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How Preferences Shape the Welfare and Employment Effects of Trade*

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Abstract

We set up a trade model with two countries, two sectors, and one production factor, which features a home-market effect due to the existence of trade costs. We consider search frictions and firm-level wage bargaining in the sector producing differentiated goods and a perfectly competitive labor market in the sector producing a homogeneous good. Consumers have price-independent generalized-linear preferences over the two types of goods, covering homothetic and quasilinear preferences as two limiting cases. We show that trade between two countries that differ in their population size leads to an expansion of the differentiated goods sector and a contraction of the homogeneous good sector in the larger economy. This induces the larger country to net-export differentiated goods at the cost of a higher economy-wide rate of unemployment in the open economy (with the effects reversed for the smaller country). The welfare effects of trade depend on the preference structure. Looking at the two limiting cases, we show that the larger country is likely to benefit from trade if preferences are homothetic, whereas losses from trade are possible if preferences are quasilinear. The opposite is true in the smaller country. This reveals an important role of preferences for the welfare effects of trade in the presence of labor market imperfection, a result we further elaborate on in two extensions, in which we consider more general preferences and differences of countries in their per-capita income levels.

Keywords: Preferences; Search frictions; Wage bargaining; Trade structure; Welfare and employment effects

JEL Classification: F12, F15, F16, D11

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1 Introduction

The question of how labor market imperfection shapes the welfare and employment effects of trade has played a prominent role in economic research since Brecher's (1974) seminal work on the role of minimum wages in a Heckscher-Ohlin model. Due to strong public discontent about the negative consequences of globalization for domestic workers, this question has gained momentum over the last 15 years. Building on various forms of labor market imperfection, recent theoretical work has been successful in identifying new, so far unexplored channels through which trade can affect economy-wide unemployment and the distribution of income with important consequences for the expected welfare effects. In this paper, we take a different perspective and show that the effects of trade not only depend on the form of labor market imperfection but also on the type of consumer preferences. That preferences matter and can give a demand-side explanation for international trade flows is well-known from Krugman's (1979; 1980) foundation of a new trade theory. However, the role of preferences for the existence of gains or losses from trade in the presence of labor market distortion has so far not been in the focus of economic research.

To fill this gap, we set up a prototype model of trade featuring a home-market effect, with two countries, two sectors of production, and labor as the only input factor. Similar to Helpman and Krugman (1985), we assume that one sector produces differentiated goods under monopolistic competition, which are subject to trade costs in the open economy. The other sector produces a homogeneous good under perfect competition that can be shipped to the foreign country at zero costs. Our home-market model differs from previous ones in two important respects. On the one hand, we consider price-independent generalized-linear (so-called PIGL) preferences, which have been put forward by Muellbauer (1975, 1976) and refer to the most general class of preferences admitting a representative consumer and thereby avoiding complications from aggregating consumer demand over heterogeneous households. The subclass of parametric PIGL preferences considered here has the advantage of delivering an explicit solution for the direct utility function (see Boppart, 2014), which is particularly useful to avoid an otherwise potentially complicated integrability problem.[1] Whereas the preferences do not have Gorman form in general, they cover homothetic and quasilinear preferences – and thus two widely used examples of Gorman-form preferences – as limiting cases (see Egger and Habermeyer, 2019). On the other hand, we consider search frictions in the sector of differentiated goods and assume that wages in this sector are set by bargaining between the firm and a continuum of workers (cf. Stole and Zwiebel, 1996; and the correction in Bruegemann et al., 2018). The assumption that the labor markets differ in the two sectors implies that higher wages bring along a higher risk of unemployment. This feature of our model is akin to the distinction of good and bad jobs in Acemoglu (2001) and gives the preferences a particular role, since they shape the

[1] It is well understood from Samuelson (1950) and Hurwicz and Uzawa (1971) that associating consumer demand derived from indirect utility with the solution of a maximization problem of rational households requires integrability of demand functions. Hurwicz and Uzawa (1971) have worked out sufficient conditions to solve the integrability problem, relying on properties of the Slutsky matrix. In the context of parametric PIGL preferences, Boppart (2014) has shown that these conditions are fulfilled for homogeneous goods, whereas a proof for a continuum of differentiated goods is so far missing. Here, we circumvent the problem by focussing on a subclass of PIGL preferences, which delivers an explicit solution for the direct utility function.

risk attitude of households and thus the wage compensation demanded by them to accept the possibility of an unfavorable outcome of unemployment when seeking employment in the production of differentiated goods.

We use our framework to study the role of preferences for the effects of trade on unemployment and welfare. As a result of the labor market distortion, wages are higher in the sector of differentiated than in the sector of homogeneous goods. The probability to find employment in the sector of differentiated goods is directly linked to the wage premium paid in this industry by an indifference condition that makes applying for jobs in the two sectors equally attractive for workers prior to the revelation of who is successfully matched with a firm. The exact link between the wage premium and the employment probability established by this indifference condition depends on household preferences. If households have quasilinear preferences, they are risk-neutral and hence for a given wage premium the employment probability in the sector of differentiated goods can be fairly small. Things are different if households have homothetic (log-transformed Cobb-Douglas) preferences which make them risk-averse. In the case of risk-averse households the employment probability must be fairly high for a given wage premium in order to make applying for jobs in the sector of differentiated goods attractive for them. Differentiating quasilinear and homothetic preferences by the risk attitudes of households is not an ad hoc assumption but follows from looking at two limiting cases of the class of parametric PIGL preferences put forward by our analysis. Due to differences in the risk attitudes, a given change in the fraction of workers applying for jobs in the sector of differentiated goods can have quite different effects on nominal income for the two types of preferences. With the employment probability unchanged, a higher fraction of workers seeking employment in the sector of differentiated goods will reduce income if preferences are quasilinear, whereas it increases income under homothetic preferences, provided that the unemployment compensation for those who do not find a job is not too generous.[2]

With this fundamental insight at hand, we then turn to the open economy and consider trade between two countries that are fully symmetric except for their population sizes. In line with the literature on home-market effects, we show that the sector of differentiated goods expands in the larger country and contracts in the smaller country, with the opposite being true for the sector producing the homogeneous good. As a consequence, the larger country will net-export the differentiated good and net-import the homogeneous good in the open economy. With a larger fraction of workers seeking employment in the sector of differentiated goods, the larger country experiences an increase in economy-wide unemployment. This is, because the risk of unemployment for an individual worker seeking employment in the sector of differentiated goods is the same in the closed and the open economy, whereas the fraction of workers prone to this risk has increased in the larger country when trade induces specialization and thus a change in the production pattern. However, the increase in unemployment does not necessarily imply a welfare loss. We can distinguish three effects: First, households in both economies benefit from lower

[2]Focussing on the two limiting cases of our parametric PIGL preferences in the main part of the analysis is attractive to separate the role of risk attitudes for the link between production structure and the level of income from additional effects due to changes in the second moment of income distribution, which becomes relevant for the structure of consumer demand and welfare if preferences do not have Gorman form.

import prices (which in the case of a movement from the closed to the open economy fall from infinity to a finite positive value). Second, provided that an increase in the fraction of workers seeking employment in the sector producing differentiated goods is associated with an increase in nominal income, trade generates an income gain in the larger and an income loss in the smaller country. This is the case if preferences are homothetic and unemployment compensation is not too generous, whereas the opposite is true if preferences are quasilinear. Third, welfare is influenced by a variety effect, which can be decomposed into two partial effects, namely an increase in the fraction of firms producing differentiated goods in the larger country and an increase or decrease in the global mass of firms producing differentiated goods. The combined variety effect is positively linked to the effect of trade on nominal income and can therefore also be positive or negative for either economy.

Taking stock, our model produces the well-known result that lower trade costs exhibit a direct positive welfare effect in both countries by lowering the costs of imports. In contrast, the income and variety effects differ in the two economies and can only be positive for one of them. If preferences are quasilinear, the income and variety effects are to the detriment of the larger economy and it is possible that these negative effects dominate the gains associated with a fall in the costs of imports so that the larger country loses from trade. In this case, the larger country experiences double losses, because, as outlined above, its economy-wide rate of employment decreases as well. Things are different in the smaller country, which due to its specialization on the production of the homogeneous good will experience double gains from trade if preferences are quasilinear. However, if preferences are homothetic and unemployment compensation is not too generous, welfare gains are guaranteed in the larger country, despite an increase in the economy-wide rate of unemployment. At the same time, the smaller country can be worse off in the open economy, despite a decrease in the economy-wide rate of unemployment. This points to an important role of preferences (and more specifically the risk attitudes implied by these preferences) for determining the welfare effects of trade in settings featuring labor market imperfection.

We consider two extensions of our model. In a first one, we analyze the case of non-Gorman preferences, implying that the distribution of income matters for consumer demand and welfare. With differentiated goods being luxuries and the homogeneous good being a necessity from the households' point of view, a larger income dispersion increases demand for differentiated goods and lowers demand for the homogeneous good. Of course, if preferences do not have Gorman form, the representative consumer in our model does not have a normative interpretation, so that the choice of a proper welfare function is a priori not clear. Choosing a utilitarian perspective, we show that welfare exhibits social inequality aversion, implying that an increase in income dispersion lowers social welfare. This effect is counteracted, however, by new entry of firms in the now larger market for differentiated goods, which increases welfare due to the households' love of variety. In the open economy, changes in the dispersion of income imply that a higher level of income is no longer sufficient for gains from trade to materialize in the larger country. This confirms our insight from the benchmark model that the form of preferences is crucial for the welfare consequences of trade.

In a second extension, we account for differences of countries in their per-capita income levels. We generate a priori differences in per-capita income by considering differences of the two countries in the labor endowments of households – while abstracting from differences in the total effective labor supply of the two economies. With this modification at hand, we show that trade does not change the labor allocation in the two economies if preferences are homothetic, leaving the economy-wide rate of unemployment at its autarky level and establishing gains from trade due to a fall in the costs of imported goods. Things are different if preferences are quasilinear. In this case, the richer country net-exports the differentiated good and net-imports the homogeneous good. This leads to an increase in economy-wide unemployment and can lead to an overall welfare loss, because the negative income and variety effects counteract the welfare stimulus from lower costs of imports.

Assessing the effects of trade in a setting that features search frictions in the sector producing differentiated goods, our model contributes to a sizable literature dealing with labor market distortions in open economies. Starting with Brecher (1974), this literature has aimed at improving our understanding about the role of labor market institutions as a determinant of international trade flows and as an important factor influencing the effects of trade on employment and welfare (cf. Davidson et al., 1988; Davis, 1998; Kreickemeier and Nelson, 2006). Whereas the focus in recent years has shifted towards models featuring heterogeneous firms and only a single sector of production (cf. Egger and Kreickemeier, 2009, 2012; Helpman et al., 2010; Amiti and Davis, 2012), advancements have also been made in trade models with multiple sectors and differences of these sectors in their labor market institutions (cf. Bastos and Kreickemeier, 2009; Egger et al., 2015). Most closely related to our model in this respect is Helpman and Itskhoki (2010) who consider, as we do, a two-sector trade model featuring a home-market effect. However, similar to other existing work, they do not look at the role of preferences for the employment and welfare effects of trade.

Pointing to potential welfare loss from trade, the analysis in this paper adds to an old and well established debate about the conditions, under which such losses can materialize (see Graham, 1923, for an early example and Helpman, 1984, for a thorough literature review). In multi-sector models disadvantageous specialization in the open economy is usually put forward as a key explanation of why trade can be to the detriment of an economy. Whereas the results from our model are well in line with this argument, we deviate from the widespread view that disadvantageous specialization requires external economies of scale in at least one industry. Excluding external economies of scale, we show that losses from trade can also be the result of a labor market distortion and may exist even if a country expands the sector offering 'good jobs' (in the terminology of Acemoglu, 2001). Provided that specialization in the open economy leads to an expansion of a sector prone to unemployment, increasing the number of good jobs can come at the cost of a higher fraction of workers not finding a job at all. This can generate welfare loss, with preferences playing a crucial role for such disadvantageous specialization to materialize in our model.

Postulating that households have PIGL preferences, this paper also contributes to a strand of literature, which points out that important new insights on the motives for trade, its structure, and consequences can be obtained when deviating from the widespread assumption of homothetic utility. Building on the insight of Linder (1961) that demand-side factors are important determinants of international trade flows, Krugman (1979, 1980), Markusen (1986), and Flam and Helpman (1987) have provided first theoretical accounts of the role of preferences. The main insight from this early research is that a substantial fraction of trade remains unexplained when only considering supply-side motives for its existence (see Markusen, 2013). Matsuyama (2000), Fajgelbaum et al. (2011), and Foellmi et al. (2018) have further contributed to the analysis by distinguishing high- and low-quality goods and by adding a discrete choice element to allow for an aggregation of consumer demand over heterogeneous households even if preferences do not have Gorman form.[3] Fieler (2011) and Caron et al. (2014) consider generalized CES preferences, whereas Bertoletti and Etro (2017) and Matsuyama (2015, 2018) consider a class of preferences that establish a "generalized separable" demand system (see Pollak, 1972). These preferences have the particular advantage to allow for aggregation of demand over various industries with differing price elasticities and are therefore well equipped for studying quantitative general equilibrium trade models. Lacking a representative consumer, the preferences are, however, less suited for aggregating consumer demand over households with differing income levels.

