpful from a technical point of view, normalizing expenditures to 1 in an economy of constant returns to scale in manufacturing implies a constant nominal GDP and accordingly, with an increasing number of blueprints, decreasing profits in the intermediate sector as well as constant wages.

Technology

Following Ethier [1982], we assume that x is produced competitively out of an increasing variety of intermediates j, each involved with quantity $x(j)$, according to a Dixit and Stiglitz [1977] specification:

$$c_x = \left[\int_0^n x(j)^\alpha dj\right]^{\frac{1}{\alpha}}, \quad \alpha \in (0,1) \tag{9}$$

Intermediate demand is chosen as to maximize profits and hence has to equal the marginal rate of substitution and the ratio of input prices for every combination of two intermediates j and j' (where $\epsilon \equiv 1/(1-\alpha)$):

$$\begin{aligned}
\left[\frac{x(j)}{x(j')}\right]^{\alpha-1} &= \frac{p(j)}{p(j')} \\
x(j) &= \left[\frac{p(j')}{p(j)}\right]^{\epsilon} x(j'). \qquad (10)
\end{aligned}$$

Using (10) to substitute for $p(j')x(j')$ in $E_x = \int_0^n p(j')x(j')dj' = \sigma$ yields the demand for each variety $x(j)$:

$$\begin{aligned}
\int_0^n \frac{p(j)^{\epsilon}}{p(j')^{\epsilon-1}} x(j)dj' &= \sigma \\
x(j)p(j)^{\epsilon} \int_0^n p(j')^{1-\epsilon}dj' &= \sigma \\
x(j) &= \frac{p(j)^{-\epsilon}}{\int_0^n p(j')^{1-\epsilon}dj'}\sigma. \qquad (11)
\end{aligned}$$

The demand for brand j has a constant elasticity of ϵ (> 1), i.e. more interchangeable intermediates (a c.p. increase in α) result in a more elastic demand.

Note that by (9) and (11), p_x is linear in each $p(j)$:

$$\begin{aligned}
c_x &= \left[\int_0^n \left(\frac{p(j)^{-\epsilon}\sigma}{\int_0^n p(j')^{1-\epsilon}dj'}\right)^{\alpha} dj\right]^{\frac{1}{\alpha}} \\
&= \frac{\left[\int_0^n p(j)^{1-\epsilon}dj\right]^{\frac{1}{\alpha}}}{\int_0^n p(j')^{1-\epsilon}dj'}\sigma \\
&= \sigma\left[\int_0^n p(j)^{1-\epsilon}dj\right]^{\frac{1}{\epsilon-1}}. \qquad (12)
\end{aligned}$$

Since $p_x c_x = \sigma$, we have $p_x = \left[\int_0^n p(j)^{1-\epsilon}dj\right]^{\frac{1}{1-\epsilon}}$. Accordingly, any mark-up pricing in the intermediate sector, weighted by some factor of equal mark-ups, directly shifts to the final sector and causes distortions once the mark-up factors in p_x and p_y differ.

For an intermediate brand to be producible, the research sector must come up with the according blueprint in the first place. In performing R&D,

researchers "stand on the shoulders of giants" and rely on public knowledge from previous research. We adopt the Romer [1990] specification for the R&D technology with knowledge spillovers and set knowledge capital equal to the number of existing varieties, n:

$$\dot{n} = \frac{nL_R}{a}, \quad a > 0 \tag{13}$$

Since ρ is assumed to be positive, the present value of utility in (1) is finite.

Labeling $\hat{n} \equiv g$, there are $L_R = ag$ workers employed in the research sector. We assume further that labor has an input coefficient equal to 1 in the production of intermediates, $x(j) = L_x(j)$, $L_x \equiv \int_0^n L_x(j)dj$. The y producers simply employ L_y workers, each of whom is assumed to produce one unit of output without using further inputs.

Erosion of market power

Initially, newly invented varieties will always be supplied by a monopolist. The notion of competition is to erode the market power of incumbents. For ease of exposition, we concentrate on the two polar cases of mark-up factors. Competition occurs by means of perfect substitutes introduced to the economy. In this case, we assume that firms compete in prices and thus face zero profits.

Firms enjoying monopoly power will maximize operating profits against the static demand given by (11), i.e. set $x'(p) = x(p)/(w - p)$. Solving for the monopoly price yields

$$p^m = \frac{w}{\alpha}. \tag{14}$$

With constant returns to scale, zero profits in competitive markets are maintained by charging a price equal to average operating cost:

$$p^c = w \tag{15}$$

In what follows, $\underline{n}$ and θ denote the number of competitive intermediate markets and the accordant share, respectively.

3 Introducing competition

We consider the introduction of perfect substitutes due to imitation (Arnold [1995]) and industrial policy. A main difference between the two is that in a setting with continued imitation, firms anticipate the threat of loosing monopoly power while the industrial policy framework is taken as given, at least to the extent that there is no anticipation of shocks when firms invest in new products. Transforming monopolistic into perfectly competitive markets finally leads to a closed economy paradigm of a situation in which a country opens up to international trade and suddenly faces perfect substitutes in some sectors (Tang and Wälde [2001]). The model presented here may also shed some light on why large scale privatization may have ambiguous effects on economic growth in different countries.

Imitation

With ongoing imitation, monopoly profits accrue in every period until a perfect substitute is released. This arrival rate is exogenous, i.e. we deny purposive imitation to be a strategic choice for the firms. The imitation technology is assumed to imply

$$\dot{\underline{n}} = \psi(n - \underline{n}) \tag{16}$$

where ψdt is the imitation probability. Monopolists capture the threat of loosing their market power by adding ψ to the discount rate in every moment in time. Albeit intuitive, this can be shown as follows. Denote by $\Psi(t, \tau+\Delta)$ the probability that no imitation occurs in a time span $(t, \tau+\Delta)$. Exogenously given imitation implies the lack of memory, i.e. $\Psi(t, \tau + \Delta) = \Psi(t, \tau)\Psi(\tau, \tau + \Delta)$. By definition, the latter factor equals $1 - \psi\Delta$, with Δ a short period of time. Thus,

$$\lim_{\Delta \to 0} \frac{\Psi(\tau, \tau + \Delta) - \Psi(t, \tau)}{\Delta} = \frac{d\Psi(t, \tau)}{d\tau} = -\Psi(t, \tau)\psi$$

The solution to this variable coefficient first order differential equation yields the probability of still earning monopoly rents τ periods after t. Recognizing $\Psi(t, t) = 1$, it reads $\Psi(t, \tau) = \exp[-\int_t^\tau \psi dz]$.

Industrial policy

In this section, we address the impact of the market structure of an economy on its growth path. That is, we consider the implications of a set of (preliminary given) rules for market entry, bureaucracy requirements, adoption of technologies, patents, etc. that indirectly determines the share of markets in which firms operate in a competitive environment.

With respect to the y good, we explore the existence of goods that are supplied competitively no matter what the considered set of rules looks like. Recapitulate that our first idea of y was a good whose production is barely capital intensive. Thus, there are probably no large bureaucratic burdens to overcome and their production process is likely to be easily replicable. Additionally, y may incorporate goods whose patent rights may not be protected or blueprints that are traditionally known. Also, there may exist industries with risk-free and otherwise barely costly start-up procedures and some sectors where mark-up pricing can not be tolerated "politically".

Departures from the standard model

Technically speaking, we depart from the standard Grossman-Helpman model in two related aspects. On the one hand, there is (severe) competition in the supply of a fraction of Dixit-Stiglitz varieties. On the other hand, we avoid the artificial absence of static monopoly distortions implied by equal mark-up pricing in the standard model. This is accomplished by assuming the permanent existence of a competitive sector.

Without both a potential growth enhancing escape competition effect and static distortions, a benevolent social planner would not have any incentive to maintain competition. If it was not for positive R&D spillovers through public knowledge capital, allocation in the standard variety model economy were efficient. Here in contrast, consumers always spend a constant fraction $(1-\sigma)$ of total expenditures to purchase c_y. This can easily be seen by multiplying (11) by $p(j)$ and summing up over all n varieties j to get spending on c_x:

$$\int_0^n p(j)x(j)dj = \int_0^n \frac{\sigma p(j)^{1-\epsilon}}{\int_0^n p(j')^{1-\epsilon}dj'}dj = \sigma \frac{\int_0^n p(j)^{1-\epsilon}dj}{\int_0^n p(j')^{1-\epsilon}dj'} = \sigma \qquad (17)$$

Thus, even in the absence of competition in any of the j-markets ($\theta = 0$), the market power of firms in the economy differs whereby some of the relative prices do not adequately reflect marginal cost. Accordingly, households base the splitting of total consumption on distorted prices and monopoly profits always give rise to the well-known trade-off between static welfare losses and dynamic gains from an increased incentive to innovate (see the note given after equation 11).

4 The imitation model

Static equilibrium

Denote by v and w the value of a variety and the wage rate, respectively. In equilibrium, an active research sector must yield zero profits to avoid both an incentive for entry and exit. Accordingly, free entry to R&D requires at most zero profits,

$$wa \geq nv \;\equiv V^{-1} \tag{18}$$

with equality whenever $\dot{n} > 0$. In any period t, v equals the present value of expected future profits, π:

$$v(t) \equiv \int_t^\infty e^{-(\rho+\psi)(\tau-t)}\pi(\tau)d\tau \tag{19}$$

Taking the derivative with respect to t, $\dot{v(t)} = -\pi(t) + \int_t^\infty \pi(\tau)e^{-(\rho+\psi)(\tau-t)}(\rho + \psi)d\tau$, and substituting for the definition of v, we get the no arbitrage condition that characterizes capital market equilibrium:

$$\begin{aligned} (\rho+\psi)v(t) &= \pi(t) + \dot{v}(t) \\ \pi(t) + \dot{v}(t) - \psi v(t) &= \rho v(t) \end{aligned} \tag{20}$$

In expectations, the value of dividend payments (π) and changes in the value of capital due to growth and imitation ($\dot{v} - \psi v$) have to equal the return on a riskless bond ($\rho v = rv$).

Now consider monopoly profits. Note that by (14) and (15), $\int_0^n p(j)^{1-\epsilon}dj = w^{1-\epsilon}\underline{n} + \left(\frac{w}{\alpha}\right)^{1-\epsilon}(n - \underline{n})$. Hence, revenues in the competitive intermediate markets add up to

$$\underline{E}_x \equiv \int_0^{\underline{n}} p(j)x(j)dj = \int_0^{\underline{n}} \frac{\sigma}{\underline{n} + \alpha^{\epsilon-1}(n-\underline{n})}dj = \frac{\sigma\theta}{\theta(1-\alpha^{\epsilon-1}) + \alpha^{\epsilon-1}}.$$

Accordingly, the sum of revenues for the monopolists is given by

$$\overline{E}_x \equiv \int_{\underline{n}}^{n} p(j)x(j)dj = \frac{\sigma(n-\underline{n})\alpha^{\epsilon-1}}{\underline{n}+\alpha^{\epsilon-1}(n-\underline{n})} = \frac{\sigma(1-\theta)\alpha^{\epsilon-1}}{\theta(1-\alpha^{\epsilon-1})+\alpha^{\epsilon-1}}.$$

Using $\overline{n}xp = \overline{E}_x$, we may express each monopolists profit, $\pi = (1-\alpha)px$, in terms of θ:

$$\pi = \frac{\sigma(1-\alpha)}{\left[1-\theta(1-\alpha^{1-\epsilon})\right]n} \tag{21}$$

Monopoly profits thus decline both in the share of competitive intermediate markets as well as in the number of available blueprints and increase with the desire for the y good.