The remainder of the paper is organized as follows. In Section 2, we discuss the building blocks of our model and in Section 3, we analyze the main mechanisms in the closed economy. In Section 4, we investigate trade between two countries that differ in their population size and study the effects of trade on production structure, economy-wide employment, and welfare. In Section 5, we consider non-Gorman preferences and investigate the effects of trade in rich and poor countries. Section 6 concludes with a summary of our results.

2 The model: basics

2.1 Endowment and preferences

We consider a static economy that is populated by a continuum of households with mass H, which in their role as workers inelastically supply $\lambda > 1$ units of labor input for the production of goods. We can interpret λ as worker productivity which is the same for all households. Households have price-independent generalized-linear (so-called PIGL) preferences over two goods, which are represented by a direct utility function of the form

$$\mathcal{U}(X_i, Y_i) = \frac{1}{\varepsilon}(X_i)^{\varepsilon}\left[\left(\frac{Y_i}{\beta}\right)^{\frac{\varepsilon}{1-\varepsilon}} - \beta\right]\left[\left(\frac{Y_i}{\beta}\right)^{\frac{1}{1-\varepsilon}} - Y_i\right]^{-\varepsilon} - \frac{1-\beta}{\varepsilon}, \tag{1}$$

where $\varepsilon, \beta \in (0,1)$ are two constants, Y_i is a homogeneous good, and X_i is a CES aggregate over a continuum of differentiated goods:

$$X_i = \left[\int_{\omega\in\Omega} x_i(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right]^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

[3]Tarasov (2012) considers a model with '0-1' preferences over a continuum of goods to study how price changes in the process of globalization affect welfare of different income groups. He shows that welfare consequences of price adjustments exert asymmetric effects if, due to nonhomothetic preferences, income groups differ in their expenditure shares.

with $x_i(\omega)$ being the consumption level of variety ω and $\sigma > 1$ being the constant elasticity of substitution between the varieties from set Ω. The utility function in Eq. (1) is well-defined only if $X_i > 0$. As pointed out by Muellbauer (1975, 1976), PIGL preferences are the most general class of preferences that deliver a representative consumer and therefore avoid an aggregation problem over households with differing levels of income. Whereas PIGL preferences are usually represented by an indirect utility function, Boppart (2014) shows that for a subclass of these preferences an explicit solution for the direct utility function exists. Egger and Habermeyer (2019) discuss the parameter assumptions needed to arrive at the utility function in Eq. (1) and explain that this utility function has the particularly nice feature of covering homothetic (log-transformed Cobb-Douglas) preferences and quasilinear preferences by the limiting cases of $\varepsilon = 0$ and $\varepsilon = 1$, respectively.

Solving the standard protocol of utility maximization delivers individual demand functions

$$Y_i = \beta \left(\frac{e_i}{P_Y} \right)^{1-\varepsilon} \quad \text{and} \quad x_i(\omega) = \frac{e_i}{P_X} \left(\frac{p(\omega)}{P_X} \right)^{-\sigma} \left[1 - \beta \left(\frac{e_i}{P_Y} \right)^{-\varepsilon} \right], \tag{3}$$

respectively, where e_i is the expenditure level of individual i, P_Y is the price of the homogeneous good, $p(\omega)$ is the price of variety ω of the differentiated good, and $P_X \equiv \left[\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}$ is a CES index over the prices of all these varieties. From Eq. (3), we see that the Engel curve of homogeneous good Y_i is concave, making this good a *necessity* with its value share of consumption decreasing in the expenditure level. In contrast, the Engel curves of differentiated goods $x_i(\omega)$ are convex making these goods *luxuries*. Aggregating over households, gives market demand functions

$$Y = \int_{i \in \mathcal{H}} Y_i di = \beta \frac{H\bar{e}}{P_Y} \left(\frac{\bar{e}}{P_Y} \right)^{-\varepsilon} \psi, \tag{4}$$

$$x(\omega) = \int_{i \in \mathcal{H}} x_i(\omega) di = \frac{H\bar{e}}{P_X} \left(\frac{p(\omega)}{P_X} \right)^{-\sigma} \left[1 - \beta \left(\frac{\bar{e}}{P_Y} \right)^{-\varepsilon} \psi \right], \tag{5}$$

where $\bar{e} \equiv H^{-1} \int_{i \in \mathcal{H}} e_i di$ is the average expenditure level of households and $\psi \equiv H^{-1} \int_{i \in \mathcal{H}} (e_i/\bar{e})^{1-\varepsilon} di$ is a dispersion index that is defined on the unit interval and captures how the distribution of household expenditures affects the value shares of consumption. The dispersion index reaches a maximum level of one if the distribution of expenditures is egalitarian or if the distribution of household expenditure is irrelevant for aggregate demand because Engel curves are linear, which applies to the two limiting cases of homothetic and quasilinear preferences.

2.2 Technology and the firms' problem

Firms in the sector of the homogeneous good enter the market at zero cost and hire workers at a common wage rate w_Y per unit of labor input. Workers need one unit of their labor input to produce one unit of the homogeneous good, which is sold under perfect competition. This establishes $w_Y = P_Y$. Firms producing differentiated goods have to develop a blueprint, which comes at the cost of f units of the homogeneous good and gives them access to a unique variety that can be sold under monopolistic competition. To produce their output firms hire workers, who manufacture one unit of the differentiated good with each unit of their labor input. Hiring and wage setting in the sector of differentiated goods is a two-stage problem. At stage one, firms install vacancies at the cost of one unit of the homogeneous good and search for workers filling these vacancies. There are search frictions and the assignment of workers to jobs is solved through random matching (cf. Pissarides, 2000; Helpman and Itskhoki, 2010; Felbermayr and Prat, 2011). For those vacancies successfully filled, firms and workers form a bilateral monopoly at stage two and distribute the production surplus generated in the workplace through Stole and Zwiebel (1996) bargaining.[4] We solve the firm's hiring and wage setting problem through backward induction and begin with stage two.

The bargaining problem at stage two is reminiscent of the multilateral problem in Helpman and Itskhoki (2010), with the difference that we allow for asymmetric bargaining power of workers and firms. The asymmetric bargaining protocol is already discussed by Stole and Zwiebel (1996) and it has been applied to a model similar as ours by Egger and Habermeyer (2019). Our problem is simpler though, because we assume that all workers employed by a firm provide the same level of labor input λ. Following Stole and Zwiebel (1996), we can characterize the solution of the bargaining problem by a splitting rule, which determines how the production surplus achieved by an agreement is distributed between the bargaining parties; and an aggregation rule, describing how infra-marginal production surpluses add up to the firm's total surplus from multilateral bargaining with all of its workers. Bargaining with a mass $l(\omega)$ of workers, firm ω's total bargaining surplus is given by

$$\pi(\omega) = \int_0^{l(\omega)} \kappa[\ell|l(\omega)]\hat{r}(\ell)d\ell, \tag{6}$$

where $\hat{r}(\ell) = D^{\frac{1}{\sigma}}(\lambda\ell)^{1-\frac{1}{\sigma}}$ are revenues achieved with employment level ℓ, D is a common demand shifter, and

$$\kappa[\ell|l(\omega)] \equiv \frac{1}{\alpha\ell}\left(\frac{\ell}{l(\omega)}\right)^{\frac{1}{\alpha}} \tag{7}$$

is a probability measure that determines the fraction of infra-marginal production surplus the firm can acquire in its wage negotiation with workers. This probability measure declines in the workers' *relative* bargaining power $\alpha > 0$. Solving the integral in Eq. (6) gives

$$\pi(\omega) = \frac{\sigma}{\sigma + \alpha(\sigma - 1)} D^{\frac{1}{\sigma}} \left[\lambda l(\omega)\right]^{1-\frac{1}{\sigma}} = \frac{\sigma}{\sigma + \alpha(\sigma - 1)} r(\omega), \tag{8}$$

where the second equality sign uses the definition $r(\omega) \equiv \hat{r}[l(\omega)]$.

[4] Bruegemann et al. (2018) show that, in contrast to common belief, the Stole and Zwiebel (1996) bargaining protocol does not give wage and profit profiles that coincide with the Shapley values. They suggest using a Rolodex Game instead of the non-cooperative game put forward by Stole and Zwiebel to achieve equivalence of the bargaining outcome with the Shapley values.

If an agreement in the wage negotiation between the firm and a worker is not achieved, the worker becomes unemployed and receives an unemployment compensation of $\gamma\lambda w_Y$, where $\gamma \in (0,1)$ is a common replacement rate. Higher unemployment compensation improves the disagreement income of workers in their wage negotiations and thus the rent accrued by workers in the bargaining with the firm. The influence of unemployment compensation on wages is reflected in the splitting rule determining how to distribute the production surplus between the firm and its workers. This splitting rule is given by

$$\frac{\partial \pi(\omega)}{\partial l(\omega)} = \lambda \frac{w_X(\omega) - \gamma w_Y}{\alpha}, \tag{9}$$

where $w_X(\omega)$ is the wage rate for each unit of labor input paid by firm ω. Eqs. (8) and (9) jointly determine the solution for the firm's bargaining problem at stage two. Thereby, firms accrue a constant fraction $\rho \equiv \sigma/[\sigma + \alpha(\sigma - 1)] < 1$ of revenues in the wage bargaining with workers, which is decreasing in the relative bargaining power of workers, α.

Equipped with the solution for the bargaining problem, we can now determine the outcome of the firm's hiring problem. Recollecting from above that firms have to invest f units of the homogeneous good to start production and one unit of the homogeneous good for each vacancy installed, this solution is found by maximizing profits $\Pi(\omega) \equiv \rho r(\omega) - q^{-1} P_Y l(\omega) - P_Y f$ with respect to $l(\omega)$, where $q < 1$ is the probability that a vacancy can be filled, which in the case of random matching is exogenous to the individual firm and the same for all producers. The first-order condition for the firm's profit-maximizing choice of $l(\omega)$ is given by

$$\frac{d\Pi(\omega)}{dl(\omega)} = \frac{\sigma - 1}{\sigma} \frac{\rho r(\omega)}{l(\omega)} - \frac{P_Y}{q} = 0. \tag{10}$$

Accounting for Eqs. (8), (9), and recollecting that $P_Y = w_Y$ then gives the outcome of hiring and wage-setting for firms producing differentiated goods:

$$w_X(\omega) = \frac{\alpha + \gamma\lambda q}{\lambda q} w_Y, \qquad \Pi(\omega) = \frac{\rho r(\omega)}{\sigma} - P_Y f. \tag{11}$$

Since all firms producing differentiated goods employ the same technology and pay the same wage, they are symmetric producers. This allows us to drop firm index ω from now on.

2.3 Industry-wide outcome in the sector of differentiated goods

Eq. (11) has been derived under the assumption that firms producing differentiated goods can attract the intended mass of applicants at a wage rate w_X. To see under which condition this is the case, we have to determine the labor market outcome in the sector of differentiated goods. For this purpose, we note that the supply of workers in the sector of differentiated goods is given by the product of the mass of households, H, and the fraction of these households seeking employment in the sector of differentiated goods, h. The ratio between the mass of workers seeking employment, hH, and the total mass of vacancies installed, Q, is pinned down by a Cobb-Douglas matching function and given by $hH/Q = m(1-u)^{-1}$, where m is a positive constant that measures matching efficiency, and $1-u$ is the share of workers successfully matched to a firm and thus the employment rate in the sector of differentiated goods. In the Appendix, we provide a microfoundation of this outcome and show that the matching technology considered here can be interpreted as a special case of the matching technology in Helpman and Itskhoki (2010). The probability of filling a vacancy is given by $q = hH(1-u)/Q = m$ and thus independent of the employment rate in our model. Setting $m = \lambda^{-1}$ proves particularly useful for our purposes, because it allows us to get rid of uninteresting constants. This additional simplification generates a negative relationship between matching efficiency and labor productivity, which can be justified by assuming that workers with higher and more specialized abilities are more difficult to place in the labor market.[5]

With this matching technology at hand, we can solve for the employment rate in the sector of differentiated goods, using the indifference condition for production workers, who can either enter the sector of the homogeneous good, which promises an income of w_Y per unit of labor input, or enter the sector of differentiated goods, which promises for each unit of labor input an income $w_X = (\alpha+\gamma)w_Y$ with probability $1-u$ and an unemployment compensation of γw_Y with probability u. Assuming that unemployment compensation is financed by a proportional tax on all types of income, including the transfer payment to the unemployed (see Egger and Kreickemeier, 2012), taxation does not influence the sector, workers choose for offering their labor input. Considering the utility function in Eq. (1) and individual demand functions in Eq. (3), we can solve the indifference condition of workers for

$$1-u = \frac{1-\gamma^{\varepsilon}}{(\alpha+\gamma)^{\varepsilon}-\gamma^{\varepsilon}}, \tag{12}$$

where $w_X = (\alpha+\gamma)w_Y$ and $q = m = \lambda^{-1}$ have been used. Eq. (12) reveals that an interior solution with $0 < u < 1$ requires $\alpha > 1-\gamma$, and hence that the sector of differentiated goods offers a wage premium $\tilde{\alpha} \equiv \alpha + \gamma > 1$. Provided that such an outcome exists, a higher relative bargaining power of workers, α, increases the wage premium, and therefore the employment rate has to fall in order to restore indifference of workers to enter the two sectors. We can complete the characterization of the industry equilibrium by noting that free entry of firms into the sector of differentiated goods establishes the zero-profit condition $\rho r = \sigma P_Y f$.