Two remarks concerning (21) are in order. Firstly note that in an economy with a constant number of blueprints, (19) guarantees the value of an innovation to remain strictly positive:

$$v(t) = \frac{\sigma(1-\alpha)}{\left[1-\theta(1-\alpha^{1-\epsilon})\right]n(\rho+\psi)} \tag{22}$$

Secondly, since $\frac{\partial v}{\partial n} < 0$, the right hand side of (22) is an upper bound for the value of blueprints in a growing economy in any point of time.

Next, rewrite labor demand in the production sectors:

$$\begin{aligned}
\overline{L}_x &= \frac{\overline{E}_x}{p} = \frac{(1-\theta)\alpha^{\epsilon}}{\theta(1-\alpha^{\epsilon-1})+\alpha^{\epsilon-1}}\frac{\sigma}{w} \\
\underline{L}_x &= \frac{\theta}{\theta(1-\alpha^{\epsilon-1})+\alpha^{\epsilon-1}}\frac{\sigma}{w} \\
L_y &= \frac{1-\sigma}{w}
\end{aligned} \tag{23}$$

Finally, each household is assumed to supply L units of labor inelastically, hence labor market clearing requires

$$L = ag + L_x + L_y \tag{24}$$

Using (23) in (24) yields

$$L = ag + \frac{\sigma}{w}\left[\frac{\theta(1-\alpha^{\epsilon})+\alpha^{\epsilon}}{\theta(1-\alpha^{\epsilon-1})+\alpha^{\epsilon-1}} + \frac{1-\sigma}{\sigma}\right]. \tag{25}$$

A static equilibrium is simply a labor allocation satisfying (18), (20) and (25).

Dynamic Equilibrium

If the economy grows at positive rate, the free entry in R&D condition (18) holds with equality such that, in this case, $V = (wa)^{-1}$. Additionally, by (13), the growth rate of knowledge is restricted to non negative values. Taken together, solving (25) for g gives

$$\begin{aligned} g &= \max\left\{0,\ \frac{L}{a} - \frac{\theta\left[(1-\alpha^{\epsilon})\sigma + (1-\sigma)(1-\alpha^{\epsilon-1})\right] + \alpha^{\epsilon}\sigma + (1-\sigma)\alpha^{\epsilon-1}}{\theta(1-\alpha^{\epsilon-1}) + \alpha^{\epsilon-1}} V\right\} \\ &= \max\left\{0,\ \frac{L}{a} - \frac{1-\theta[1-\alpha^{1-\epsilon}-(1-\alpha)\sigma] - (1-\alpha)\sigma}{1-\theta(1-\alpha^{1-\epsilon})} V\right\}. \end{aligned} \tag{26}$$

Since the coefficient of V is positive (consider the upper row in (26)), the first argument of the max operator becomes binding iff $V \geq \tilde{V}$ where $\tilde{V}$ is defined such that it stashes away the second argument:

$$\tilde{V}(\theta) \equiv \frac{L}{a}\frac{1-\theta(1-\alpha^{1-\epsilon})}{1-\theta[1-\alpha^{1-\epsilon}-(1-\alpha)\sigma] - (1-\alpha)\sigma} \tag{27}$$

By construction, $V \geq \tilde{V}$ yields zero growth whereas $V < \tilde{V}$ potentially results in the positive growth rate given by the second argument of the max operator in (26). On the relevant unit interval, $\tilde{V}(\theta)$ is continuous, strictly decreasing and strictly convex,

$$\begin{aligned} \tilde{V}'(\theta) = \frac{L}{a}\frac{\alpha^{\epsilon-1}\sigma(\alpha-1)}{(.)^2} &< 0 \\ \tilde{V}''(\theta) = \frac{L}{a}\frac{\alpha^{\epsilon-1}\sigma(1-\alpha)\left[2(.)(1-\alpha^{\epsilon})\sigma + (1-\sigma)(1-\alpha^{\epsilon-1})\right]}{(.)^4} &> 0 \end{aligned}$$

with $\tilde{V}(0) = L/a[1-(1-\alpha)\sigma]^{-1}$ and $\tilde{V}(1) = L/a$.

The dynamics of the system are fully characterized by the equations of motion for V and θ. Since $\hat{V} = -\hat{v} - g$ by definition and $\hat{v} = \rho + \psi - \pi/v$ in the capital market equilibrium, see (20), after inserting monopoly profits (21), we get the law of motion for V:

$$\hat{V} = \frac{(1-\alpha)\sigma}{1-\theta(1-\alpha^{1-\epsilon})}V - (\rho+\psi) - g. \tag{28}$$

The share of competitive intermediates evolves according to $\dot{\theta} = \underline{\dot{n}}/n - \theta g$. Taking the imitation technology (16) into account yields the law of motion for θ:

$$\hat{\theta} = \frac{1-\theta}{\theta}\psi - g \tag{29}$$

Accordingly, if the number of existing blueprints remains constant, the increase in the share of competitive markets is the smaller, the larger θ (using (16) in $\hat{\theta} = \underline{\hat{n}}$ yields $\dot{\theta} = (1-\theta)\psi$).

To begin with, concentrate on $g = 0$, i.e. $V \geq \tilde{V}$. According to (28), V is constant on

$$\bar{V}_0(\theta) = (\rho + \psi)\frac{1 - \theta(1 - \alpha^{1-\epsilon})}{(1-\alpha)\sigma}, \quad V = 0 \tag{30}$$

$$\bar{V}_0' = \frac{(\rho + \psi)(\alpha^{1-\epsilon} - 1)}{(1-\alpha)\sigma} > 0 \tag{31}$$

where $\bar{V}_0(0) = (\rho + \psi)/[(1-\alpha)\sigma]$ and $\bar{V}_0(1) = (\rho + \psi)/[(1-\alpha)\alpha^{\epsilon-1}\sigma]$. Equivalently, by (29), the $\dot{\theta} = 0$ locus in the zero growth area reads $\bar{\theta}_0 = 1$. The slopes of $\bar{V}_0$ and $\tilde{V}$ involve that $\bar{V}_0(\theta)$ is irrelevant as long as $\tilde{V}(1) > \bar{V}_0(1)$. That is, an economy has the potential for long-run growth if the resource base is large enough to satisfy

$$\frac{L}{\alpha a} > \frac{\rho + \psi}{(1-\alpha)\alpha^{\epsilon}\sigma}. \tag{A1}$$

If, however, $\bar{V}_0(0) \geq \tilde{V}(0)$, $\bar{V}_0(\theta)$ is relevant on the whole unit interval. Accordingly,

$$\frac{L}{\alpha a} < \frac{(\rho + \psi)[1 - (1-\alpha)\sigma]}{(1-\alpha)\alpha\sigma} \tag{A2}$$

describes a resource base too low to generate a positive growth steady state.

As both $\tilde{V}$ and $\hat{V}$ are strictly monotone, if (A1) and (A2) do not hold, they uniquely intersect on the unit interval, and hence the possibility for positive growth exists.

Next, consider the strictly positive growth area, i.e. $V < \tilde{V}$. By (28), V is constant on

$$\bar{V}(\theta) \equiv \left(\rho + \psi + \frac{L}{a}\right) \frac{1 - \theta(1 - \alpha^{1-\epsilon})}{1 - \theta[1 - \alpha^{1-\epsilon} - (1-\alpha)\sigma]}, \; V = 0 \qquad (32)$$

where $\bar{V}(0) = \rho + \psi + L/a$ and $\bar{V}(1) = \left(\rho + \psi + \frac{L}{a}\right)\left[1 + (1-\alpha)\alpha^{\epsilon-1}\sigma\right]^{-1}$ $(< \bar{V}(0))$. For $\theta \in (0,1)$, the $\dot{V} = 0$ locus is strictly decreasing and strictly convex (see appendix 7.1).

By (29), the $\dot{\theta} = 0$-locus reads

$$\bar{\theta}(\theta) \equiv \left(\frac{L}{a} - \frac{1-\theta}{\theta}\psi\right) \frac{1 - \theta(1 - \alpha^{1-\epsilon})}{1 - \theta\left[1 - \alpha^{1-\epsilon} - (1-\alpha)\sigma\right] - (1-\alpha)\sigma} \qquad (33)$$

$$= \tilde{V}(\theta) - \frac{1-\theta}{\theta}\psi \frac{1 - \theta(1 - \alpha^{1-\epsilon})}{1 - \theta\left[1 - \alpha^{1-\epsilon} - (1-\alpha)\sigma\right] - (1-\alpha)\sigma} \qquad (34)$$

where $\bar{\theta}(1) = L/a$ and $\bar{\theta}(\theta) = 0$ once on the unit interval, $\bar{\theta}^{-1}(0)_1 = \left(L/(\alpha\psi) + 1\right)^{-1}$, and once on the irrelevant negative real line, $\bar{\theta}^{-1}(0)_2 = (1 - \alpha^{1-\epsilon})^{-1}$.

Note that by (34), $\bar{\theta} \leq \tilde{V}(\theta)$ on $(0,1]$ with equality if $\theta = 1$. Some technical remarks characterizing the run of $\bar{\theta}(\theta)$ are in order. Firstly, (33) exhibits two discontinuity points on the real line, namely $\theta = 0$ and $\theta = [1 - (1-\alpha)\sigma]/[1 - \alpha^{1-\epsilon} - (1-\alpha)\sigma] < 0$. Secondly, at the former point (see appendix 7.2),

$$\lim_{\theta \to 0^{+(-)}} \bar{\theta}(\theta) = -(+)\infty \qquad (35)$$

while at the latter,

$$\lim_{\theta \to \frac{1-(1-\alpha)\sigma}{1-\alpha^{1-\epsilon}-(1-\alpha)\sigma}^{+(-)}} \bar{\theta}(\theta) = (-)\infty. \qquad (36)$$

Thirdly, with $N(\theta) \equiv 1 - \theta[1 - \alpha^{1-\epsilon} - (1-\alpha)\sigma] - (1-\alpha)\sigma$,

$$\begin{aligned} \bar{\theta}'(\theta) = \frac{1}{N(\theta)} \Bigg\{ & \left(\frac{L}{a} - \frac{1-\theta}{\theta}\psi\right) \times \\ & \times \frac{N(\theta)(\alpha^{1-\epsilon} - 1) + \left[1 - \theta(1 - \alpha^{1-\epsilon})\right]\left[1 - \alpha^{1-\epsilon} - (1-\alpha)\sigma\right]}{N(\theta)} + \\ & + \frac{\psi}{\theta^2}\left[1 - \theta(1 - \alpha^{1-\epsilon})\right] \Bigg\} \end{aligned} \qquad (37)$$

and hence

$$\bar{\theta}'(1) = \psi - (1-\alpha)\alpha^{\epsilon-1}\sigma\frac{L}{a}. \qquad (38)$$

Preliminaries

In this section, we will discuss the run of both the $\dot{\theta} = 0$ and $\dot{V} = 0$ locus under different parameter constellations. We will start with an economy that has sufficient resources to satisfy (A1) and afterwards turn to less endowed economies.

Under (A1), $\bar{\theta}(1)' < 0$ since then $L/(\alpha a) > \psi/[(1-\alpha)\alpha^\epsilon\sigma]$. Thus, (38) and (35) imply that $\bar{\theta}$ exhibits a maximum on $(0, 1)$. To see that this maximum is unique, consider (37) which restricts the number of extreme values for $\bar{\theta}$ to at most two, since, if they indeed exist, they are implicitly given by the roots of the second order polynomial (the derivation is delegated to appendix 7.5):

$$\left\{1 - \theta\left[1 - \alpha^{1-\epsilon} - (1-\alpha)\sigma\right] - (1-\alpha)\sigma\right\}\left[1 - \theta(1 - \alpha^{1-\epsilon}\right]\psi = (1-\alpha)\alpha^{1-\epsilon}\sigma\left[\theta\frac{L}{a} - (1-\theta)\psi\right]\theta \qquad (39)$$

Both sides of this equation are quadratic with positive leading coefficients (explicitly shown in appendix 7.3), thus both are rising on the left as well as on the right. Define the left-hand side as $\zeta(\theta)$ and the right-hand side as $\xi(\theta)$.

Then, $\zeta(0) = \alpha^{2\epsilon-1}\psi > 0$ and $(\zeta')^{-1}(0) < 0$ (see appendix 7.3). With respect to $\xi(\theta)$, $\xi(0) = 0$ and $(\xi')^{-1}(0) = \psi/[2(L/a + \psi)] > 0$. Taken together $\bar{\theta}$ has indeed two extreme values ($\exists\theta$ s.t. $\zeta = \xi \neq 0$), and one is located on $((\zeta')^{-1}(0), 0)$. Thus, if there is a maximum on the unit interval, it is unique. Note that ζ and ξ intersect at $\theta = 1$, iff the resource base satisfies $L/(\alpha a) = \psi/[(1-\alpha)\alpha^\epsilon\sigma]$ which is lower than required by (A1). We know from appendices 7.3 and 7.4 that $\zeta(\theta)'$ does not depend on L/a whereas $\partial\xi'(\theta)/\partial(L/a) > 0$ if $\theta > 0$. Accordingly, if (A1) holds, there exists an intersection and hence a maximum on $((\xi')^{-1}(0), 1)$.
Moreover, observing $1 - (1-\alpha)\sigma < \alpha^{1-\epsilon}$, (A1) implies $\bar{V}(0) < \tilde{V}(0)$ and directly gives $\bar{V}(1) < \tilde{V}(1) = \bar{\theta}(1)$. Recognizing (35) and $\bar{V}(0) > 0$, the latter observations imply an intersection of $\bar{V}(\theta)$ and $\bar{\theta}(\theta)$ on the unit interval. In addition, under (A1) they imply that the number of intersections on the unit interval must be odd. Recognizing the common multiplicative factor in (33) and (32), which vanishes for $\bar{\theta}^{-1}(0)_2 < 0$, another intersection is located on $(-\infty, 0)$. Since equating both loci after canceling the common term clearly does not yield a cubic polynomial at the least (see the up-coming equation 40), it has to be that the interior steady state is unique.

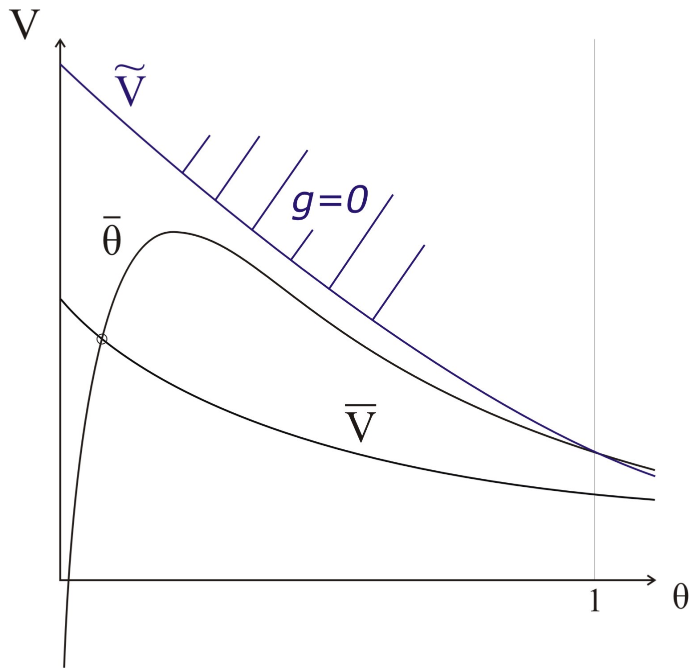


Figure 1: An economy satisfying (A1).

It is located in the $g > 0$ region, see (34). The situation in which the resource base is "large" in the sense that it suffices (A1) is depicted in figure 1.

As an aside, note that resources $L/(\alpha a) < \psi/[(1-\alpha)\alpha^{\epsilon}\sigma]$ imply that $\bar{\theta}$ does not exhibit an extremum on the unit interval ($\bar{\theta}'(1) > 0$ in combination with the discussion on the run of $\bar{\theta}$ above). In this case , there exists no intersection of $\bar{V}(\theta)$ and $\bar{\theta}(\theta)$ in the relevant interval since the condition that ruled out the interior maximum of $\bar{\theta}$ also rules out $\bar{\theta}(1) \geq \bar{V}(1)$, which would require

$$\frac{L}{a} \quad \geq \quad \frac{\rho + \psi}{\alpha^{\epsilon - 1}(1-\alpha)\sigma} \quad > \quad \frac{\psi}{\alpha^{\epsilon-1}(1-\alpha)\sigma}.$$

Such a "small" resource base economy is depicted in figure 2.
We are yet missing resource bases ranging between the previous mentioned boundaries, i.e.

$$\frac{\psi}{(1-\alpha)\alpha^{\epsilon}\sigma} < \frac{L}{\alpha a} < \frac{\psi + \rho}{(1-\alpha)\alpha^{\epsilon}\sigma}. \tag{A3}$$

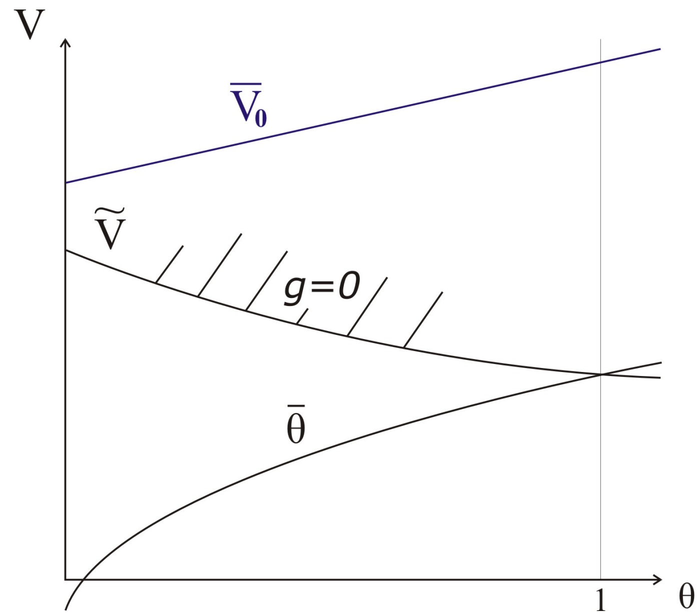


Figure 2: An economy satisfying (A2).

Here, $\bar{\theta}$ again exhibits an unique maximum on the unit interval. The corresponding highest V value will exceed L/A since $\bar{\theta}(1) = L/a$ and $\bar{\theta}'(1) < 0$. Also, $\bar{V}(1) > L/a$ since assuming the contrary implies a contradiction:

$$\begin{aligned} \bar{V}(1) \;=\; \frac{\rho + \psi + \frac{L}{a}}{1 + (1-\alpha)\alpha^{\epsilon-1}\sigma} &\;\leq\; \frac{L}{a} \\ \frac{\rho + \psi}{(1-\alpha)\alpha^{\epsilon}\sigma} &\;\leq\; \frac{L}{\alpha a} \end{aligned}$$

As mentioned above, equating $\bar{\theta}$ and $\bar{V}$ offers at most two intersections on $(0, \infty)$. Thus, if both loci intersect on the unit interval, they do so twice since $\bar{V}(0) > \bar{\theta}(0)$ and $\bar{V}(1) > L/a$. If they do not, we are back in the case with no interior intersection described in the last paragraph. If they do, however, by (34) there exist two interior steady states which, by (34) are located in the positive growth region. The two cases are depicted in figures 3 and 4.

We further explore this possibility by equating (32) and (33). After dropping an irrelevant negative solution, potential steady states are implicitly given by (derivation see appendix 7.6):

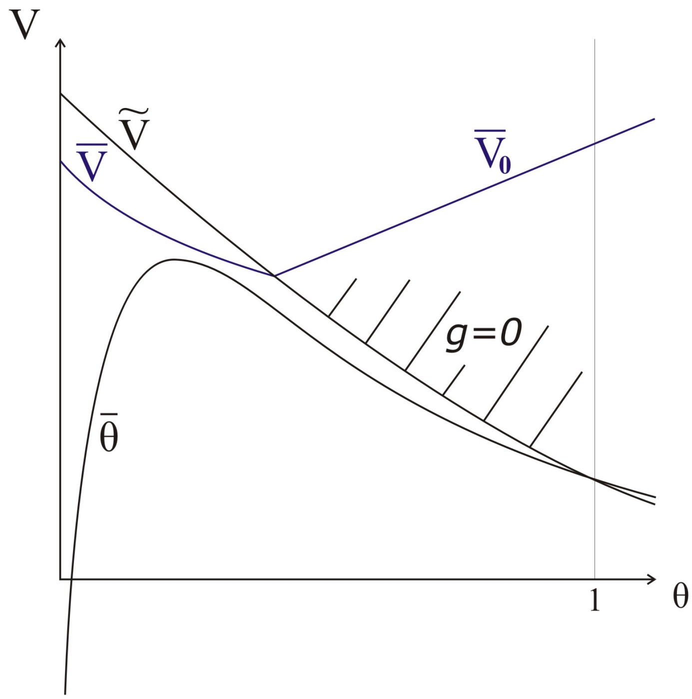


Figure 3: An economy satisfying (A3) without intersection of $\bar{V}$ and $\bar{\theta}$.

$$\theta^2 \rho A + \theta \left\{ \frac{L}{a}(1-\alpha)\sigma + \psi(1-\alpha^{1-\epsilon}) - \rho\left[1-(1-\alpha)\sigma\right] \right\} - \psi = 0 \tag{40}$$

$$(1-\theta)\psi = \theta^2 \rho A + \theta \left\{ \frac{L}{a}(1-\alpha)\sigma - \psi\alpha^{1-\epsilon} - \rho\left[1-(1-\alpha)\sigma\right] \right\} \tag{41}$$

where $A \equiv 1 - \alpha^{1-\epsilon} - (1-\alpha)\sigma < 0$. For (41) to hold in the relevant area,

$$\frac{L}{\alpha a} > \frac{\psi + \rho\alpha^{\epsilon-1}\left[1-(1-\alpha)\sigma\right]}{(1-\alpha)\alpha^{\epsilon}\sigma} \tag{42}$$

since otherwise the second coefficient of $r(\theta)$ is negative or null, and hence $l(\theta)$ and $r(\theta)$ could never balance in sign. Define the left-hand side of (41) as $l(\theta)$ and the right-hand side as $r(\theta)$. Then, $l(\theta)$ is a downward sloping straight line through $l(0) = \psi$ and $l(1) = 0$. $r(\theta)$ has a negative leading coefficient, thus is hump shaped, and passes through the origin with positive slope (the