2.4 Production structure and disposable labor income

We complete the discussion of the main building blocks of our model by elaborating on how changes in the production structure affect the average level and dispersion of *disposable* labor income with a particular focus on the role of preferences for this outcome. Due to our assumption that all types of income are subject to the same income tax, average disposable labor income and thus the average household

[5]As briefly discussed in the Appendix, the results from our analysis extend to more general matching technologies, with further derivation details available from the authors upon request.

consumption expenditure is given by

$$\overline{e} = w_Y \lambda \left\{1 + h[(1-u)\tilde{\alpha} - 1]\right\}. \tag{13}$$

Eq. (13) points to a trade-off an increase in the fraction of workers producing differentiated goods has on average disposable income. On the one hand, a higher h leads to an increase in the fraction of workers receiving the wage premium offered by luxury producers. On the other hand, it increases the economy-wide rate of unemployment, $U \equiv uh$, and thus the share of labor input not productively used in the economy. In general, $(1-u)\tilde{\alpha} >,=,< 1$ is possible, so that allocating more workers to the sector of differentiated goods can have a positive or negative effect on average disposable household income, depending on whether the first or the second effect dominates. Since the effect that changes in the fraction of workers seeking employment in the production of differentiated goods have on average disposable labor income is essential for the welfare effects of trade, it is useful to shed light on the role of preferences for the ranking of $(1-u)\tilde{\alpha} >,=,< 1$. The following lemma summarizes this role.

Lemma 1 *If preferences are quasilinear (and thus $\varepsilon = 1$), we have $(1-u)\tilde{\alpha} < 1$. In all other cases $(1-u)\tilde{\alpha} >,=,< 1$ is possible, with $(1-u)\tilde{\alpha} > 1$ achieved for sufficiently high levels of α. In the limiting case of homothetic preferences (and thus $\varepsilon = 0$), $(1-u)\tilde{\alpha} > 1$ extends to all possible $\alpha > 1-\gamma$ if $\gamma < \exp[-1]$.*

Proof Formal proof in the Appendix

Whereas production of differentiated goods promises a wage premium if households are successfully matched with firms, applying for jobs in the sector producing differentiated goods comes at the risk of being not successfully matched and receiving only unemployment compensation. The households' risk attitudes and hence the evaluation of the risk of job loss depend on their preferences (or, more specifically, on preference parameter ε). If $\varepsilon = 1$ preferences are quasilinear and households are risk-neutral. In this case, the constraint in Eq. (12), which makes workers indifferent between the two sectors, reduces to a condition equalizing the expected disposable income from job search in the two sectors: $(1-u)\tilde{\alpha}w_Y + u\gamma w_Y = w_Y$. Because of their risk neutrality, households accept a relatively low probability of a successful match and thus a relatively high rate of unemployment, when seeking employment in the sector of differentiated goods, leading to $(1-u)\tilde{\alpha} < 1$. Things are different if households are risk-averse due to $\varepsilon < 1$, with the degree of risk aversion maximized in our model if $\varepsilon = 0$ makes preferences homothetic. In this case, households applying for jobs in the sector producing differentiated goods must be compensated for accepting the risk of unemployment. With the wage premium $\tilde{\alpha} > 1$ fixed, risk aversion leads to a fall in the unemployment rate, thereby increasing $(1-u)\tilde{\alpha}$. In the case of homothetic preferences $(1-u)\tilde{\alpha} > 1$ is achieved for all $\tilde{\alpha} > 1$ and thus for all $\alpha > 1-\gamma$, if unemployment compensation is not too generous, i.e if $\gamma < \exp[-1]$.[6] This is the parameter domain we focus on in the

[6] For a given wage premium $\tilde{\alpha}$, a higher replacement rate γ increases household income in the event of unemployment, and hence unemployment rate u has to increase in order to restore indifference condition (12). This provides an intuition for an upper limit of γ needed to ensure $(1-u)\tilde{\alpha} > 1$ for all possible levels of $\tilde{\alpha}$ if preferences are homothetic.

subsequent analysis in order to emphasize the important role played by the degree of risk aversion when contrasting the two limiting cases of quasilinear and homothetic preferences.

With the insights regarding the relationship of production structure and average disposable household income (expenditures) at hand, we now turn to the dispersion index of disposable household income, which can be computed according to

$$\psi = \left[\frac{(1-\tau)w_Y\lambda}{\bar{e}}\right]^{1-\varepsilon}\left\{1+h\left[(1-u)\tilde{\alpha}^{1-\varepsilon}+u\gamma^{1-\varepsilon}-1\right]\right\}$$

where $\tau \in (0,1)$ is the common income tax rate that is determined by the condition of a balanced budget of the government:

$$\tau \equiv \frac{hu\gamma}{1+h\left[(1-u)\tilde{\alpha}+u\gamma-1\right]}. \tag{14}$$

Tax rate τ increases in the fraction of workers seeking employment in the production of differentiated goods, h. This is because a higher h is associated with higher economy-wide unemployment, U, implying that the now fewer employed production workers have to finance the compensation for a larger mass of unemployed. Substituting tax rate τ and $\bar{e}$ into ψ establishes

$$\psi = \frac{1+h\left[(1-u)\tilde{\alpha}^{1-\varepsilon}+u\gamma^{1-\varepsilon}-1\right]}{\left\{1+h\left[(1-u)\tilde{\alpha}+u\gamma-1\right]\right\}^{1-\varepsilon}}, \tag{15}$$

where $\psi = 1$ holds in the case of Gorman form preferences, which are associated with the limiting cases of $\varepsilon = 0$ and $\varepsilon = 1$. This points to the important result that higher degrees of risk aversion do not exert a monotonic effect on dispersion index ψ. This is, because the dispersion index does not capture the second moment of income distribution but the impact of income distribution on the structure of consumer demand. With quasilinear or homothetic preferences, aggregate consumer demand does not depend on the distribution of disposable household income – provided that even the households with the lowest income consume both goods. To ensure that this is the case, condition $(1-\tau)\gamma\lambda > \beta^{1/\varepsilon}$ must be fulfilled. This condition depends on the endogenous, yet to be determined, fraction of workers seeking employment in the production of differentiated goods, h, which is different for the closed and the open economy.[7]

3 The closed economy

To determine the fraction of workers seeking employment in the sector of differentiated goods, h, we can make use of two important insights from our analysis. The first one is that combining Eqs. (8) and (9), we can compute $[(\sigma-1)/\sigma]\rho r/l = \lambda[w_X - \gamma w_Y]/\alpha$ and can thus express the wage bill of firms as

[7]To derive a sufficient parameter constraint for this condition to hold, we can note from above that τ reaches a maximum at $h = 1$, which we denote by $\bar{\tau}$. Making use of Eqs. (12) and (14) we compute $\bar{\tau} \equiv \{1+\tilde{\alpha}(1-\gamma^{\varepsilon})/[\gamma(\tilde{\alpha}^{\epsilon}-1)]\}^{-1} < 1$, and hence $(1-\bar{\tau})\gamma\lambda > \beta^{1/\varepsilon}$ gives a sufficient condition in exogenous model parameters for the intended result that all households purchase both types of goods.

$\lambda l w_X = \lambda l \gamma w_Y + \rho r \alpha(\sigma - 1)/\sigma$. This captures the outcome of wage bargaining (plus constant markup pricing) and, noting that M firms enter and $hH(1-u)$ workers find a job, allows us to determine a positive link between the share of workers seeking employment in the sector of differentiated goods and the mass of firms producing them according to

$$hH\lambda w_Y(1-u) = \frac{\sigma - 1}{\sigma}\rho M r, \tag{16}$$

where $w_X = (\alpha + \gamma) w_Y$ has been considered. A second relationship between h and M follows from the market clearing condition for differentiated goods, can be derived from Eq. (5), and is given by

$$H\lambda w_Y \left(1 - \beta\lambda^{-\varepsilon}\right) + H\lambda w_Y B(h) = Mr, \tag{17}$$

where $B(h) \equiv h\big[(1-u)\tilde{\alpha} - 1\big] + \beta\lambda^{-\varepsilon}\big[1 - T(h)\big]$ is derived in the Appendix and captures the additional effect on consumer demand from the labor market distortion and the tax-transfer scheme implemented to compensate the unemployed. Rent-sharing increases *market* income of an endogenous fraction of $h(1-u)$ workers, who find employment in the sector of differentiated goods and therefore benefit from a wage premium $\tilde{\alpha} > 1$. This gives term $h\big[(1-u)\tilde{\alpha} - 1\big]$ as a first component of $B(h)$. The second component captures the demand effect through endogenous changes in the dispersion of disposable household income, because workers seeking employment in the sector of differentiated goods can experience higher or lower income than in the homogeneous goods sector, depending on their employment status, and because the tax-transfer system makes disposable income more egalitarian. The combined dispersion effect is captured by $\beta\lambda^{-\varepsilon}\big[1 - T(h)\big]$, with

$$T(h) \equiv \left\{1 + h\left[(1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon} - 1\right]\right\}\left(\frac{1 + h[(1-u)\tilde{\alpha} - 1]}{1 + h[(1-u)\tilde{\alpha} + u\gamma - 1]}\right)^{1-\varepsilon}. \tag{18}$$

In the limiting case of homothetic preferences, we have $\lim_{\varepsilon\to 0} T(h) = 1 + h[(1-u)\tilde{\alpha} - 1] > 0$ and thus $\lim_{\varepsilon\to 0} B(h) = h[(1-u)\tilde{\alpha} - 1](1-\beta)$, whereas in the limiting case of quasilinear preferences, we have $\lim_{\varepsilon\to 1} T(h) = 1$ and thus $\lim_{\varepsilon\to 1} B(h) = h[(1-u)\tilde{\alpha} - 1]$. In both scenarios, $B(h)$ captures a pure efficiency effect due to changes in the level of average disposable household income, while changes in the dispersion of income do not exert an additional effect in the case of Gorman form preferences. However, this efficiency effect is not the same for homothetic and quasilinear preferences. As pointed out by Lemma 1, in the case of quasilinear preferences $(1-u)\tilde{\alpha} < 1$ holds for all possible parameter configurations, and hence the demand for differentiated goods is reduced by the labor market distortion, because the negative employment effect dominates the positive wage effect of those successfully matched to firms. In contrast, with homothetic preferences, $(1-u)\tilde{\alpha} > 1$ is achieved for all possible $\alpha > 1 - \gamma$ if unemployment compensation is not too generous, establishing $B(h) > 0$ (despite of $T(h) > 1$). If preferences do not have Gorman form, demand for differentiated goods is furthermore influenced by the dispersion of disposable household income. Whereas this complicates the analysis considerably, the model remains nicely tractable for $\varepsilon = 1/2$. In this case, we have $T(h) < 1$ (using the indifference condition in Eq. (12)), so that the combined dispersion effect on demand for differentiated goods is positive. This suggests that the increase in the dispersion of market income dominates the decrease in the dispersion of disposable income due to the tax-transfer system and implies that demand for differentiated goods is further increased by the labor market distortion.

Combining Eqs. (16) and (17) allows us to solve for the equilibrium fraction of workers seeking employment in the sector of differentiated goods. The respective solution is given by the condition $\Gamma(h) = 0$, with

$$\Gamma(h) \equiv 1 - h\left[1 + \left(\frac{\sigma}{\sigma - 1} - \gamma\right)(1-u)\right] - \beta\lambda^{-\varepsilon}T(h). \tag{19}$$

We show in the Appendix that $\Gamma(h) = 0$ has a unique solution in h. Combining this solution with Eq. (16) and the zero-profit condition $\rho r = \sigma P_Y f$ determines the equilibrium mass of firms producing differentiated goods, M. For Gorman form preferences, we get explicit solutions for h and M. For the limiting case of homothetic preferences, we compute

$$h = \frac{\frac{1-\beta}{1-u}}{\frac{\sigma}{\sigma-1}\frac{1}{\rho} - \frac{1-\beta}{1-u}\left[(1-u)\tilde{\alpha} - 1\right]}, \qquad M = \frac{\frac{(1-\beta)H\lambda}{(\sigma-1)f}}{\frac{\sigma}{\sigma-1}\frac{1}{\rho} - \frac{1-\beta}{1-u}\left[(1-u)\tilde{\alpha} - 1\right]}, \tag{20}$$

whereas in the case of quasilinear preferences, we obtain

$$h = \frac{\frac{\lambda-\beta}{\lambda(1-u)}}{\frac{\sigma}{\sigma-1}\frac{1}{\rho} - \frac{1}{1-u}\left[(1-u)\tilde{\alpha} - 1\right]}, \qquad M = \frac{\frac{(\lambda-\beta)H}{(\sigma-1)f}}{\frac{\sigma}{\sigma-1}\frac{1}{\rho} - \frac{1}{1-u}\left[(1-u)\tilde{\alpha} - 1\right]}. \tag{21}$$

Higher levels of per-capita labor endowment λ make for a given allocation of workers all households richer and increase the expenditures for differentiated goods. In the case of homothetic preferences the expenditure shares of differentiated goods are independent of λ, so that the now higher demand for differentiated goods is offset by the now higher supply of labor producing them, leaving the fraction of workers seeking employment in the sector of differentiated goods, h, unaffected. Things are different in the case of quasilinear preferences. With expenditure shares for differentiated goods increasing in λ, more workers are needed in the sector of differentiated goods to fulfill the now higher consumer demand for these goods. As a consequence, h has to increase to restore market clearing. Irrespective of the preferences, more firms will enter the now larger market for differentiated goods.