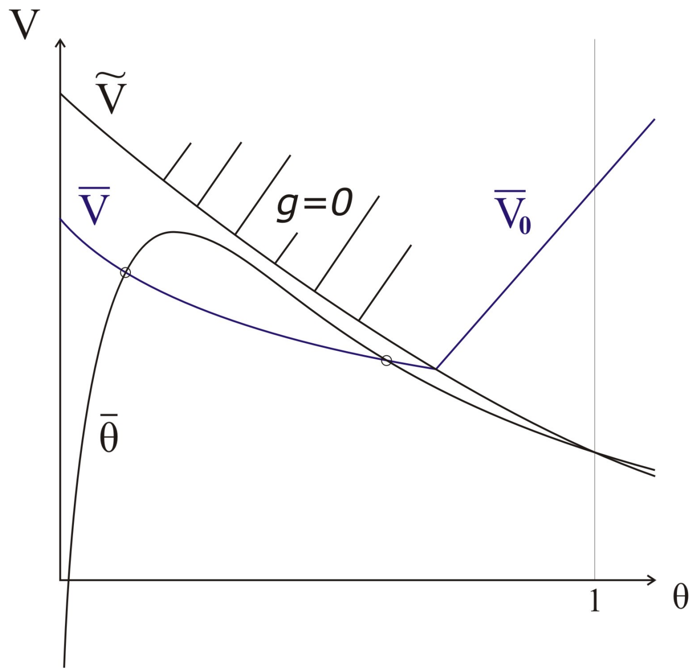


Figure 4: An economy satisfying (A3) with two intersections of $\bar{V}$ and $\bar{\theta}$

necessary condition for two interior intersections of $\bar{\theta}(\theta)$ and $\bar{V}(\theta)$ given by (42) amounts to require $r'(0) > 0$. Also, with a resource base ranging between the borders given by (A3), $r(1) < 0$ (see appendix 7.7). That is, if $r(\theta)$ intersects with $l(\theta)$, the (usually) two intersections are located on $\theta \in (0, 1)$. The possible cases for these "intermediate" economies are depicted in figure (7.6).

Solving (A1) and (A2) for ψ, we have

$$\frac{L}{\alpha a}\left[(1-\alpha)\alpha^{\epsilon}\sigma\right] - \rho \quad > \quad \psi \tag{43}$$

$$\frac{L}{\alpha a}\frac{(1-\alpha)\alpha\sigma}{1-(1-\alpha)\sigma} - \rho \quad < \quad \psi. \tag{44}$$

In figure 6, we plot (43) and (44) in $(L/(\alpha a), \psi)$-space to get a first impression of the endowments leading to no interior intersections of the zero growth loci for θ and V (region I) and one unique intersection (region II), respectively.

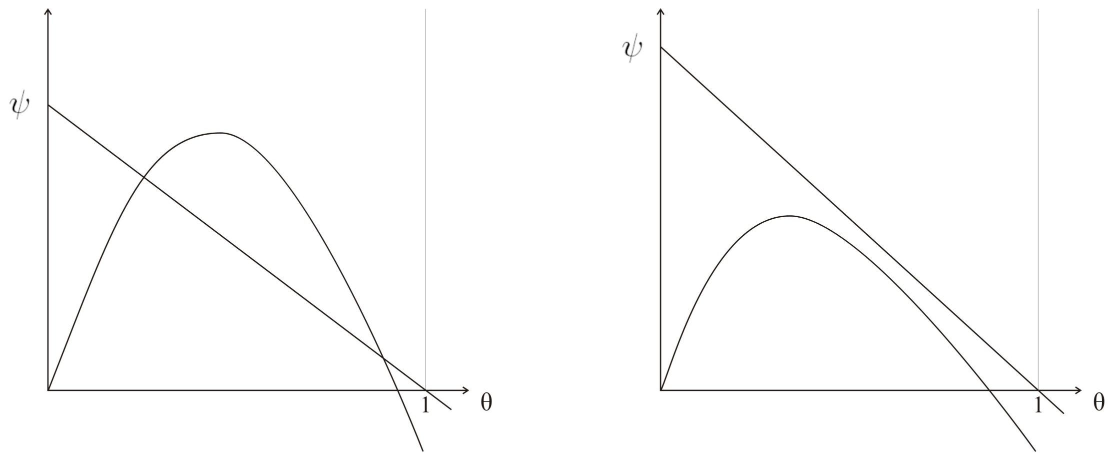


Figure 5: Two or no interior intersections of $\bar{\theta}(\theta)$ and $\bar{V}(\theta)$.

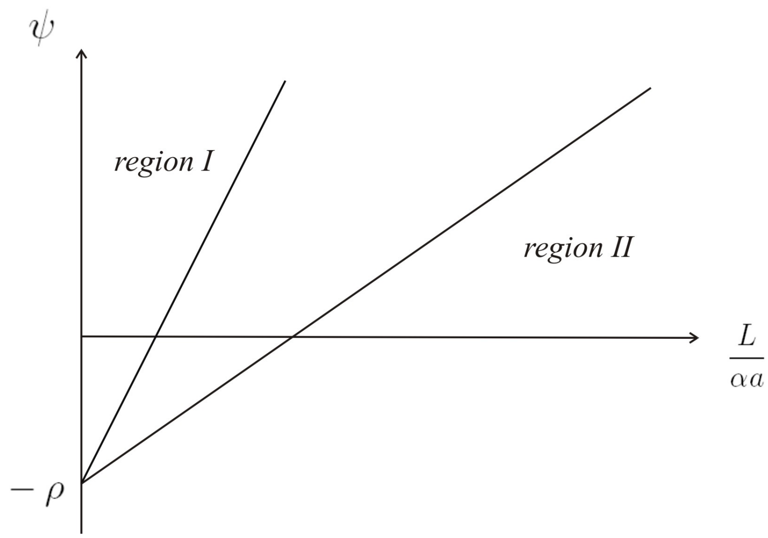


Figure 6: Parameters implying no (region I) and one intersection (region II) of $\bar{\theta}$ and $\bar{V}$.

Two interior steady states may only occur in the region between region I and region II. Solving (40) for θ, and labeling the second coefficient of the polynomial by B, two solutions emerge if the radicand in $\sqrt{B^2+4\psi\rho A}$ is positive.

Accordingly, $B^2 + 4\psi\rho A = 0$ implicitly defines a correspondence $\psi(L/(\alpha a))$ that cuts off the region where no interior steady state occurs:

$$\left\{\frac{L}{a}(1-\alpha)\sigma + \psi(1-\alpha^{1-\epsilon}) - \rho\left[1-(1-\alpha)\sigma\right]\right\}^2 + 4\psi\rho A = 0 \tag{45}$$

Implicit differentiation of (45) gives the slope of the cut-off line as

$$\frac{\partial\psi}{\partial\left(\frac{L}{\alpha a}\right)} = -\frac{(1-\alpha)\alpha\sigma}{1-\alpha^{1-\epsilon}+\frac{2\rho[1-\alpha^{1-\epsilon}-(1-\alpha)\sigma]}{B}} \quad > 0. \tag{46}$$

The inequality sign hereby follows from $B > 0$ which is satisfied for $L/(\alpha a)$ in the range between the critical values given by (A1) and (A2):

$$\begin{aligned}\frac{L}{\alpha a} \quad &\geq \quad \frac{\left\{\left[1-(1-\alpha)\sigma\right]\rho - (1-\alpha^{1-\epsilon})\psi\right\}\alpha^{\epsilon-1}}{(1-\alpha)\alpha^{\epsilon}\sigma} \\ &= \frac{\left[1-(1-\alpha)\sigma\right]\rho - (1-\alpha^{1-\epsilon})\psi}{(1-\alpha)\alpha\sigma}.\end{aligned}$$

If B were 0, i.e. $L/(\alpha a)$ were equal to the border given by (A2) with equality, the slope and also the value for ψ were 0 ($4\psi\rho A = 0$). Figure 7 qualitatively includes the correspondence implicitly given by (6) in the previous considered separation of parameter space (a numerical example is provided in appendix 7.8). Two interior steady states occur in the hatched area.

We conclude that, from a theoretical point of view, two interior steady states may occur if the resource base is just not large enough to generate long-run positive growth in the presence of imitation. The calculation in appendix 7.9 provides a numerical example.

For further reference, we make the following definition:

Definition 1 (Scale of the resource base)**.** *A resource base implying no (two, one) interior intersection of $\bar{V}(\theta)$ and $\bar{\theta}(\theta)$ is henceforth called "low" ("intermediate", "large").*

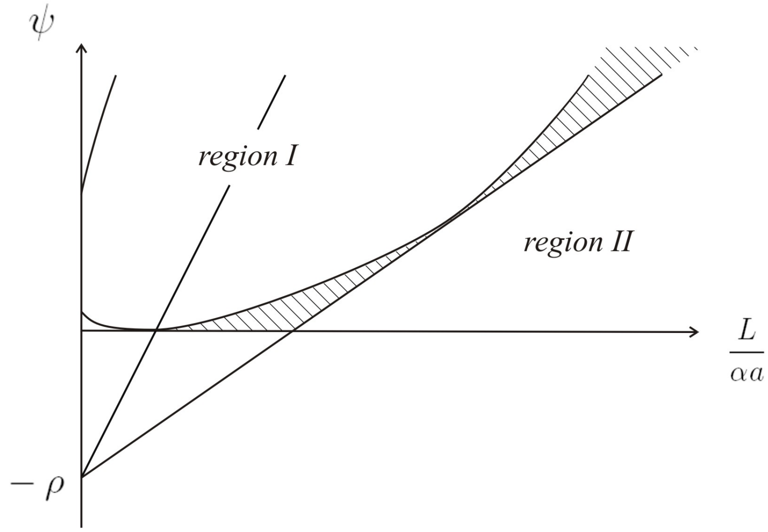


Figure 7: Two interior steady states occur in the hatched area.

We will solve the model for all three possibilities by considering the associated phase diagrams in turn. In the intermediate case in which $\bar{V}(\theta)$ and $\bar{\theta}(\theta)$ do not intersect, the dynamics are analogous to those resulting in figure 3.

Solution

Observe firstly that the $\dot{V} = 0$ locus is always instable:

$$\left.\frac{\partial \dot{V}}{\partial V}\right|_{\bar{V}_{(0)}} = \frac{(1-\alpha)\sigma}{1-\theta(1-\alpha^{\epsilon-1})} V \quad > 0$$

Accordingly, starting from below or above $\bar{V}_{(0)}$ would imply $V \to \infty$ and $V \to 0$ in the long run.

Lemma 1. *Paths that ultimately implying either $V = 0$ or $V \to \infty$ violate rational expectations.*

Proof: In line with the reasoning in Grossman and Helpman [1991]. Firstly, if V were to approach infinity, $nv \to 0$ by definition. Since $g = 0$, this requires $v \to 0$, but according to (22), $v > 0$. Secondly, $V = 0$ also offers a contradiction since, as noted before, in a growing economy v is smaller than the (positive) right-hand side of (22), hence $V > 0$.

Secondly, let $\Theta(\theta, V) \equiv \dot{\theta} = (1-\theta)\psi - \theta g(\theta, V)$. By definition, $\Theta(\theta, \bar{\theta}(\theta)) = 0$. As $d\Theta/dV > 0$, $\hat{\theta} > 0$ (< 0) above (below) $\bar{\theta}$.

Small resource base

A small resource base economy is defined as an economy that does not exhibit an interior common point of rest of both V and θ. As depicted in figure 8, both variables however do share two common points of rest on the axes, i.e. $(1, \bar{V}_0(1))$ and $(\bar{\theta}(0), 0)$.

Proposition 1 (Small resource base economy)**.** *An unique steady state equilibrium exists:* $\bar{V}(\theta^*) = \bar{\theta}(\theta^*) = (1, \bar{V}_0(1))$. *The associated growth rate is zero. The steady state is a globally stable saddle point.*

Proof: We utilize our previous work on the run and location of the relevant loci. Applying lemma 1, only steady state $\bar{\theta}(\theta^*) = (1, \bar{V}_0(1))$ remains. It is located in the zero growth area. By definition of $\bar{V}_0$, trajectories crossing $\bar{V}_0$ have zero slope at the point of intersection, $dV/d\theta = \dot{V}/\dot{\theta} = 0$. Above $\bar{V}_0$, $dV/d\theta > 0$. Accordingly, there exists a trajectory leading directly into steady state. Trajectories above imply $V \to \infty$ and thus are ruled out by lemma 1. Then again, starting just below, the dynamics of the system drive the economy towards the no-growth border since $\bar{V}$ is instable and the no-growth area is located to the left of the stable $\bar{\theta}$. Continuing (or starting) just below $\tilde{V}$, V continues to decrease while the share of competitive markets increases until the economy hits $\bar{\theta}$. Here, $dV/d\theta \to -\infty$. Eventually, as $\bar{\theta}$ is continuously increasing, the economy passes through $\bar{\theta}$ and tends towards the $(\bar{\theta}(0), 0)$ steady state, all along accompanied by a steady decrease in both the share of competitive markets and V. Trajectories that start below $\tilde{V}$ and hit the abscissa to the left of $\bar{\theta}$ also end up in $(\bar{\theta}(0), 0)$.

The saddle path is increasing since v decreases while n remains constant. The value of a monopolistically produced good in turn decreases since the associated profits decrease in the share of competitive markets, see (21). As the share of competitively priced goods increases, monopolists will find it harder to place their high priced varieties.

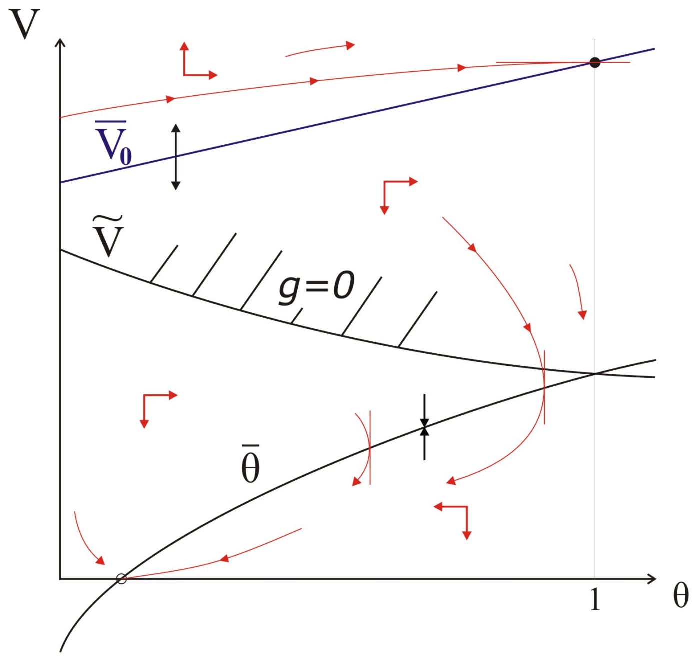


Figure 8: Dynamics in a small resource base economy

Large resource base

Figure 9 was drawn using our previous findings on the run of the relevant loci in the large resource base economy. Here, $\bar{\theta}(\theta)$ and $\bar{V}(\theta)$ separate the relevant phase plane in four regions. As shown in the beginning of this section, $\hat{V} > 0$ (< 0) above and below $\bar{V}$ while $\hat{\theta} < 0$ (> 0) above and below $\bar{\theta}$. Thus, starting above both zero growth loci and turning clockwise around the intersection, trajectories are pointing to the north-east, north-west, south-west, south-east. These dynamics are represented by the arrows. Two trajectories pass through the fixpoint, i.e. it is a saddle point. The trajectories that pass through $\bar{\theta}$ ($\bar{V}$) do so with infinite (zero) slope ($dV/d\theta = \dot{V}/\dot{\theta}$). Accordingly, the saddle path has the depicted shape. It is convex since $\dot{\theta} < 0$ and $\dot{V} > 0$ between $\bar{V}$ and $\bar{\theta}$, thus

$$\begin{aligned}
\dot{\hat{V}} &= (1-\alpha)\sigma \frac{\left[1-\theta(1-\alpha^{1-\epsilon})\right]\dot{V} + \dot{\theta}(1-\alpha^{1-\epsilon})V}{(.)^2} > 0 \\
\dot{\hat{\theta}} &= -\frac{\psi}{\theta} < 0
\end{aligned}$$

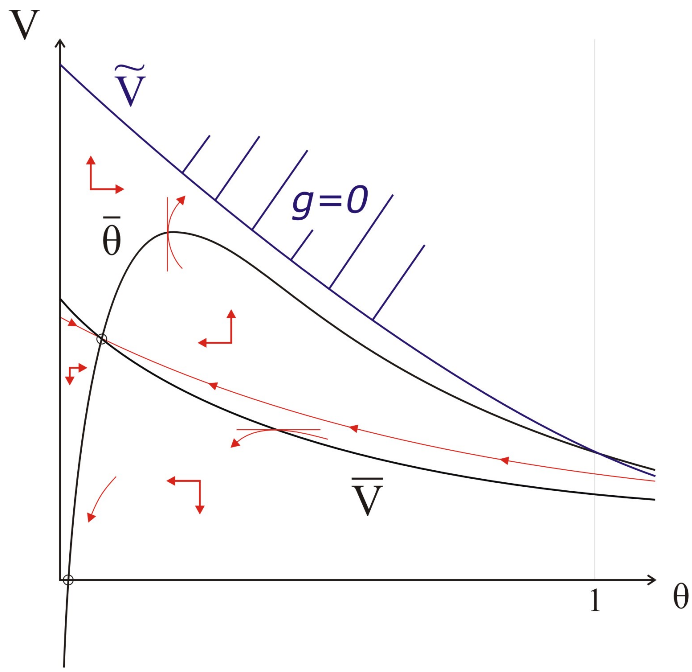


Figure 9: Dynamics in a large resource base economy

and $\dot{\hat{V}} - \dot{\hat{\theta}} > 0$ implying that all trajectories in the considered area are strictly convex. Taken together, we may state

Proposition 2 (Large resource base economy)**.** *There exists an unique steady state equilibrium: $\bar{V}(\theta^*) = \bar{\theta}(\theta^*)$. The associated growth rate is positive. The steady state is a globally stable saddle point.*

We next turn to the intermediate resource base economy and consider the case of two interior steady states as previously depicted in figure 4.

Intermediate resource base

Let us denote the left and right hand side intersection of $\bar{\theta}$ and $\bar{V}$ by steady state I and II, respectively. Note that these two intersections are not the only singular points. As θ is also constant on $\theta = 1$, see (29), $(\bar{V}_0(1), 1)$ is a third singular point, henceforth called steady state III. Apparently, dynamics

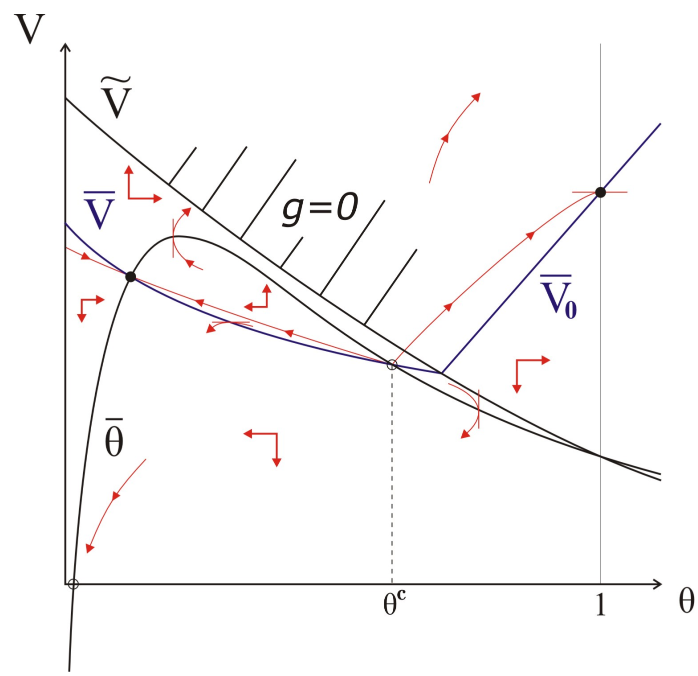


Figure 10: Dynamics in an intermediate resource base economy

in the neighborhood of steady state I are identical to those analyzed in the previous section, i.e. starting in the south of steady state I and moving clockwise, trajectories point to the north-east, north-west, south-west, south-east. This dynamic behavior characterizes steady state I as saddle point. The stable arm starting from the ordinate is decreasing, passing "through" steady state I and II. This last feature has to hold since every trajectory starting at either $\bar{\theta}$ or $\bar{V}$ between steady state I and II would imply $V \rightarrow \infty$ and $V \rightarrow 0$, respectively and can thus not be part of the trajectory leading towards steady state I. As indicated by the arrows in figure 10, starting in the south of steady state II and moving clockwise around it, trajectories point to the north-east, south-east, south-west, north-west. Thus, steady state II is totally unstable (it is no center). This implies that infinitely many paths depart from its immediate vicinity, from which we may pick one that points towards $V \rightarrow \infty$ and one that points towards the intersection of $\bar{\theta}(0)$ and the abscissa. The two chosen paths thus form a funnel in which one trajectory ends up in steady state III, i.e. the saddle path.

Thus, if we consider an initial share of competitive markets $\theta(0) < \theta^c$, where θ^c is the θ value belonging to steady state II, there are three possibilities. If we were to start above (below) the saddle path, the dynamics of the system would ultimately imply $V \to \infty$ ($V = 0$). Both cases are ruled out by lemma 1. Hence, the economy immediately has to start on the saddle path. It then gradually converges towards steady state I and hence enjoys long-run positive growth. Then again, if we consider an initial share of competitive markets $\theta(0) > \theta^c$, paths above and below the trajectory pointing towards steady state III are ruled out by lemma 1. Accordingly, it has to be that the economy converges on the saddle path towards its long-run zero growth equilibrium. Collecting all the information from the above section, we have just shown

Proposition 3 (Intermediate resource base economy)**.** *There exists a critical value θ^c for the initial share of competitive markets. If $\theta(0) < \theta^c$ ($\theta(0) > \theta^c$), the economy converges towards steady state I (III). Steady state I (III) has the properties described in proposition 2 (1).*

Accordingly, in an intermediate resource base economy, the long-run development crucially depends on the initial degree of competition. Note that if we were to start with a competitive share slightly higher than θ^c, the economy would initially enjoy a positive growth rate. After some time however, the economy hits $\tilde{V}$ and will find itself in zero-growth transition. If steady state III is reached, all monopolies have vanished and the economy is finally stuck in the no-growth trap.

By now, we considered product market competition due to costless imitation. In what follows, we reinterpret the baseline model as to describe the impact of industrial policy from a macro perspective.

5 The industrial policy model

As described earlier in section 3, the impact of industrial policy may be such that some industries operate in a competitive environment while others do not. We focus on the impact of a constant set of initial rules that shapes the degree of competition to abstract from anticipation issues. If we stick to the standard assumption of infinitively valid patents, the overall share of competitive markets in the innovating sector will obviously tend towards zero in the long-run. Our assumptions concerning the utility function however guarantee that the trade-off between welfare losses and the incentive to innovate remains. Accordingly, we are able to further explore this trade-off by comparing the welfare implications of different steady states. We demonstrate that the static welfare losses may actually be large enough to offset the positive gains from an increasing number of varieties. This finding is quite intuitive: as any positive growth rate requires market power in the first place, if economies are only able to generate a small growth rate, they may then find it worthwhile to forgo economic growth and thereby avoid welfare losses due to market power.