A higher relative bargaining power of workers α can increase or decrease the fraction of workers seeking employment in the sector of differentiated goods. A higher α must lower employment rate $1-u$ to restore indifference of workers between the two sectors. All other things equal, a higher fraction of workers must therefore seek employment in the sector of differentiated goods to fulfill a given demand. This effect can be counteracted if an increase in average disposable household income, due to an increase in $(1-u)\tilde{\alpha}$, induces households to increase their demand for the homogeneous good, causing a reallocation of labor away from the sector of differentiated goods. This second effect needs not to work against the first one, because average disposable household income can fall in α and because with quasilinear preferences income changes leave demand for the homogeneous good unaffected, according to Eq. (3). However, in general it is a priori not clear, which of the two effects dominates, so that $dh/d\alpha$ can be positive or negative. Whereas we cannot rule out positive effects of a stronger labor market distortion on the fraction of workers seeking employment in the sector of differentiated goods, the mass of firms producing them, M, unambiguously decreases in α in the two limiting cases captured by Eqs. (20) and (21). This is, because a higher wage premium increases the costs of production, and therefore makes entry less attractive for firms. Whereas an increase in average disposable household income would counteract this effect, it does not dominate because the respective demand stimulus is mitigated by an income loss of those workers becoming newly unemployed in the sector of differentiated goods.

Changes in the fraction of workers seeking employment in the sector of differentiated goods and changes in the mass of firms producing them are important determinants of welfare effects. In the two limiting cases of $\varepsilon = 0$ and $\varepsilon = 1$ preferences have Gorman form, giving the representative consumer a normative interpretation. This allows us to consider utility of the representative consumer as a proper welfare function, establishing[8]

$$V_{CD}(\overline{e}, P_Y, P_X) \equiv \ln\left(\frac{\overline{e}}{P_Y^{\beta} P_X^{1-\beta}}\right), \qquad V_{QL}(\overline{e}, P_Y, P_X) \equiv \frac{\overline{e}}{P_X} - \beta\frac{P_Y}{P_X} - 1 + \beta \tag{22}$$

in the case of homothetic (log-transformed Cobb-Douglas) and quasilinear preferences, respectively. Substituting Eqs. (13), (20), and (21), and accounting for $P_X = \frac{\sigma}{\sigma-1}\frac{w_Y}{\rho}M^{\frac{1}{1-\sigma}}$, we can express welfare as a function of λ and α. Intuitively, welfare increases with per-capita labor endowment λ for two reasons: On the one hand, an increase in per-capita labor endowment makes all households richer. In the case of quasilinear preferences this direct effect is counteracted by an indirect effect, because the reallocation of labor towards the production of differentiated goods leads to an aggregate income loss. However, differentiating $\overline{e}$, it is easily verified that the indirect effect cannot dominate. On the other hand, a higher λ induces more firms to enter the sector of differentiated goods, which leads to a fall in the CES price index P_X and thereby stimulates welfare. Regarding the effect of a higher wage premium, we show in the Appendix that welfare unambiguously decreases in α if preferences are quasilinear. This is, because a stronger labor market distortion decreases average disposable household income, lowers the mass of firms producing differentiated goods, and increases the prices charged by the remaining firms. All three effects are detrimental for social welfare. If preferences are homothetic, average disposable household income can increase in α, thereby counteracting negative effects from a lower mass of firms and higher prices for each differentiated variety. In this case, a stronger labor market distortion can be a stimulus for social welfare.

Whereas specifying a welfare function in the case of Gorman preferences is straightforward, choosing

[8] The (price-invariant) representative level of expenditures is defined by Muellbauer (1975) as the expenditure level that gives the same expenditure shares for the homogeneous good and differentiated goods as observed for the whole economy. It is given by $e_r = \overline{e}\psi^{-\frac{1}{\varepsilon}}$, and the household with this income level is therefore called representative consumer. With Gorman form preferences, we have $e_r = \overline{e}$, and we can compute the welfare functions in Eq. (22) by determining indirect utility of the representative household, using Eqs. (1)-(3).

a proper welfare function is less obvious if preferences do not have Gorman form, because the representative consumer does not bear a normative interpretation in this case (see Muellbauer, 1975, 1976). One possibility put forward by Egger and Habermeyer (2019) is to take a utilitarian perspective and we follow this approach in Section 5, where we discuss how the results from our analysis change when $\varepsilon \in (0,1)$. This completes the discussion of the closed economy.[9]

4 The open economy

In the open economy, we consider trade between two countries that are symmetric in all respects, except for their population size: $H \neq H^*$, where an asterisk is used to indicate foreign variables and to distinguish them from home variables. Trade in the homogeneous good is free of costs, and hence wage w_Y is the same in the two economies, provided that production is diversified in either of the two economies. We discuss the parameter domain supporting diversification below. Trade in differentiated goods is subject to iceberg trade costs, implying that $t^{\frac{1}{\sigma-1}} > 1$ units of the good must be shipped in order for one unit to arrive in the foreign country.

4.1 Characterization of the open economy equilibrium

Under diversification, the open economy equilibrium can be characterized by combining the outcome of wage bargaining with the zero-profit conditions and goods market clearing for differentiated goods in the two economies. Following the steps of the closed economy, we find that wage bargaining (plus constant markup pricing) establishes a proportional link between the fraction of workers seeking employment in the sector of differentiated goods, h, and the mass of firms producing them, M. We obtain

$$hH\lambda w_Y(1-u) = \frac{\sigma-1}{\sigma}\rho M r, \qquad h^*H^*\lambda w_Y(1-u) = \frac{\sigma-1}{\sigma}\rho M^* r^* \tag{23}$$

for home and foreign, respectively. Contrasting Eqs. (16) and (23), we see that trade leaves the link between h and M established by wage bargaining unaffected. This result is intuitive because Eqs. (8) and (9) are the same in the closed and the open economy. Furthermore, firm-level revenues in home and foreign, r and r^*, respectively, are linked by the zero-profit conditions $\rho r = \sigma P_Y f$, $\rho r^* = \sigma P_Y f$. Accordingly, firm-level revenues are the same in the two economies, provided that production is diversified and that trade of the homogeneous good is costless. Market clearing in the sector of differentiated goods

[9] With a utilitarian perspective, we propose that welfare is equal to the average utility level of households. Average household utility corresponds to V_{QL} in Eq. (22) if $\varepsilon = 1$. However, from Jensen's inequality it follows that V_{CD} in Eq. (22) is larger than average household utility if $\varepsilon = 0$. Due to inequality aversion, a social planner would prefer an egalitarian income distribution, and would therefore implement V_{CD} if a lump-sum tax-transfer system were available. Whereas the difference between V_{CD} and utilitarian welfare is notable, it is of no further interest for the analysis in this paper.

gives for home and foreign

$$\begin{aligned} H\lambda w_Y\left(1-\beta\lambda^{-\varepsilon}\right)+H\lambda w_Y B(h) &= \frac{Mrt}{1+t}+\frac{M^*r^*}{1+t}, \\ H^*\lambda w_Y\left(1-\beta\lambda^{-\varepsilon}\right)+H^*\lambda w_Y B(h^*) &= \frac{M^*r^*t}{1+t}+\frac{Mr}{1+t}, \end{aligned} \tag{24}$$

respectively.

Combining Eqs. (23) and (24) and accounting for the zero-profit conditions, we can solve for the equilibrium values of h and h^* in the open economy. These values are determined by a system of two equations

$$h^* = \frac{1}{\eta}\Phi(h), \qquad h = \eta\Phi(h^*), \tag{25}$$

with $\eta \equiv H^*/H$,

$$\Phi(x) \equiv x + \frac{\sigma-1}{\sigma}\frac{\rho(1+t)}{1-u}\Gamma(x), \quad x = h, h^*, \tag{26}$$

and $\Gamma(\cdot)$ being defined in Eq. (19). The first expression in (25) makes use of market clearing for differentiated goods at home and therefore gives the response of h to changes in h^* that is necessary to restore market clearing in home. The second expression in (25) makes use of market clearing for differentiated goods abroad and therefore gives the response of h^* to changes in h that is necessary to restore market clearing in foreign.

We illustrate the open economy equilibrium for the case of symmetric countries ($\eta = 1$) in Figure 1. There, we depict the two equations in (25) in (h, h^*)-space by the two curves $\Phi(h)$ and $\Phi(h^*)$, respectively. The negative slope of the two curves is assumed for now and further discussed below. $\Phi(h)$ has an intercept with the vertical axis at $\Phi(0) = \frac{\sigma-1}{\sigma}\frac{\rho(1+t)}{1-u}(1-\beta\lambda^{-\varepsilon})$ and this intercept is denoted by $f_1(t)$, with $f_1'(t) > 0$. Due to symmetry of the two trading partners, the intercept of $\Phi(h^*)$ with the horizontal axis is also given by $f_1(t)$. Furthermore, $\Phi(h)$ has an intercept with the horizontal axis if $h + \frac{\sigma-1}{\sigma}\frac{\rho(1+t)}{1-u}\Gamma(h) = 0$ has a solution in h. For $-\frac{1-u}{1+t}\left[t\left(\frac{\sigma}{\sigma-1}-\gamma\right)-\tilde{\alpha}\right]-\beta\lambda^{-\varepsilon}T(1) < 0$ a solution exists and it lies on the unit interval.[10] We denote this solution by $f_2(t)$, with $f_2'(t) < 0$, and it is unique due to our assumption that $\Phi(h)$ has a negative slope.

We can now make use of Figure 1 to discuss existence, uniqueness, and stability of the open economy equilibrium. Showing existence of the open economy equilibrium is simple for the case of symmetric countries, because we see in Figure 1 that the two curves $\Phi(h)$ and $\Phi(h^*)$ intersect in point A at the 45°-line, where the fractions of workers seeking employment in the sector of differentiated goods are the same in the two economies and are given by their autarky levels. This establishes $h = h^* = h_a$, with subscript a used to indicate an autarky variable. To show uniqueness of the intersection point, we can

[10] To see this, we can substitute Eq. (19) for $\Gamma(\cdot)$ and evaluate $\Phi(x)$ at $x = 0$ and $x = 1$. This gives $\Phi(0) = \frac{\sigma-1}{\sigma}\frac{\rho(1+t)}{1-u}(1-\beta\lambda^{-\varepsilon}) > 0$ and $\Phi(1) = \frac{\sigma-1}{\sigma}\frac{\rho(1+t)}{1-u}\left\{-\frac{1-u}{1+t}\left[t\left(\frac{\sigma}{\sigma-1}-\gamma\right)-\tilde{\alpha}\right]-\beta\lambda^{-\varepsilon}T(1)\right\}$, respectively.

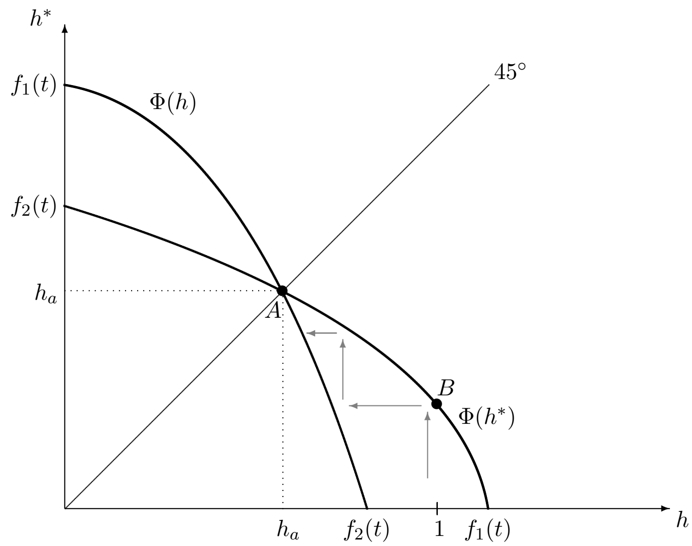

Figure 1: *Equilibrium in the open economy with symmetric countries*

define a critical

$$\underline{t}(x) \equiv -\frac{2\sigma}{\sigma-1}\frac{1-u}{\rho}\Gamma'(x)^{-1} - 1, \quad x = h, h^* \tag{27}$$

such that $\Phi'(h) < -1$ holds if $t > \underline{t}(h)$, whereas $\Phi'(h^*) < -1$ holds if $t > \underline{t}(h^*)$. Noting that $t > \max\{\underline{t}(0), \underline{t}(1)\}$ is sufficient for $\Phi'(h) < -1$ to extend to all $h \in (0,1)$ and for $\Phi'(h^*) < -1$ to extend to all $h^* \in (0,1)$,[11] it follows from $t > \max\{\underline{t}(0), \underline{t}(1)\}$ that curve $\Phi(h)$ is steeper than curve $\Phi(h^*)$, proving uniqueness of intersection point A on the unit interval. Finally, stability of the open economy equilibrium in point A follows from its uniqueness and is illustrated by the grey arrows in Figure 1. Of course, the analysis so far has been confined to diversification equilibria, and one may suspect that an equilibrium with full specialization of production in one of the two economies also exists, as indicated, for instance, by a point like B. However, this is not true, because the requirement of market clearing rules out such an outcome provided that $f_2(t) < 1$. This follows from the direction of the grey arrows in Figure 1.