Static and dynamic equilibrium

The industrial policy model follows from the imitation setting without ongoing imitation. Accordingly, we simply adopt the free entry condition (18) and the profits of monopolists, (21). Without anticipation of future policy shocks, the value of any monopolistically produced brand and the capital market equilibrium is given by (19) and (20) with $\psi = 0$, i.e.

$$v(t) \equiv \int_t^\infty e^{-\rho(\tau - t)} \pi(\tau) d\tau \tag{47}$$

$$\rho v(t) = \pi(t) + \dot{v}(t). \tag{48}$$

Solving the labor market clearing condition (25) for g yields the growth rate given in (26) and again, the (θ, V) space is separated in a growth and no growth area by $\tilde{V}(\theta)$, see (27). The laws of motion for θ and V given by (28) and (29) simplify to

$$\hat{V} = V \frac{\sigma(1-\alpha)\alpha^{\epsilon-1}}{\theta(1-\alpha^{\epsilon-1}) + \alpha^{\epsilon-1}} - g - \rho \tag{49}$$

$$\hat{\theta} = -g. \tag{50}$$

Solution

Drawing on the work in the previous sections, the zero growth loci for V both in the $g = 0$ and $g > 0$ area can easily be obtained by setting $\psi = 0$ in

(30) and (32):

$$
\begin{aligned}
\bar{V}_0(\theta) &\equiv \rho\frac{1-\theta(1-\alpha^{1-\epsilon})}{(1-\alpha)\sigma} && (51)\\
\bar{V}(\theta) &\equiv \left(\rho+\frac{L}{a}\right)\frac{1-\theta(1-\alpha^{1-\epsilon})}{1-\theta[1-\alpha^{1-\epsilon}-(1-\alpha)\sigma]} && (52)
\end{aligned}
$$

The run of both functions was previously derived in the imitation model and does not change significantly by choosing $\psi = 0$. Additionally, V is constant on $V = 0$. Recapitulating (29), in a growing economy and with perfect patent protection for newly invented brands, $\dot{\theta} = 0$ only at $\theta = 0$. If $V(\theta) \geq \tilde{V}(\theta)$, θ remains constant. For the time being, we provisionally assume that the economy fulfills the necessary prerequisites for a long-run positive growth equilibrium: $\tilde{V}(0) > \bar{V}(0)$, i.e. $L/a > [1-(1-\alpha)\sigma]\rho/[(1-\alpha)\sigma]$.

Suppose there exists an intersection of $\bar{V}(\theta)$ and $\tilde{V}(\theta)$, i.e. a share of competitive markets defined by $\bar{V}(\theta^*) \equiv \tilde{V}(\theta^*)$. By definition of $\tilde{V}(\theta)$, $(\theta^*, \bar{V}(\theta^*))$ is characterized by $g = \dot{V} = 0$. If $0 < \theta^* < 1$, the locus of a point with this features is exclusively characterized by $\bar{V}_0(\theta)$. Accordingly, $\bar{V}(\theta^*) \equiv \tilde{V}(\theta^*) = \bar{V}_0(\theta^*)$. Equating (27) with (51) and (52), respectively, and dropping an irrelevant negative solution yields a unique remaining positive intersection of $\tilde{V}$, $\bar{V}$ and $\bar{V}_0$ at

$$
\begin{aligned}
\theta^* &= \frac{\left(\frac{L}{a\rho}+1\right)[\sigma\alpha^{\epsilon-1}(1-\alpha)]-\alpha^{\epsilon-1}}{1-\alpha^{\epsilon-1}+\sigma\alpha^{\epsilon-1}(1-\alpha)}\\
&= \frac{[1-\sigma(1-\alpha)]\left(\frac{L}{a\rho}+1\right)}{[1-\sigma(1-\alpha)]-\alpha^{1-\epsilon}}. && (53)
\end{aligned}
$$

Note that if $\tilde{V}(0) < \bar{V}(0)$, $\theta^* \notin (0,1)$ since the numerator and the denominator of θ^* exhibit different signs. If the provisional parameter restriction holds, it directly implies the numerator to be strictly positive such that θ^* is located on the unit interval iff $L/(a\rho) < \alpha^{1-\epsilon}/[(1-\alpha)\sigma]$.

Then, $\tilde{V}$, $\bar{V}_0$ and $\bar{V}$ intersect in the relevant area and show the run depicted in figure 11. Clearly, this is the industrial policy analogue of the intermediate resource base economy from the chapter on imitation. Equivalently, if the resource base were lower (larger), we would get the small (large) economy analogue.

In what follows, we focus on the intermediate case:

$$\frac{1-(1-\alpha)\sigma}{(1-\alpha)\sigma} < \frac{L}{a\rho} < \frac{\alpha^{1-\epsilon}}{(1-\alpha)\sigma} \tag{A3$'$}$$

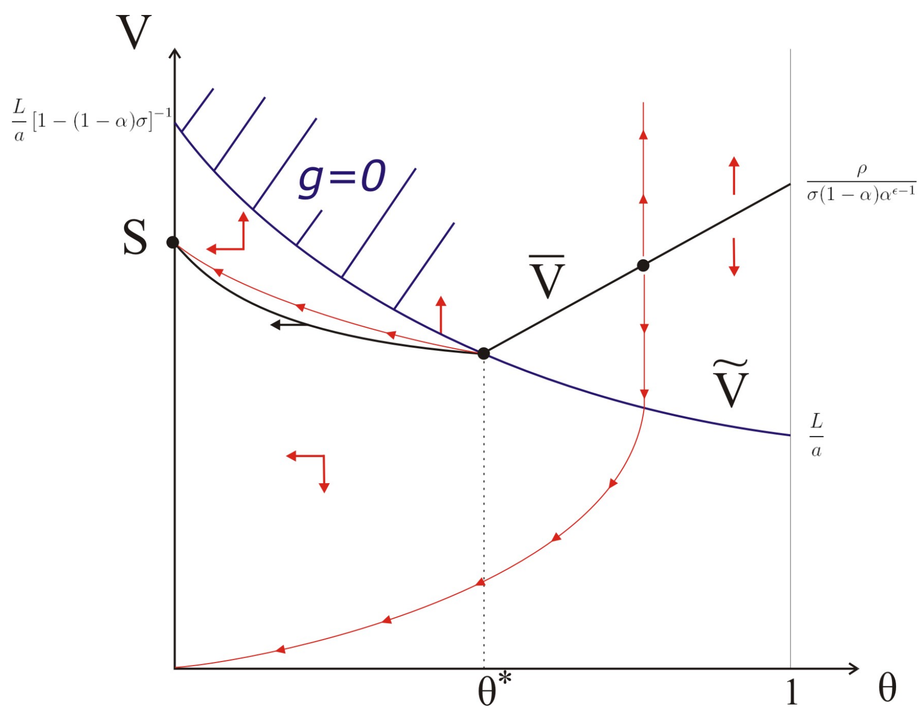


Figure 11: Phase diagram under A3$'$

By (28), the piecewise defined zero growth locus for V is instable, $\hat{V}$ $>$ $(<)$ 0 if V $>$ $(<)$ $\bar{V}_{(0)}$. Since θ is constant above $\tilde{V}$ and decreasing everywhere below, we get the dynamics represented by the arrows in figure 1. Clearly, there are multiple steady states. One is located in the $g > 0$ area, namely $(0, \rho + \frac{L}{a}) \equiv S$. Also, each point on $\bar{V}_0$ above $\tilde{V}$ is a point of rest, likewise is $(0, 0)$.

Dynamic equilibrium

Consider a situation in which the initial set of policies or "market rules" is such that a fraction $\theta(0)$ of markets in the x-sector operates competitively while the incumbent firms in each of the remaining markets enjoy monopoly power.

We will describe the dynamic equilibrium firstly for $\theta(0) \geq \theta^*$ and afterwards consider $\theta(0) < \theta^*$.

Lemma 2. *If $\theta(0) \geq \theta^*$, the economy instantly gets stuck in a no growth equilibrium with $\theta = \theta(0)$ and $V = \bar{V}_0(\theta(0))$.*

Proof: If this were not the case, given $\theta(0)$, both $V > \bar{V}_0(\theta(0))$ and $V < \bar{V}_0(\theta(0))$ would violate rational expectations since a) in the first case, due to the instability of $\bar{V}_0$, $V \to \infty$ and b), in the second case, the economy would approach $(0,0)$, i.e. $V = 0$ in the long run. Both cases are ruled out by lemma 1.

Lemma 3. *If $\theta(0) < \theta^*$, there exists an increasing, strictly convex saddle path leading to an unique steady state, $(0, \bar{V}(0))$. The associated growth rate is positive.*

Proof: As indicated by the arrows in figure 11, $\chi \equiv V/\theta$ increases everywhere above $\bar{V}$ and decreases below. Again, trajectories above $\tilde{V}$ and below $\bar{V}$ imply $V \to \infty$ and $V = 0$, respectively, and according to lemma 1 violate rational expectations. In the area between, $\hat{V} > 0$ and $\dot{\theta} > 0$, i.e.

$$\dot{\hat{\chi}} = V \left\{ \hat{V} \frac{(1-\alpha)\alpha^{\epsilon-1}}{\theta(1-\alpha^{\epsilon-1}) + \alpha^{\epsilon-1}} - \frac{(1-\alpha)\alpha^{\epsilon-1}(1-\alpha^{\epsilon-1})\dot{\theta}}{[\theta(1-\alpha^{\epsilon-1}) + \alpha^{\epsilon-1}]^2} \right\} > 0.$$

Thus, the saddle path is strictly convex. It has to run "through" $(\theta^*, \tilde{V}(\theta^*))$ just as departing from an intersection with either $\tilde{V}$ or $\bar{V}$ would again eventually yield $V \to \infty$ and $V = 0$, respectively. Lemma 1 completes the argument.

S exhibits the (positive) Grossman-Helpman growth rate (consider (26) and steady state S value for V):

$$\begin{aligned} g_S &= \frac{L}{a} - \frac{\alpha^{\epsilon}\sigma + (1-\sigma)\alpha^{\epsilon-1}}{\alpha^{\epsilon-1}} \left(\rho + \frac{L}{a}\right) \\ &= \left[\frac{L}{a}(1-\alpha) - \alpha\rho\right]\sigma - (1-\sigma)\rho \end{aligned} \tag{54}$$

As opposed to the absence of transitional dynamics in the standard setting, the economy does not jump directly in the steady state, but starts on the point on the saddle path determined by the initial share of competitive markets. If the share is not too high, i.e. $\theta(0) \leq \theta^*$, the economy grows at positive rate and converges to its long run equilibrium. This implies an increasing number of monopolistic markets and accordingly, the share of competitive markets tends towards zero. Compared to standard Grossman-Helpman, the long run growth rate is driven down by the households desire to consume goods from the (static) y sector which diverts resources away from the innovative branch of the economy ($\sigma < 1$ appears in the first term in equation 54).

Growth vs. no Growth

Discounted utility can be calculated by evaluating equation (1) at steady state values for any θ implying the absence of transition, which according to lemmas 2 and 3 are $\theta(0) \in \{0, [\theta^*, 1]\}$. Make use of one to one relation between factor input and output, $\underline{nx} = \underline{L}_x$ and $\overline{nx} = \overline{L}_x$, to write $c_x = (\underline{nx}^\alpha + \overline{nx}^\alpha)^{\frac{1}{\alpha}}$ as

$$c_x = n^{\frac{1-\alpha}{\alpha}} \left[\theta^{1-\alpha} \underline{L}_x^\alpha + (1-\theta)^{1-\alpha} \, \overline{L}_x^\alpha\right]^{\frac{1}{\alpha}}. \tag{55}$$

In what follows, we compare the positive growth steady state S without transitional dynamics and the zero growth steady state $T \equiv (\theta = 1, V = \rho/[\sigma(1-\alpha)\alpha^{\epsilon-1}]$. In any steady state without transition, indirect present value utility reads

$$\begin{aligned}
U &= \int_0^\infty \exp(-\rho t) \left[\sigma \ln c_x + (1-\sigma) \ln c_y\right] dt \\
&= \int_0^\infty \exp(-\rho t) \times \\
&\quad \times \left\{\sigma \left[\frac{1-\alpha}{\alpha} \ln n + \frac{1}{\alpha} \ln \left(\theta^{1-\alpha} \underline{L}_x^\alpha + (1-\theta)^{1-\alpha} \overline{L}_x^\alpha\right)\right] + (1-\sigma) \ln L_y\right\} dt \\
&= \frac{\sigma}{\rho} \frac{1-\alpha}{\alpha} \left[\ln n(0) + \frac{g}{\rho}\right] + \frac{1}{\rho} \left\{\frac{\sigma}{\alpha} \ln \left[(1-\theta) \overline{L}_x^\alpha + \theta^{1-\alpha} \underline{L}_x^\alpha\right] + (1-\sigma) \ln L_y\right\}.
\end{aligned}$$

In S, $\theta = 0$ and hence

$$\rho \left[U^S - \frac{\sigma}{\rho} \frac{1-\alpha}{\alpha} \ln n(0)\right] = \sigma \ln \overline{L}_x^S + (1-\sigma) \ln L_y^S + \frac{\sigma}{\rho} \frac{1-\alpha}{\alpha} g^S. \tag{56}$$

In T, $\theta = 1$ and $g = 0$:

$$\rho\left[U^T - \frac{\sigma}{\rho}\frac{1-\alpha}{\alpha}\ln n(0)\right] = \sigma \ln \underline{L}_x^T + (1-\sigma)\ln L_y^T \tag{57}$$

where $\underline{L}_x^T + L_y^T$ has to equal L. Maximizing the right-hand side by choosing L_x^T yields the efficient labor allocation in the static economy:

$$\frac{\sigma}{1-\sigma} = \frac{L_x^T}{L_y^T} \tag{58}$$

Lemma 4. *Labor is efficiently allocated in T. In S, too much labor is devoted to the y sector.*

Proof: In T, all markets are competitive. Thus, $L_x^T = \sigma/p = \sigma L$ since the capital market has zero value which implies $wL = 1$ by the instantaneous budget constraint, see (2). Equivalently, $L_y^T = (1-\sigma)/w = (1-\sigma)L$, and according to (58) labor is allocated efficiently. In S, all markets in sector x are monopolistic and $L_x^S = \sigma\alpha/w = \alpha\sigma a V^s$. If the economy grows with positive rate, $L - ag$ units of labor may be divided in the two sectors x and y. According to (57) and g^s given by (54), an efficient allocation that maximizes U^s would have to satisfy $L_x = \sigma(L - ag)$. Comparing L_x^S to this benchmark amounts to comparing αV to $L/a - g$. The former is $\alpha(L/a + \rho)$ while the latter reads

$$\frac{L}{a} - \frac{L}{a}\sigma(1-\alpha) + \rho\left[1 - \sigma(1-\alpha)\right] = \left[1-\sigma(1-\alpha)\right]\left(\frac{L}{a} + \rho\right).$$

Since $\alpha < 1$, $\alpha(1-\sigma) < 1-\sigma$ and hence $1 - \sigma(1-\alpha) > \alpha$. This completes the proof to lemma 4.

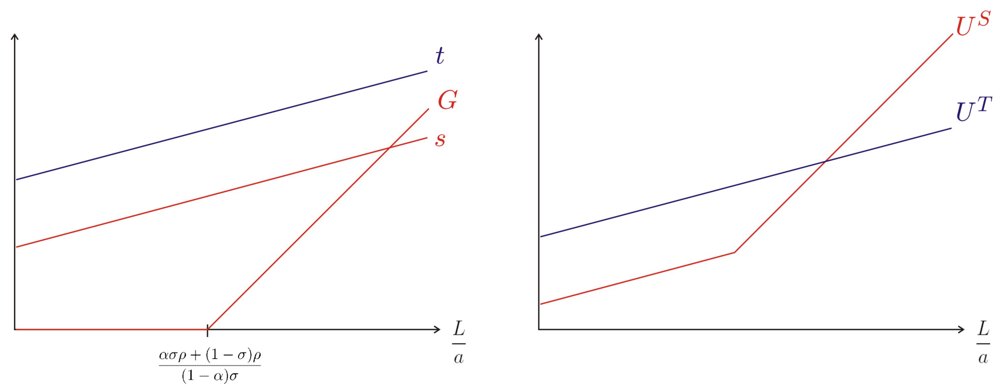
Accordingly,

$$s(\cdot) \equiv \sigma \ln \bar{L}_x^S + (1-\sigma)\ln L_y^S > \sigma \ln \underline{L}_x^T + (1-\sigma)\ln L_y^T \equiv t(\cdot). \tag{59}$$

Let $G(\cdot) \equiv \sigma(1-\alpha)/(\alpha\rho)g^S$. With this abbreviation, the right-hand sides of (56) and (57) read $s(\cdot) + G(\cdot)$ and $t(\cdot)$, respectively.

Proposition 4 (Size of static distortions)**.** *If the long-run growth rate is sufficiently low, static distortions due to market power can be large enough to offset welfare gains from growth.*

Proof: We plotted t, s and G against L/a in the left panel of figure 5. The right panel of this figure shows the associated present value of utility according to (56) and (57). As $t(\cdot)$ is discrete larger than $s(\cdot)$, see (59), while $G(\cdot)$ is continuous in L/a, there exist cases in which $g > 0$ and $U^S < U^T$.



6 Conclusions

In this article, we have considered competitive markets in the standard increasing variety growth model. Competition in existing varieties was motivated by either costless imitation (section 4) or by industrial policy rules (section 5). In the former model, we concluded that the impact of the profit decreasing channel of competition crucially depends on the size of the resource endowment of the economy. In analogy to Grossman and Helpman [1991] we find that economies with sufficiently large endowments exhibit a positive growth rate in the long-run. In equilibrium, both monopolistic and competitive markets exist. Economies with a resource base too small to sustain long-run growth end up exclusively with competitive markets. In contrast to the standard model, there exists an intermediate range in which the long-run growth crucially depends on the initial share of competitive markets. Below some critical value, the economy behaves like a richly endowed economy whereas with too much initial competition it will ultimately get stuck in a no growth trap. If competitive and monopolistic markets coexist in the long-run, the varying degrees of market power give rise to static welfare losses. Decreasing these losses however comes at the expense of decreasing the incentives to innovate. Accordingly, by introducing asymmetry, allowing for competition is one way to establish the often considered trade-off between static and dynamic goals in the standard variety growth model. We explored this trade-off in the industrial policy version of the baseline model. Comparing positive and zero growth steady states, we showed that the static welfare losses may actually be large enough to offset the benefits of long-run growth. This situation occurs once the positive growth rate is "small". Then, taking losses due to market power does not pay off even in the long run.

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7 Appendix

7.1 On the run of $\bar{V}$

$$\begin{aligned}
\bar{V}'(\theta) &= \left(\rho+\psi+\frac{L}{a}\right)\times \\
&\times \frac{\{1-\theta\left[1-\alpha^{1-\epsilon}-(1-\alpha)\sigma\right]\}\left(\alpha^{1-\epsilon}-1\right)-\left[1-\theta(1-\alpha^{1-\epsilon})\right]\left[\alpha^{1-\epsilon}-1(1-\alpha)\sigma\right]}{(.)^2} \\
&= \left(\rho+\psi+\frac{L}{a}\right)\frac{(1-\alpha)(\alpha^{1-\epsilon}-1)\sigma\theta-(1-\alpha)\sigma\left[1-\theta(1-\alpha^{1-\epsilon})\right]}{(.)^2} \\
&= \left(\rho+\psi+\frac{L}{a}\right)\frac{(\alpha-1)\sigma}{(.)^2} \;<0 \\
\bar{V}''(\theta) &= \left(\rho+\psi+\frac{L}{a}\right)(\alpha-1)\sigma\frac{1-\alpha-(1-\alpha)\sigma}{(.)^4} \;>0
\end{aligned}$$

7.2 On the run of $\bar{\theta}(\theta)$

Using a somewhat sloppy notation, at the first discontinuity point ($\theta = 0$),

$$\lim_{\theta\to 0^{+(-)}}\bar{\theta}(\theta)=\frac{1}{1-(1-\alpha)\sigma}\left(\frac{L}{a}-\lim_{\theta\to 0^{+(-)}}\frac{1-\theta}{\theta}\psi\right)=-(+)\infty$$

At the latter, i.e. $\theta = \frac{1-(1-\alpha)\sigma}{1-\alpha^{1-\epsilon}-(1-\alpha)\sigma}$,

$$\lim_{\theta\to \frac{1-(1-\alpha)\sigma}{1-\alpha^{1-\epsilon}-(1-\alpha)\sigma}^{+(-)}}\bar{\theta}(\theta)=-\left(\frac{L}{a}+\frac{\alpha^{1-\epsilon}}{1-(1-\alpha)\sigma}\right)\times$$

$$\times\lim_{\theta\to \frac{1-(1-\alpha)\sigma}{1-\alpha^{1-\epsilon}-(1-\alpha)\sigma}^{+(-)}}\left[\frac{(1-\alpha)\sigma\alpha^{1-\epsilon}}{1-\theta[1-\alpha^{1-\epsilon}-(1-\alpha)\sigma]-(1-\alpha)\sigma}\right]=(-)\infty$$

7.3 On $\zeta(\theta)$

As mentioned in the main text, $\zeta(\theta)$ has a positive leading coefficient. This becomes obvious once we rewrite $\zeta(\theta)$ as

$$\psi\left\{\theta^2\left[1-\alpha^{1-\epsilon}-(1-\alpha)\sigma\right]\left(1-\alpha^{1-\epsilon}\right)-\theta\left[2(1-\alpha^{1-\epsilon})-(1-\alpha)(2-\alpha^{1-\epsilon})\sigma\right]+\right.$$
$$\left.+1-(1-\alpha)\sigma\right\}$$

Next, we show that $(\zeta')^{-1}(0) < 0$, i.e. the extreme value of ζ is located on the negative real line. The derivative of $\zeta(\theta)$ with respect to θ reads

$$\zeta'(\theta) = \left\{N'(\theta)\left[1-\theta(1-\alpha^{1-\epsilon})\right] + N(\theta)(\alpha^{1-\epsilon}-1)\right\}\psi$$

Equating this expression to zero yields

$$1-\theta(1-\alpha^{1-\epsilon}) = \frac{N(\theta)}{N'(\theta)}(1-\alpha^{1-\epsilon}) \tag{60}$$

Clearly, $N(\theta) \equiv 1-\theta[1-\alpha^{1-\epsilon}-(1-\alpha)\sigma]-(1-\alpha)\sigma = 1-\theta\left[1-\alpha^{1-\epsilon}\right] - (1-\alpha)(1-\theta)\sigma > 0$. As $N'(\theta) = -(1-\alpha^{1-\epsilon})+(1-\alpha)\sigma = \alpha(\alpha^{-\epsilon}-\sigma) > 0$, the right-hand side of (60) is strictly negative. Accordingly, $(\zeta')^{-1}(0) < 0$.