The open economy equilibrium is no longer symmetric, however, if the two countries differ in their population size. For instance, if the foreign country is larger than the domestic one, we have $\eta > 1$, and in this case the foreign country features a larger market for differentiated goods. This case is illustrated in Figure 2. In the closed economy, the additional demand for labor from a larger population size is offset by a larger labor supply, leaving the fraction of workers seeking employment in the sector of differentiated goods unaffected. Accordingly, the autarky equilibrium remains to be given by point A,

[11]To see this, it is worth noting that the second derivative of $\Phi(h)$ adopts the properties of the second derivative of $\Gamma(h)$: $\Phi''(h) = \frac{\sigma-1}{\sigma}\frac{\rho(1+t)}{1-u}\Gamma''(h)$. In Appendix A.4 we discuss the properties of $\Gamma(h)$ and show in particular that $\Gamma(h)$ – and in extension $\Phi(h)$ – cannot have an extremum at the unit interval. We can therefore conclude that $\Phi'(h) < -1$ must hold for all possible $h \in (0,1)$ if $\Phi'(0) < -1$ and $\Phi'(1) < -1$.

irrespective of the prevailing differences in population size. Things are different in the open economy. From previous work on home-market effects (cf. Helpman and Krugman, 1985), we know that in a setting as ours "a country whose share of demand for a good is larger than average will have – ceteris paribus – a more than proportionally larger-than average share of world production of that good" (Crozet and Trionfetti, 2008, p.309). Therefore, in the open economy the fraction of workers seeking employment in the production of differentiated goods increases in foreign and decreases in home if $\eta > 1$. In Figure 2 the relative increase in foreign market size leads to a counter-clockwise rotation of locus $\Phi(h)$ and locus $\Phi(h^*)$ in their respective intercepts $f_2(t)$. These intercepts are unaffected because they capture the local market clearing conditions in the respective countries if worldwide production of differentiated goods is concentrated there. Accordingly, relative country size differences are irrelevant for the positions of these intercepts. Things are different for intercepts $f_1(t)$, which reflect the local market-clearing conditions in the respective countries if no local production is left. In this case, relative country size differences exhibit the largest effect. Figure 2 shows a new open economy equilibrium in point $\tilde{A}$ and illustrates that access to trade leads to an expansion of the production of differentiated goods in the country with the initially larger market for these goods and to a contraction of the production of differentiated goods in the other economy.

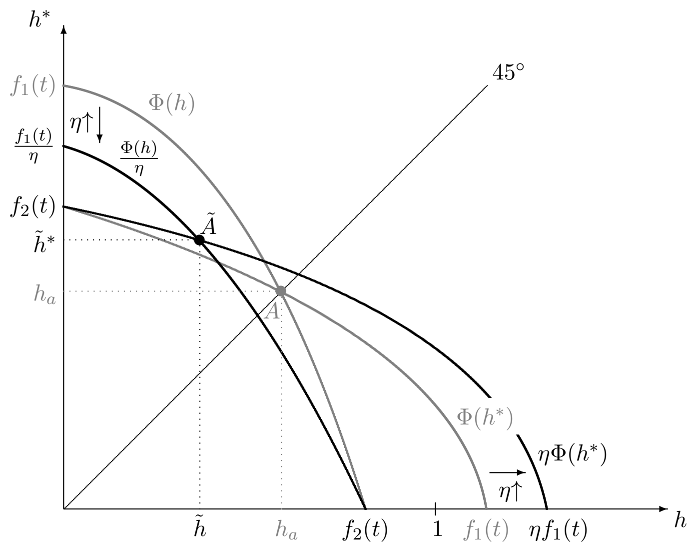

Figure 2: *Equilibrium in the open economy with asymmetric countries*

For a better understanding of how trade affects the allocation of labor, we can determine the effects of marginal changes in trade cost parameter t on h and h^*. These effects are illustrated in Figure 3. Starting point is the open economy equilibrium for asymmetric countries depicted by point $\tilde{A}$. Due to our assumption that foreign is larger than home, this equilibrium corresponds to a production pattern with $\tilde{h}^* > \tilde{h}$. The autarky equilibrium is depicted by point A and leads to a symmetric outcome in the two economies regarding the fraction of workers seeking employment in the sector of differentiated goods: $h = h^* = h_a$. An increase in the trade cost parameter from t to t' rotates locus $\frac{1}{\eta}\Phi(h)$ clockwise in point

C. To understand this effect, it is worth noting that a clockwise rotation of $\frac{1}{\eta}\Phi(h)$ captures that higher trade costs make the home market more relevant for firms and guard domestic producers in their home market from competition with foreign ones. As a consequence, for higher levels of t an increase in foreign production (reflected by an increase in h^*) induces a smaller production decrease at home (reflected by a less pronounced decline in h) to restore market clearing there. This makes locus $\frac{1}{\eta}\Phi(h)$ steeper. Locus $\frac{1}{\eta}\Phi(h)$ rotates in point C, because in this point the fraction of workers seeking employment in the sector of differentiated goods at home is at its autarky level: $h = h_a$. This establishes $\Gamma(h) = 0$, and we can conclude from Eq. (25) that in this case changes in t do not affect h^* for a given level of h. Using the same reasoning, it follows that $\eta\Phi(h^*)$ rotates counter-clockwise in point C^*, implying that higher trade costs bring the fraction of workers seeking employment in the sector of differentiated goods closer to the autarky levels of the two economies. To put it differently, higher trade costs lower the scope for specialization in the open economy.

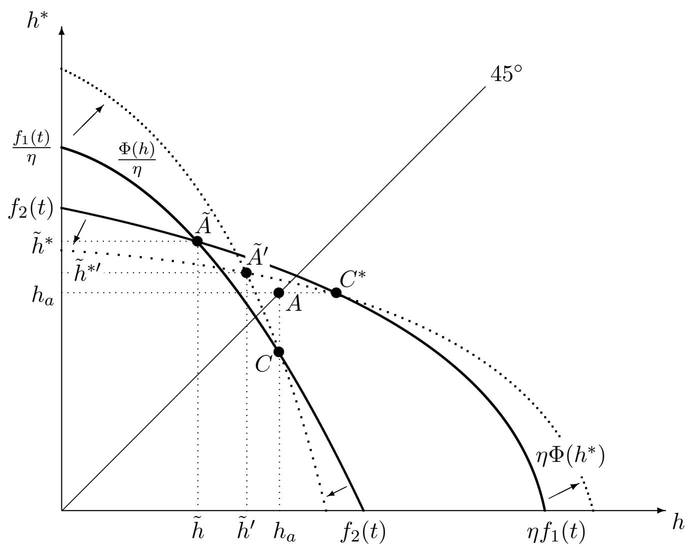

Figure 3: *Increase in trade cost parameter from* t *to* t'

With the solution for h and h^* at hand, we can make use of the outcome of wage bargaining in (23) and the zero-profit conditions $\rho r = \rho r^* = \sigma P_Y f$ to solve for the equilibrium masses of domestic and foreign producers of differentiated goods, M and M^*, respectively. As pointed out above, $h = h^*$ holds under autarky, irrespective of prevailing size differences of the two economies. Whereas the fraction of workers seeking employment in the sector of differentiated goods is the same, the two countries differ in the mass of firms producing differentiated goods in the closed economy. Since the market for differentiated goods is larger in foreign than in home if $\eta > 1$, we have $M^* > M$ in this case. Since wage bargaining (plus constant markup pricing) establishes for either country a positive link between the fraction of workers seeking employment in the sector of differentiated goods and the mass of firms producing them, we can conclude from the graphical analysis in Figure 2 that trade leads to firm entry in the larger country and to firm exit in the smaller one, thereby augmenting pre-existing differences in the mass of local firms producing differentiated goods. From Figure 3, we can further conclude that higher trade costs bring the masses of firms closer to their respective autarky levels, reducing the differences in the local mass of firms producing differentiated goods. This completes the characterization of the open economy equilibrium.

4.2 Trade pattern, unemployment and welfare

With the mass of firms determined in the previous section, we can now make use of the zero-profit conditions and compute home's total exports and imports of differentiated goods according to

$$EX_X = M\frac{1}{1+t}\frac{\sigma P_Y f}{\rho}, \qquad IM_X = M^*\frac{1}{1+t}\frac{\sigma P_Y f}{\rho}, \tag{28}$$

respectively. This implies that home is a net-importer of differentiated goods, $EX_X < IM_X$, if the foreign to domestic firm ratio $\mu \equiv M^*/M$ is larger than one. This is the case, if foreign is the larger economy, $\eta > 1$, and therefore offers the larger home market for differentiated goods. The opposite is true if home is the larger economy. In this case, $\eta < 1$ establishes $\mu < 1$ and thus $EX_X > IM_X$. This trade structure is well in line with other models featuring a home-market effect (see Helpman and Krugman, 1985). Assuming that households in the case of indifference purchase the domestic product, we have $IM_Y = 0$ and $EX_Y = IM_X - EX_X$ if $\eta > 1$ and therefore $\sum_j (EX_j + IM_j) = 2IM_X$, where $j \in \{X, Y\}$ is an industry index. In contrast, $\eta < 1$ gives $EX_Y = 0$ and $IM_Y = EX_X - IM_X$ and thus $\sum_j (EX_j + IM_j) = 2EX_X$. Also, higher trade costs lower the mass of firms that are active in the larger economy, thereby reducing the volume of trade.

The trade structure in our model is directly linked to the employment effects of trade. From the analysis in the closed economy, we know that only a fraction $1 - u$ of workers seeking employment in the sector of differentiated goods is successfully matched with a firm. Since $1 - u$ is pinned down by the condition that under diversification workers must be indifferent between employment in the production of the homogeneous good or employment in the production of differentiated goods and since this indifference condition is given by Eq. (12) and thus the same in the closed and the open economy, the economy-wide rate of unemployment, $U \equiv hu$, can be affected by trade only through adjustments in the fraction of workers seeking employment in the sector producing differentiated goods, h. This establishes the following proposition.

Proposition 1 *In the open economy, the larger country is net-exporter of differentiated goods and suffers from a higher rate of unemployment. An increase in trade costs lowers the export of differentiated goods in the larger and the import of differentiated goods in the smaller economy. The economy-wide rate of unemployment decreases in the larger and increases in the smaller economy.*

Proof The proposition follows from the analysis above.

The link between trade structure and unemployment established in Proposition 1 is a direct consequence of associating employment in the sector of differentiated goods with a higher risk of unemployment. This property of our model is akin to the distinction put forward by Acemoglu (2001) between good jobs offering high wages at the cost of a longer duration of unemployment to wait for the respective offer and bad jobs associated with low wages and a shorter duration of unemployment. The link between unemployment and wages is also well in line with the observation from the US that manufacturing, while offering higher hourly earnings than the average workplace according to data from Bureau of Labor Statistics, is prone to longer durations of unemployment (see Chien and Morris, 2016).

Since the large country is net-exporter of differentiated goods, it experiences an increase in the rate of unemployment in the open economy. However, this does not mean that trade is to the detriment of the larger economy. To see this, we can determine the welfare effects of trade. As pointed out in the analysis of the closed economy, the representative consumer in the case of PIGL preferences does not have a normative interpretation in general, implying that the choice of a proper welfare function is a priori not clear. This is different if preferences have Gorman form, and we therefore focus on the two limiting cases of homothetic and quasilinear preferences for now, while discussing the case of $\varepsilon \in (0,1)$ in Section 5.