7.4 On $\xi(\theta)$

We defined $\xi \equiv (1-\alpha)\alpha^{1-\epsilon}\sigma\left[\theta\frac{L}{a}-(1-\theta)\psi\right]\theta$. Accordingly,

$$\Xi \equiv \xi'(\theta) = (1-\alpha)\alpha^{1-\epsilon}\sigma\left[2\theta\left(\frac{L}{a}+\psi\right)-\psi\right]$$

Thus, $\partial\Xi/\partial(L/a) = 2(1-\alpha)\alpha^{1-\epsilon}\sigma\theta > 0$.

7.5 Derivation of equation 39

Equating (37) to zero yields:

$$\left[\frac{L}{a}-\frac{1-\theta}{\theta}\psi\right]\left\{N(\theta)(\alpha^{1-\epsilon}-1)+\left[1-\theta(1-\alpha^{1-\epsilon})\right]\left[1-\alpha^{1-\epsilon}-(1-\alpha)\sigma\right]\right\} = \\ = -\frac{\psi}{\theta^2}N(\theta)\left[1-\theta(1-\alpha^{1-\epsilon})\right]$$

Multiply by $-1/\theta^2$ to get

$$N(\theta)\psi\left[1-\theta(1-\alpha^{1-\epsilon})\right] = \\ = \left[\theta\frac{L}{a}-(1-\theta)\psi\right]\theta\left\{N(\theta)(1-\alpha^{1-\epsilon})-\left[1-\alpha^{1-\epsilon}-(1-\alpha)\sigma\right]+\right. \\ \left. +\,\theta(1-\alpha^{1-\epsilon})\left[1-\alpha^{1-\epsilon}-(1-\alpha)\sigma\right]\right\}$$

Inserting the definition of $N(\theta)$ simplifies the expression in parentheses to

$$\begin{aligned}\left\{1-\theta\left[1-\alpha^{1-\epsilon}-(1-\alpha)\sigma\right]-(1-\alpha)\sigma\right\}\left(1-\alpha^{1-\epsilon}\right)-\left[1-\alpha^{1-\epsilon}-(1-\alpha)\sigma\right]+\\ +\theta\left[1-\alpha^{1-\epsilon}-(1-\alpha)\sigma\right]\left(1-\alpha^{1-\epsilon}\right)=\\ =1-\alpha^{1-\epsilon}-(1-\alpha)\left(1-\alpha^{1-\epsilon}\right)\sigma-\left[1-\alpha^{1-\epsilon}-(1-\alpha)\sigma\right]=\\ =(1-\alpha)\alpha^{1-\epsilon}\sigma\end{aligned}$$

Plug this factor back in the previous expression to get equation 39.

7.6 Derivation of equation 40

$$\begin{aligned}\left[\frac{L}{a}-\frac{1-\theta}{\theta}\psi\right]\frac{1-\theta\left(1-\alpha^{1-\epsilon}\right)}{1-\theta\left[1-\alpha^{1-\epsilon}-(1-\alpha)\sigma\right]-(1-\alpha)\sigma}=\\ =\left(\rho+\psi+\frac{L}{a}\right)\frac{1-\theta\left(1-\alpha^{1-\epsilon}\right)}{1-\theta\left[1-\alpha^{1-\epsilon}-(1-\alpha)\sigma\right]}\end{aligned}$$

Let $A\equiv\left[1-\alpha^{1-\epsilon}-(1-\alpha)\sigma\right]$ (<0).

$$\begin{aligned}\left[\frac{L}{a}-\frac{1-\theta}{\theta}\psi\right]\{1-\theta A\} &= \{1-\theta A-(1-\alpha)\sigma\}\left(\rho+\psi+\frac{L}{a}\right)\\ \left[\frac{L}{a}\theta-(1-\theta)\psi\right](1-\theta A) &= [1-\theta A-(1-\alpha)\sigma]\left(\rho+\psi+\frac{L}{a}\right)\theta\end{aligned}$$

Multiply terms to get

$$\begin{aligned}\frac{L}{a}\theta(1-\theta A)-\frac{L}{a}\theta\left[1-\theta A-(1-\alpha)\sigma\right]-\psi(1-\theta)(1-\theta A)-\left[1-\theta A-(1-\alpha)\sigma\right]\psi\theta &=\\ =\left[1-\theta A-(1-\alpha)\sigma\right]\rho\theta\\ \frac{L}{a}\theta(1-\alpha)\sigma-\psi(1-\theta)A+\theta\psi(1-\alpha)\sigma &= \rho\theta-\theta^2\rho A-(1-\alpha)\sigma\rho\theta\end{aligned}$$

Collecting terms yields

$$\begin{aligned}\frac{L}{a}\theta(1-\alpha)\sigma-\psi+\psi\theta A+\theta\psi(1-\alpha)\sigma-\rho\theta+\theta^2\rho A+(1-\alpha)\sigma\rho\theta &= 0\\ \theta^2\rho A+\theta\left[\frac{L}{a}(1-\alpha)\sigma+\psi A+\psi(1-\alpha)\sigma-\rho+(1-\alpha)\sigma\rho\right] &= \psi\\ \theta^2\rho A+\theta\left\{\frac{L}{a}(1-\alpha)\sigma+\psi\left[A+(1-\alpha)\sigma\right]-\rho\left[1-(1-\alpha)\sigma\right]\right\} &= \psi\end{aligned}$$

$$\theta^2\rho A+\theta\left\{\frac{L}{a}(1-\alpha)\sigma+\psi(1-\alpha^{1-\epsilon})-\rho\left[1-(1-\alpha)\sigma\right]\right\}=\psi \qquad (61)$$

As our objective is to find conditions for relevant solutions, i.e. $\theta \in (0,1)$, we rearrange terms such that both sides become functions of θ with nice properties:

$$\theta^2 \rho A + \theta \left\{ \frac{L}{a}(1-\alpha)\sigma - \psi\alpha^{1-\epsilon} - \rho\left[1-(1-\alpha)\sigma\right] \right\} = \psi(1-\theta) \tag{62}$$

If we define the left hand side of this equation as $r(\theta)$ and the right hand side as $l(\theta)$, then $r(0) = 0$, $l(0) = \psi$ and $l(1) = 0$. Figure gives an illustration.

7.7 On $r(\theta)$

The derivative of $r(\theta)$ is

$$r'(\theta) = 2\theta\rho A + \left\{ \frac{L}{a}(1-\alpha)\sigma - \psi\alpha^{1-\epsilon} - \rho\left[1-(1-\alpha)\sigma\right] \right\}$$

Accordingly,

$$r'(0) \;\; = \;\; \frac{L}{a}(1-\alpha)\sigma - \psi\alpha^{1-\epsilon} - \rho\left[1-(1-\alpha)\sigma\right]$$

which is strictly positive if

$$\frac{L}{a} > \frac{\psi\alpha^{1-\epsilon} + \rho\left[1-(1-\alpha)\sigma\right]}{(1-\alpha)\sigma}$$

As shown in the main text, (42) holds. Multiply both sides by α and cancel α^{ϵ} on the right hand side to see that $r'(0) > 0$ in the considered region.

Assumption A3 ensures $r(1) < 0$. As

$$r(1) \;\; = \;\; \rho A + \frac{L}{a}(1-\alpha)\sigma - \psi\alpha^{1-\epsilon} - \rho\left[1-(1-\alpha)\sigma\right]$$

which is strictly negative if

$$\begin{aligned} \frac{L}{a}(1-\alpha)\sigma &< \rho[1-(1-\alpha)\sigma] + \psi\alpha^{1-\epsilon} - \rho[1-\alpha^{1-\epsilon}-(1-\alpha)\sigma] \\ \frac{L}{\alpha a} &< \frac{\psi+\rho}{(1-\alpha)\alpha^{\epsilon}\sigma} \end{aligned}$$

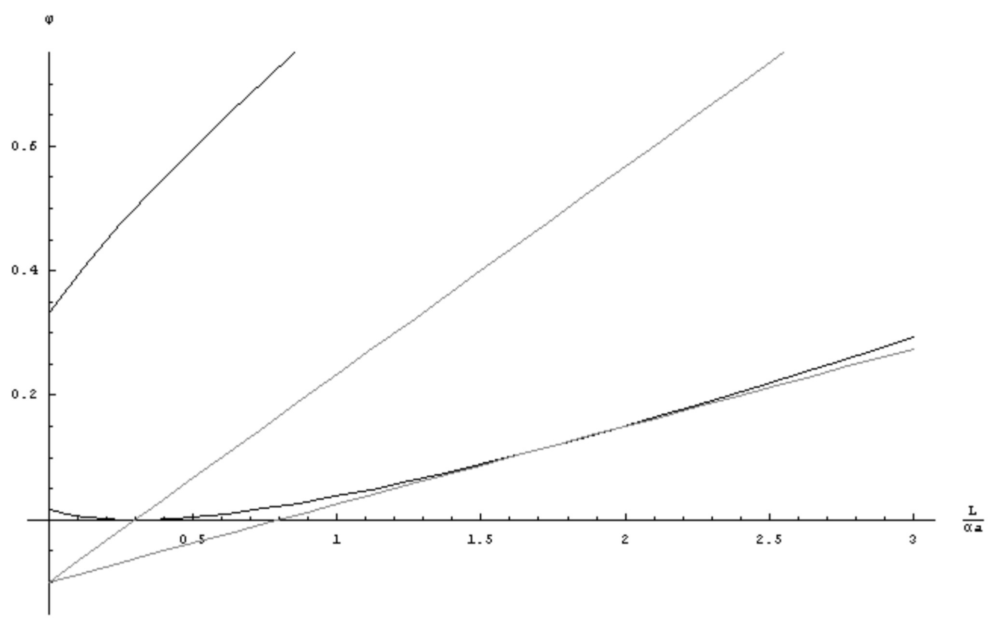

Figure 12: A numerical example for the separation of the parameter space.

7.8 On the separation of parameter space

Let $\alpha = 0.5$, $\rho = 0.1$, $\sigma = 0.5$. The separation of the parameter space for this particular case is shown in figure 12.

7.9 On the possibility of two interior steady states

Let $\alpha = 0.6$, $\rho = 0.1$, $\psi = 0.05$, $\sigma = 0.5$. Then, as depicted in figure 13, $l(\theta)$ and $r(\theta)$ intersect twice ($\theta = 0.44578$, $\theta = 0.829818$) on $(0, 1)$.

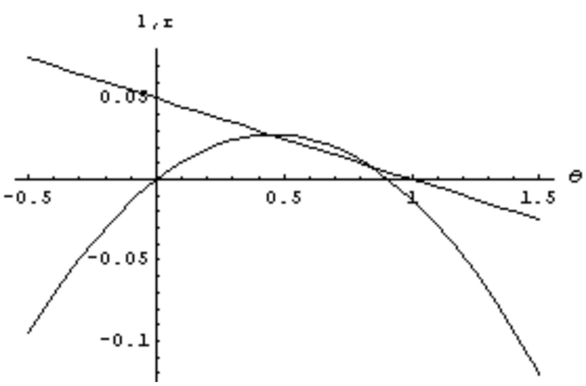

Figure 13: A numerical example.