If households have Gorman form preferences, we can combine Eqs. (23) and (24) to compute an explicit solution for the ratio of foreign to domestic firms μ as a function of the relative foreign population size η and trade cost parameter t. This gives for homothetic and quasilinear preferences

$$\mu = \frac{\eta\delta(t) - 1}{\delta(t) - \eta}, \qquad \mu = \frac{\eta\hat{\delta}(t) - 1}{\hat{\delta}(t) - \eta}, \tag{29}$$

respectively, with

$$\delta(t) \equiv t - \frac{\sigma - 1}{\sigma}\frac{\rho(1+t)}{1-u}\left[(1-u)\tilde{\alpha} - 1\right](1-\beta), \quad \hat{\delta}(t) \equiv t - \frac{\sigma - 1}{\sigma}\frac{\rho(1+t)}{1-u}\left[(1-u)\tilde{\alpha} - 1\right]. \tag{30}$$

Furthermore, using the definition of μ in Eq. (24) and accounting for the markup pricing rule in Eq. (23) and zero-profit condition $\rho r = \sigma P_Y f$, we can determine the fraction of workers seeking employment in the sector of differentiated goods and the mass of firms producing them in home. For the case of homothetic preferences, we compute

$$h = \frac{\frac{1-\beta}{1-u}}{\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t} - \frac{1-\beta}{1-u}[(1-u)\tilde{\alpha} - 1]}, \qquad M = \frac{\frac{(1-\beta)H\lambda}{(\sigma-1)f}}{\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t} - \frac{1-\beta}{1-u}[(1-u)\tilde{\alpha} - 1]}, \tag{31}$$

whereas for the case of quasilinear preferences, we obtain

$$h = \frac{\frac{\lambda-\beta}{\lambda(1-u)}}{\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t} - \frac{1}{1-u}[(1-u)\tilde{\alpha} - 1]}, \qquad M = \frac{\frac{(\lambda-\beta)H}{(\sigma-1)f}}{\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t} - \frac{1}{1-u}[(1-u)\tilde{\alpha} - 1]}. \tag{32}$$

With Eqs. (31) and (32) at hand, we can formulate the following proposition, using Eq. (22).

Proposition 2 *Let us assume that preferences have Gorman form and let us consider an open economy equilibrium with diversified production in both economies. Then, a decline in the trade cost parameter increases welfare in the larger economy, while it can increase or decrease welfare in the smaller economy* *if* $(1-u)\tilde{\alpha} > 1$. *Things are different if* $(1-u)\tilde{\alpha} < 1$. *In this case, a decline in the trade cost parameter increases welfare in the smaller economy, whereas it can increase or decrease welfare in the larger economy.*

Proof See the Appendix.

To provide an intuition for the welfare effects described in Proposition 2, we can distinguish three channels through which a decline in trade costs impacts welfare in our model. The first one is a fall in the price of differentiated goods imported from the foreign economy. This effect is captured by an increase in $(1+t)/t$ in price index $P_X = \frac{\sigma}{\sigma-1}\frac{w_Y}{\rho}\left(M\frac{\mu+t}{1+t}\frac{1+t}{t}\right)^{\frac{1}{1-\sigma}}$, and it also exists if countries are symmetric and hence in cases in which $\eta = 1$ and the fraction of workers seeking employment in the sector of differentiated goods as well as the mass of firms producing them remain at their autarky levels. If countries differ in their population size, there are two additional effects. The first one is a disposable income effect, which materializes through changes in $\overline{e} = w_Y\lambda\{1 + h[(1-u)\tilde{\alpha} - 1]\}$ and can be positive or negative. It is positive for the larger country net-exporting differentiated goods if $(1-u)\tilde{\alpha} > 1$, because in this case the wage premium received by workers newly employed by firms producing differentiated goods dominates the income loss of the newly unemployed. The opposite is true if $(1-u)\tilde{\alpha} < 1$. Disposable income effects in the two countries go into opposite directions, because the fraction of workers seeking employment in the sector of differentiated goods increases in the larger and decreases in the smaller economy.

Finally, there exists a variety effect, because existing firms change the location of production (captured by changes in μ for a given total mass of producers, $M + M^*$) and because firms enter or exit the market (captured by changes in the total mass of producers, $M + M^*$, for a given μ). This variety effect materializes through changes in price index $P_X = \frac{\sigma}{\sigma-1}\frac{w_Y}{\rho}\left(M\frac{\mu+t}{1+t}\frac{1+t}{t}\right)^{\frac{1}{1-\sigma}}$ due to changes in the composite term $M\frac{\mu+t}{1+t}$ and it can be positive or negative. In the larger country, which net-exports differentiated goods, the mass of domestic producers increases. However, the mass of foreign firms decreases and the former dominates the latter only if trade increases average disposable household income, i.e. if $(1-u)\tilde{\alpha} > 1$. In this case, the larger country net-exporting differentiated goods unambiguously benefits from a fall in the trade cost parameter. Things are different if $(1-u)\tilde{\alpha} < 1$. In this case, a negative disposable income effect and a negative variety effect counteract the positive effect of cheaper access to foreign imports, and we show in the Appendix that they can dominate if σ is sufficiently large, because for high levels of σ both the positive price effect for imported goods as well as the negative variety effect are relatively small compared to the negative income effect.

While Proposition 2 is valid for both types of Gorman form preferences, there is a difference regarding the expected trade effects for homothetic and quasilinear preferences. As pointed out by Lemma 1, quasilinear preferences establish $(1-u)\tilde{\alpha} < 1$ for all possible $\alpha > 1-\gamma$. This is because in the limiting case of $\varepsilon = 1$ households are risk-neutral and hence they find it attractive to seek employment in the sector of differentiated goods and accept a lower probability of finding a job whenever this causes an increase in their expected income. This leads to a relatively low employment rate in the sector of differentiated goods, implying that the impact of trade on economy-wide unemployment is fairly strong.

As a consequence, average disposable labor income falls in the country expanding production of differentiated goods, so that the larger country is at risk of double losses from trade due to an increase in the economy-wide unemployment and a decrease in the representative consumer's welfare level if preferences are quasilinear. Things are different in the case of homothetic (log-transformed Cobb-Douglas) preferences, because households are risk-averse and thus expect a compensation for the possibility of ending up in an unfavorable state of unemployment when applying for jobs in the sector of differentiated goods. For a given wage premium offered by firms producing differentiated goods, this results in a higher employment rate $1-u$, and thus in a moderate increase in unemployment when exporting in the open economy increases the fraction of workers seeking employment in the sector of differentiated goods, h. As put forward by Lemma 1, $(1-u)\tilde{\alpha} > 1$ is guaranteed for all $\alpha > 1-\gamma$ if $\gamma < \exp[-1]$. This implies that if preferences are homothetic and unemployment compensation is not too generous, trade is to the benefit of the larger economy, but may be detrimental for the smaller country.[12] Double losses from trade are not possible in this case.

5 Extensions

To complete the analysis in this paper, we discuss two extensions of our model. In the first one, we consider the case of $\varepsilon \in (0,1)$ and analyze to what extent the insight from the two limiting cases of homothetic and quasilinear preferences are informative about the trade effects if preferences do not have Gorman form. In the second extension, we consider differences of countries in the per-capita labor endowment of households and study whether rich or poor countries are more likely to benefit from trade liberalization.

5.1 Trade effects if preferences do not have Gorman form

As pointed out above, the representative consumer in our model does not have a normative interpretation if $\varepsilon \in (0,1)$. This makes the choice of a social welfare function somewhat arbitrary. Egger and Habermeyer (2019) suggest to take a utilitarian perspective and to use average household utility as a social welfare function. This establishes

$$V(\overline{e}, P_Y, P_X, \hat{\psi}) \equiv \frac{1}{\varepsilon}\left(\frac{P_Y}{P_X}\right)^{\varepsilon}\left[\left(\frac{\overline{e}}{P_Y}\right)^{\varepsilon}\hat{\psi} - \beta\right] - \frac{1-\beta}{\varepsilon}, \tag{33}$$

where $\hat{\psi} \equiv H^{-1}\int_{i\in\mathcal{H}} (e_i/\overline{e})^{\varepsilon}\, di$ is a dispersion index, which is equal to ψ only if $\varepsilon = 1/2$. Eq. (33) is a natural candidate for our welfare analysis and it converges to $V_{CD}(\overline{e}, P_Y, P_X)$ and $V_{QL}(\overline{e}, P_Y, P_X)$ in the limiting cases of $\varepsilon = 0$ and $\varepsilon = 1$, respectively. As extensively discussed in Egger and Habermeyer (2019), the welfare function in Eq. (33) features social inequality aversion (through $\hat{\psi} < 1$), which,

[12] In many applications to international trade, economists set unemployment compensation equal to 0 (see, e.g., Helpman et al., 2010; Egger and Kreickemeier, 2012). Due to the risk aversion of households with homothetic preferences in our model, the employment probability in the sector of differentiated goods increases to one in the limiting case $\gamma \to 0$, so that trade would not affect economy-wide unemployment and would therefore increase average disposable household income unambiguously.

however, is not the consequence of a prioritarian social planner but is rooted in the risk aversion of households imposed by the preferences in Eq. (1). Thus, the welfare function in Eq. (33) would associate a market outcome with the same level but a higher dispersion of disposable household income with a lower level of welfare, providing scope for achieving a welfare gain through redistribution of income from richer to poorer households.

In comparison to the limiting cases of homothetic and quasilinear preferences studied in the previous section, the assumption of non-Gorman form preferences opens an additional channel through which trade affects welfare in the open economy, namely through changes in the dispersion of disposable household income. Thereby, changes in the dispersion of disposable household income influence welfare through a direct and an indirect effect. The direct effect works through the social income inequality aversion and implies that welfare decreases if trade lowers $\hat{\psi}$. The indirect effect works through changes in firm entry. Because the Engel curves for luxuries are convex, while the Engel curve for the necessity is concave, an increase in the dispersion of disposable household income increases consumer demand for differentiated goods and therefore leads to additional firm entry through a decline in ψ. This firm entry lowers price index P_X relative to price P_Y with positive welfare implications, according to Eq. (33). To keep things simple, we look at the case of $\varepsilon = 1/2$, implying that the two dispersion measures are equal: $\psi = \hat{\psi}$. In this case, we have $(1-u)\sqrt{\tilde{\alpha}} + u\sqrt{\gamma} = 1$ from Eq. (12) and thus

$$\sqrt{\frac{\bar{e}}{P_Y}}\psi = \sqrt{\lambda}T(h) = \sqrt{\lambda\frac{1+h[(1-u)\tilde{\alpha}-1]}{1+h[(1-u)\tilde{\alpha}+u\gamma-1]}}, \tag{34}$$

with $T'(h) < 0$. Furthermore, the constraint that even unemployed households consume the differentiated good, $(1-\tau)\lambda\gamma > \beta^2$, establishes $\sqrt{\bar{e}/P_Y}\psi = \sqrt{\lambda}T(h) > \beta$. Combining the market clearing condition in Eq. (24) with the zero-profit condition $\rho r = \sigma P_Y f$, we further compute

$$M\frac{\mu+t}{1+t} = \frac{H\lambda\rho}{\sigma f}\left\{1 + h\left[(1-u)\tilde{\alpha}-1\right] - \beta\left(\sqrt{\lambda}\right)^{-1}T(h)\right\}. \tag{35}$$

Substituting into the price index for differentiated goods, we then obtain the welfare function

$$V(\cdot) = 2\left(\sqrt{\frac{\sigma}{\sigma-1}\frac{1}{\rho}}\right)^{-1}\left(\frac{H\lambda\rho}{\sigma f}\right)^{\frac{1}{2(\sigma-1)}}\hat{V}(h) - 2(1-\beta), \tag{36}$$

with

$$\hat{V}(h) \equiv \left\{\left[1 + h[(1-u)\tilde{\alpha}-1] - \beta\left(\sqrt{\lambda}\right)^{-1}T(h)\right]\frac{1+t}{t}\right\}^{\frac{1}{2(\sigma-1)}}\left[\sqrt{\lambda}T(h) - \beta\right]. \tag{37}$$

Noting from Figure 3 that $dh/dt < 0$ if home is a net-exporter of differentiated goods, we can conclude that $(1-u)\tilde{\alpha} > 1$ is no longer sufficient for gains from trade in the larger economy. If σ is sufficiently large, the detrimental impact of trade on the level and dispersion of disposable household income (captured by a lower $\sqrt{\lambda}T(h) - \beta$) may dominate the gains from a lower import price and a positive variety effect. This strengthens our insights from the main text that the specific form of preferences plays a crucial role for the welfare effects of trade in our model.

5.2 Trade effects in the case of rich and poor countries

We now consider trade between two countries that differ in the labor endowments of households but feature the same total effective labor supply, $H\lambda = H^*\lambda^*$. Households with a larger labor endowment receive higher disposable income and their country is thus associated with the richer economy. With differences in the households' labor endowments, the outcome of wage bargaining (plus constant markup pricing) and the market clearing conditions for differentiated goods change to

$$hH\lambda w_Y(1-u) = \frac{\sigma-1}{\sigma}\rho M r, \qquad h^*H^*\lambda^* w_Y(1-u) = \frac{\sigma-1}{\sigma}\rho M^* r^* \tag{23$'$}$$

and

$$\begin{aligned} H\lambda w_Y(1-\beta\lambda^{-\varepsilon}) + H\lambda w_Y B(h) &= \frac{Mrt}{1+t} + \frac{M^*r^*}{1+t}, \\ H^*\lambda^* w_Y(1-\beta(\lambda^*)^{-\varepsilon}) + H^*\lambda^* w_Y B^*(h^*) &= \frac{M^*r^*t}{1+t} + \frac{Mr}{1+t}, \end{aligned} \tag{24$'$}$$

respectively, where $B^*(h^*)$ is defined in analogy to $B(h)$ with λ^* replacing λ. Combining Eqs. (23′) and (24′), we compute

$$h^* = \Phi(h), \qquad h = \Phi^*(h^*), \tag{25$'$}$$

with $\Phi(h)$ given by Eq. (26), $\Phi^*(h^*) \equiv h^* + \frac{\sigma-1}{\sigma}\frac{\rho(1+t)}{1-u}\Gamma^*(h^*)$, and $\Gamma^*(h^*)$ defined in analogy to $\Gamma(h)$, with λ^* replacing λ.

System (25′) gives two equations in two unknowns, which can be combined to solve for the equilibrium values of h and h^* in the open economy. For this purpose, we make use of Figure 4, where the open economy equilibrium for the case of two symmetric countries is given by point A (similar to Figure 1). A richer labor endowment of households in the foreign country ($\lambda^* > \lambda$) increases the home market for differentiated goods there, provided that higher average disposable household income increases demand for differentiated goods, which is the case if $\varepsilon > 0$. Then, the fraction of workers producing differentiated goods is already under autarky higher in foreign than at home, which can be seen from contrasting h_a^* in point A' with h_a in point A. In the open economy equilibrium (point $\tilde{A}$), the difference between h and h^* is further increased, because foreign specializes on the production of differentiated goods in line with the idea of a home-market effect put forward by Helpman and Krugman (1985).[13]

With the equilibrium labor allocation at hand, we can derive the mass of firms producing differentiated goods from the outcome of wage bargaining in Eq. (23′) and the zero-profit conditions $\rho r = \rho r^* = \sigma P_Y f$. Provided that $\varepsilon > 0$, the richer country hosts a larger mass of firms producing differentiated

[13]The equilibrium is derived for the case of diversification of production in both economies. With a reasoning similar to the one in the main text, one can show that such an outcome is guaranteed for sufficiently high trade costs.

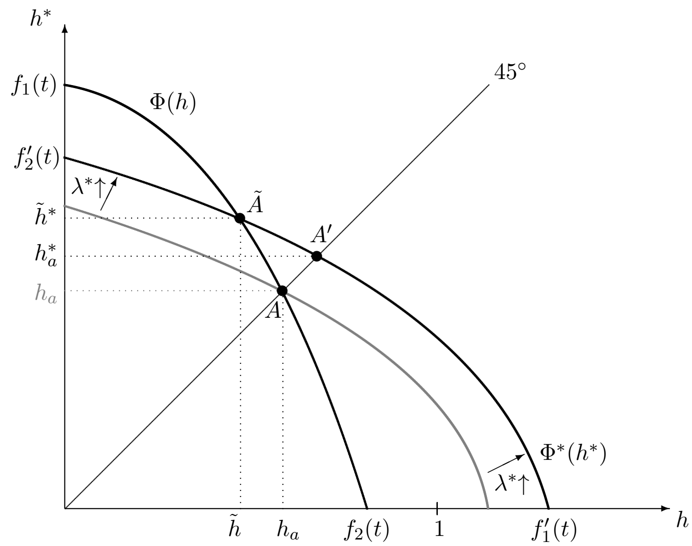

Figure 4: *Open economy equilibrium if foreign is richer than home ($\lambda^* > \lambda$)*

goods, and hence becomes net-exporter of these goods in the open economy. Similar to the baseline scenario with country asymmetries rooted in different population sizes, net-exporting differentiated goods comes at the cost of a higher economy-wide unemployment rate. To determine the welfare effects of trade, we proceed as in the main text and focus on the two limiting cases representing Gorman form preferences. From Eq. (31), we see that for symmetry of the two countries in aggregate labor supply $H\lambda = H^*\lambda^*$, h and M are the same in the two economies and do not differ from their autarky levels (due to $\mu = 1$) if preferences are homothetic ($\varepsilon = 0$). In this case, trade leaves unemployment unaffected and increases welfare in both economies, according to Eq. (22). With quasilinear preferences ($\varepsilon = 1$), differences in the households' labor endowments generate differences of the two economies in their demand for differentiated goods. This establishes $h^* > h$ and $M^* > M$ if $\lambda^* > \lambda$, implying that the richer country net-exporting differentiated goods not only suffers from an increase in the economy-wide rate of unemployment but may also experience welfare losses from trade if σ is sufficiently large (see the Appendix).

6 Conclusion

We have developed a two-country model of trade with differentiated and homogeneous goods using labor as the only production input. The model features a home-market effect due to trade costs of differentiated goods. Whereas the labor market in the homogeneous goods sector is perfectly competitive, there are search frictions and firm-level wage bargaining in the sector of differentiated goods. This generates involuntary unemployment, whose extent at the economy-wide level is linked to the fraction of workers seeking employment in the sector of differentiated goods. The exact form of this link depends on consumer preferences, which are assumed to be from the PIGL class and cover homothetic and quasilinear preferences as two limiting cases.

In the open economy, the larger of the two countries specializes on the production of differentiated goods and net-exports these goods. Since seeking employment in the sector of differentiated goods is prone to the risk of unemployment, trade increases the economy-wide rate of unemployment in the larger economy. In the case of quasilinear preferences, trade lowers average disposable household income and exerts a negative variety effect in the larger country, so that social welfare can be reduced there, although the prices of imported goods are reduced. Things are different in the smaller country, which benefits from trade. If preferences are homothetic, trade induces an increase of average disposable household income and generates a positive variety effect in the larger economy, provided that unemployment compensation is not too generous. This adds to the gains from lower import prices, implying that the larger country benefits from trade, despite an increase in the economy-wide rate of unemployment. At the same time, the smaller country can lose from trade, because the negative income and variety effects work against the gains from lower import prices.

In an extension of our analysis, we study non-Gorman preferences and show that in this case changes in the dispersion of income exert an additional impact on welfare, which is missing under homothetic and quasilinear preferences. The impact of changes in the dispersion of income is twofold. On the one hand, a higher income dispersion increases demand for differentiated goods, which are luxuries in our model. This implies that higher income dispersion leads to firm entry and therefore induces indirect welfare gains due to a love-of-variety effect. On the other hand, from a utilitarian perspective welfare exhibits social inequality aversion, so that higher income dispersion reduces welfare through a direct effect. In the open economy, the assumption of non-Gorman preferences implies that an increase in the level of income is no longer sufficient for welfare gains from trade. In a second extension, we consider differences of the two countries in their per-capita labor endowments and show that such differences may lead to welfare loss in the richer economy if preferences are quasilinear. In contrast, welfare gains are guaranteed for both countries if preferences are homothetic, because with homothetic utility per-capita income levels do not matter for aggregate consumer demand, implying that trade does not change the production structure in the open economy.

To improve the exposition of our analysis, we have imposed several simplifying assumptions, which are not crucial for our results. For instance, allowing for differentiated goods in only one sector and associating output of the other sector with a homogeneous good is useful for the analysis of asymmetric countries. However, as long as the wage premium as well as the risk of unemployment are larger in the sector associated with the production of luxuries and as long as the elasticity of substitution between necessities is sufficiently high, the main mechanisms of our model remain valid in a modified setting, in which the differences of the two sectors are less pronounced. Also, allowing for heterogeneous firms in the production of differentiated goods would not alter our results in a qualitative way. Whereas extensions in these directions are straightforward, we leave a detailed analysis of them to the interested reader.

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A Theoretical appendix

A.1 Microfoundation for the search and matching model

Starting point is the static search and matching model proposed by Helpman and Itskhoki (2010),[14] where the number of matches of workers with firms, L, is determined as a Cobb-Douglas function of the mass of vacancies generated by firms, Q, and the mass of workers seeking employment in the sector of differentiated goods, hH (see Pissarides, 2000, for an extensive discussion of the Cobb-Douglas matching function):

$$L = \hat{m} Q^{\chi} (hH)^{1-\chi}, \quad 0 < \chi < 1. \tag{A.1}$$

Thereby, parameter $\hat{m}$ is a positive constant that measures the efficiency of the matching process. Establishing a vacancy comes at the cost of one unit of the homogeneous good. Assuming that not all vacancies can be successfully filled, hiring costs per worker can be expressed by $q^{-1}w_Y$, where $q \equiv L/Q < 1$ is the probability to fill a vacancy. Denoting the probability of finding a job by $1 - u < 1$ the number of successful matches can be expressed as $L = hH(1-u)$. Substituting into Eq. (A.1), we can write

$$\frac{Q}{hH} = m^{-1}(1-u)^{\frac{1}{\chi}}, \tag{A.2}$$

where $m \equiv \hat{m}^{\frac{1}{\chi}}$. In the main text, we consider the limiting case of $\chi \to 1$ and $m = \lambda^{-1}$, which then establishes Eq. (12) as the indifference condition of workers. To see that looking at the limiting case does

[14] Helpman and Itskhoki (2010) also discuss an extension of their model to a dynamic setting, and we therefore refer readers interested in such dynamic effects to their paper.

not change the main insights from our analysis, we can determine employment rate $1-u$ for the more general case of $m<1$ (needed for $q<1$) and $\chi<1$. In this case, the employment rate $1-u$ is implicitly determined by

$$1-u=\frac{1-\gamma^{\varepsilon}}{\left[\frac{\alpha}{m\lambda}(1-u)^{1/\chi-1}+\gamma\right]^{\varepsilon}-\gamma^{\varepsilon}}, \tag{A.3}$$

which delivers $d(1-u)/d\alpha<0$ and $d(1-u)/d\gamma<0$ as in the baseline specification. Furthermore, the insight from the main text regarding the ranking of $(1-u)\tilde{\alpha}>,=,<1$ also extends to the more general case. This completes our discussion of the matching technology.

A.2 Proof of Lemma 1

Multiplying Eq. (12) by $\tilde{\alpha}$ gives $(1-u)\tilde{\alpha}=\tilde{\alpha}(1-\gamma^{\varepsilon})/(\tilde{\alpha}^{\varepsilon}-\gamma^{\varepsilon})$ and thus $(1-u)\tilde{\alpha}-1=\tilde{\alpha}[(1-\gamma^{\varepsilon})/(\tilde{\alpha}^{\varepsilon}-\gamma^{\varepsilon})]-1\equiv\Psi(\tilde{\alpha})$. We compute $\Psi(1)=0$, $\lim_{\tilde{\alpha}\to\infty}\Psi(\tilde{\alpha})=\infty$, and

$$\Psi'(\tilde{\alpha})=\frac{\Psi(\tilde{\alpha})+1}{\tilde{\alpha}}\left[1-\frac{\varepsilon\tilde{\alpha}^{\varepsilon}}{\tilde{\alpha}^{\varepsilon}-\gamma^{\varepsilon}}\right], \qquad \Psi''(\tilde{\alpha})=-\frac{\varepsilon\tilde{\alpha}^{\varepsilon}}{\tilde{\alpha}(\tilde{\alpha}^{\varepsilon}-\gamma^{\varepsilon})}\Psi'(\tilde{\alpha})+\frac{\varepsilon^2\tilde{\alpha}^{\varepsilon}\gamma^{\varepsilon}[\Psi(\tilde{\alpha})+1]}{\tilde{\alpha}^2(\tilde{\alpha}^{\varepsilon}-\gamma^{\varepsilon})^2} \tag{A.4}$$

From the derivatives of $\Psi(\tilde{\alpha})$, we can safely conclude that if $\Psi(\tilde{\alpha})$ has an extremum at $\tilde{\alpha}>1$, this extremum must be unique and a minimum, implying that $\Psi(\tilde{\alpha})>0$ holds for sufficiently high levels of α (with $\alpha=\tilde{\alpha}-\gamma$). Furthermore $\Psi'(1)\geq 0$ follows if $\gamma\leq(1-\varepsilon)^{\frac{1}{\varepsilon}}\equiv\underline{\gamma}(\varepsilon)$ and, in this case, $\Psi'(\tilde{\alpha})>0$ and thus $\Psi(\tilde{\alpha})>0$ holds for all $\tilde{\alpha}>1$ or, equivalently, for all $\alpha>1-\gamma$. Accounting for $\underline{\gamma}'(\varepsilon)<0$, $\lim_{\varepsilon\to 0}\underline{\gamma}(\varepsilon)=\exp[-1]$, and $\lim_{\varepsilon\to 1}\underline{\gamma}(\varepsilon)=0$ then establishes Lemma 1.

A.3 Derivations details for $B(h)$ and Eqs. (17) and (18)

From Eq. (5) it follows that total expenditures for differentiated goods are equal to

$$\int_{\omega\in\Omega}p(\omega)x(\omega)d\omega=H\overline{e}\left[1-\beta\left(\frac{\overline{e}}{P_Y}\right)^{-\varepsilon}\psi\right]. \tag{A.5}$$

Substituting Eq. (13) for $\overline{e}$ and Eq. (15) for ψ, we can express economy-wide demand for differentiated goods as

$$\begin{aligned}\int_{\omega\in\Omega}p(\omega)x(\omega)d\omega &= Hw_Y\lambda\left\{1+h[(1-u)\tilde{\alpha}-1]\right\}-\beta Hw_Y\lambda^{1-\varepsilon}T(h)\\ &= Hw_Y\lambda(1-\beta\lambda^{-\varepsilon})+Hw_Y\lambda B(h),\end{aligned} \tag{A.6}$$

where the first equality sign uses the definition of $T(h)$ in Eq. (18), while the second equality sign uses the definition of $B(h)$ in the main text. Setting $\int_{\omega\in\Omega}p(\omega)x(\omega)d\omega=Mr$ finally establishes the market clearing condition in Eq. (17). This completes the proof.

A.4 Determination of h and M in the closed economy

In the main text, we argue that $\Gamma(h) = 0$ has a unique solution on the unit interval. To see this, we can make use of the definition of $\Gamma(h)$ in Eq. (19) and first note that $\Gamma(0) = 1 - \beta\lambda^{-\varepsilon} > 0$ and that $\Gamma(1) = -\big(\frac{\sigma}{\sigma-1} - \gamma\big)(1-u) - \beta\lambda^{-\varepsilon}T(1) < 0$. Making use of the Intermediate Value Theorem, we can thus safely conclude that $\Gamma(h) = 0$ has a solution in $h \in (0,1)$. As put forward in the main text, in the two limiting cases of $\varepsilon = 0$ and $\varepsilon = 1$, we have $T(h) = 1 + h\left[(1-u)\tilde{\alpha} - 1\right]$ and $T(h) = 1$, implying that $\Gamma(h) = 0$ has an explicit and unique solution in $h \in (0,1)$. Things are less obvious if $\varepsilon \in (0,1)$. Twice differentiating $\Gamma(h)$, we obtain

$$\Gamma'(h) = -\left[1 + \left(\frac{\sigma}{\sigma-1} - \gamma\right)(1-u)\right] - \beta\lambda^{-\varepsilon}T(h)\frac{(1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon} - 1}{1 + h\big[(1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon} - 1\big]}$$
$$+ \beta\lambda^{-\varepsilon}T(h)\frac{(1-\varepsilon)u\gamma}{\big\{1 + h\big[(1-u)\tilde{\alpha} - 1\big]\big\}\big\{1 + h\big[(1-u)\tilde{\alpha} + u\gamma - 1\big]\big\}}$$

and

$$\Gamma''(h) = \beta\lambda^{-\varepsilon}T(h)\frac{(1-\varepsilon)u\gamma}{\big\{1 + h\big[(1-u)\tilde{\alpha} - 1\big]\big\}\big\{1 + h\big[(1-u)\tilde{\alpha} + u\gamma - 1\big]\big\}}$$
$$\times \left[\frac{2\big[(1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon} - 1\big]}{1 + h\big[(1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon} - 1\big]} - \frac{(2-\varepsilon)u\gamma + 2[(1-u)\tilde{\alpha} - 1]\big\{1 + h[(1-u)\tilde{\alpha} + u\gamma - 1]\big\}}{\big\{1 + h\big[(1-u)\tilde{\alpha} - 1\big]\big\}\big\{1 + h\big[(1-u)\tilde{\alpha} + u\gamma - 1\big]\big\}}\right].$$

We next show that $\Gamma'(0) < 0$ and $\Gamma'(1) < 0$. For this purpose, we can first note that $\Gamma'(0) = -1 - \big(\frac{\sigma}{\sigma-1} - \gamma\big)(1-u) - \beta\lambda^{-\varepsilon}\big[(1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon} - 1 - (1-\varepsilon)u\gamma\big]$ and thus $\Gamma'(0) < -\big(1 - \beta\lambda^{-\varepsilon}\big) - \beta\lambda^{-\varepsilon}\big[(1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon} - (1-\varepsilon)u\gamma\big]$. Positive expenditures of differentiated goods require $\gamma\lambda(1-\tau) > \beta^{1/\varepsilon}$. Noting that $\tau = 0$ if $h = 0$, we have $\beta < (\gamma\lambda)^{\varepsilon}$ and thus $1 - \beta\lambda^{-\varepsilon} > 1 - \gamma^{\varepsilon} > 0$. This implies that $(1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon} - (1-\varepsilon)u\gamma > 0$ is sufficient for $\Gamma'(0) < 0$. Second, we can note that

$$\Gamma'(1) = -\left[1 + \left(\frac{\sigma}{\sigma-1} - \gamma\right)(1-u)\right] - \beta\lambda^{-\varepsilon}\left(\frac{(1-u)\tilde{\alpha}}{(1-u)\tilde{\alpha} + u\gamma}\right)^{1-\varepsilon} Z(\tilde{\alpha}), \tag{A.7}$$

with

$$Z(\tilde{\alpha}) \equiv (1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon} - 1 - \frac{(1-\varepsilon)u\gamma\left[(1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon}\right]}{(1-u)\tilde{\alpha}\left[(1-u)\tilde{\alpha} + u\gamma\right]}. \tag{A.8}$$

If $Z(\tilde{\alpha}) \geq 0$, then $\Gamma'(1) < 0$ is immediate. If $Z(\tilde{\alpha}) < 0$, we can note that $h = 1$ gives $\tau = u\gamma/[(1-u)\tilde{\alpha} + u\gamma]$ and that $\lambda\gamma(1-\tau) > \beta^{1/\varepsilon}$ establishes

$$\beta\lambda^{-\varepsilon}\left(\frac{(1-u)\tilde{\alpha}}{(1-u)\tilde{\alpha} + u\gamma}\right)^{1-\varepsilon} < \gamma^{\varepsilon}\frac{(1-u)\tilde{\alpha}}{(1-u)\tilde{\alpha} + u\gamma}$$

and thus

$$\beta\lambda^{-\varepsilon}\left(\frac{(1-u)\tilde{\alpha}}{(1-u)\tilde{\alpha}+u\gamma}\right)^{1-\varepsilon}Z(\tilde{\alpha})>\gamma^{\varepsilon}\left\{-1+(1-u)\tilde{\alpha}\frac{(1-u)\tilde{\alpha}^{1-\varepsilon}+u\gamma^{1-\varepsilon}}{(1-u)\tilde{\alpha}+u\gamma}\right.$$
$$\left.+\frac{u\gamma}{(1-u)\tilde{\alpha}+u\gamma}\left[1-(1-\varepsilon)\frac{(1-u)\tilde{\alpha}^{1-\varepsilon}+u\gamma^{1-\varepsilon}}{(1-u)\tilde{\alpha}+u\gamma}\right]\right\}.$$

Using Eq. (12), we can note that $\big[(1-u)\tilde{\alpha}^{1-\varepsilon}+u\gamma^{1-\varepsilon}\big]/\big[(1-u)\tilde{\alpha}+u\gamma\big]\ >,=,<\ 1$ if $f(\tilde{\alpha})\equiv(1-\gamma^{\varepsilon})\tilde{\alpha}^{1-\varepsilon}+(\tilde{\alpha}^{\varepsilon}-1)\gamma^{1-\varepsilon}-(1-\gamma^{\varepsilon})\tilde{\alpha}-(\tilde{\alpha}^{\varepsilon}-1)\gamma\ >,=,<\ 0$. Thereby, we have $f(1)=0$ and $f'(\tilde{\alpha})=-(1-\gamma^{\varepsilon})[1-(1-\varepsilon)\tilde{\alpha}^{-\varepsilon}]+\varepsilon\tilde{\alpha}^{\varepsilon-1}[\gamma^{1-\varepsilon}-\gamma]$, $f''(\tilde{\alpha})=-\varepsilon(1-\varepsilon)[\tilde{\alpha}^{-\varepsilon-1}(1-\gamma^{\varepsilon})+\tilde{\alpha}^{\varepsilon-2}(\gamma^{1-\varepsilon}-\gamma)]<0$. Hence, if $f(\tilde{\alpha})$ has an extremum, it must be a maximum. Noting further that $f'(1)=-\varepsilon\big(1-\gamma^{\varepsilon}-\gamma^{1-\varepsilon}+\gamma\big)<0$ holds for all permissible levels of γ,[15] it follows that $f(\tilde{\alpha})<0$ and thus $\big[(1-u)\tilde{\alpha}^{1-\varepsilon}+u\gamma^{1-\varepsilon}\big]/\big[(1-u)\tilde{\alpha}+u\gamma\big]<1$ hold for all $\alpha>1-\gamma$ (and thus $\tilde{\alpha}>1$). Putting together, we can therefore conclude that

$$\beta\lambda^{-\varepsilon}\left(\frac{(1-u)\tilde{\alpha}}{(1-u)\tilde{\alpha}+u\gamma}\right)^{1-\varepsilon}Z(\tilde{\alpha})>-\gamma^{\varepsilon}$$

and this is sufficient for $\Gamma'(1)<0$.

Let us now turn to the second derivative of $\Gamma(h)$, for which we can note that $\Gamma''(h)\ >,=,<\ 0$ is equivalent to $F(h)>,=,<0$, with

$$F(h)\equiv2\left[(1-u)\tilde{\alpha}^{1-\varepsilon}+u\gamma^{1-\varepsilon}-(1-u)\tilde{\alpha}\right]\{1+h\left[(1-u)\tilde{\alpha}+u\gamma-1\right]\}$$
$$-(2-\varepsilon)u\gamma\left\{1+h\left[(1-u)\tilde{\alpha}^{1-\varepsilon}+u\gamma^{1-\varepsilon}-1\right]\right\}. \tag{A.9}$$

Then, $F(h)<0$ and thus $\Gamma''(h)<0$ holds if $(1-u)\tilde{\alpha}^{1-\varepsilon}+u\gamma^{1-\varepsilon}-(1-u)\tilde{\alpha}\leq0$, and in this case $\Gamma'(0)<0$ is sufficient for $\Gamma'(h)<0$ to hold for all $h>0$. To see whether this can be the case, we can note that $(1-u)\tilde{\alpha}^{1-\varepsilon}+u\gamma^{1-\varepsilon}-(1-u)\tilde{\alpha}>,=,<0$ is equivalent to $\zeta(\tilde{\alpha})\equiv(1-\gamma^{\varepsilon})\tilde{\alpha}^{1-\varepsilon}+(\tilde{\alpha}^{\varepsilon}-1)\gamma^{1-\varepsilon}-(1-\gamma^{\varepsilon})\tilde{\alpha}>,=,<0$. Then, accounting for $\zeta(1)=0$, $\zeta'(\tilde{\alpha})=-(1-\gamma^{\varepsilon})\big[1-(1-\varepsilon)\tilde{\alpha}^{-\varepsilon}\big]+\varepsilon\tilde{\alpha}^{\varepsilon-1}\gamma^{1-\varepsilon}$, $\zeta''(\tilde{\alpha})=-\varepsilon(1-\varepsilon)\big[(1-\gamma^{\varepsilon})\tilde{\alpha}^{-\varepsilon-1}+\tilde{\alpha}^{\varepsilon-2}\gamma^{1-\varepsilon}\big]<0$, and $\lim_{\tilde{\alpha}\to\infty}\zeta(\tilde{\alpha})=-\infty$, we can conclude that if $\zeta(\tilde{\alpha})$ has an extremum, it must be a maximum with positive function value. Such a maximum can only exist if $\zeta'(1)>0$. We have $\zeta'(1)=-\varepsilon(1-\gamma^{\varepsilon}-\gamma^{1-\varepsilon})>,=,<0$ if $0>,=,<1-\gamma^{\varepsilon}-\gamma^{1-\varepsilon}$. This determines a unique $\underline{\gamma}\in(0,1)$, which is implicitly given by $1-\underline{\gamma}^{\varepsilon}=\underline{\gamma}^{1-\varepsilon}$, such that $\zeta'(1)>,=,<0$ if $\gamma>,=,<\underline{\gamma}$. This implies that $\gamma\leq\underline{\gamma}$ is sufficient for $(1-u)\tilde{\alpha}^{1-\varepsilon}+u\gamma^{1-\varepsilon}-(1-u)\tilde{\alpha}\leq0$ to hold for all $\tilde{\alpha}>1$. In contrast, if $\gamma>\underline{\gamma}$, there exists a unique $\tilde{\alpha}^{0}>0$, such that $(1-u)\tilde{\alpha}^{1-\varepsilon}+u\gamma^{1-\varepsilon}-(1-u)\tilde{\alpha}>,=,<0$ if $\tilde{\alpha}^{0}>,=,<\tilde{\alpha}$.

Let us now consider a parameter configuration $(1-u)\tilde{\alpha}^{1-\varepsilon}+u\gamma^{1-\varepsilon}-(1-u)\tilde{\alpha}>0$. This requires $1-\gamma^{\varepsilon}<\gamma^{1-\varepsilon}$. Then, differentiating Eq. (A.9), we see that $F(h)$ is a monotonic function. Furthermore, evaluating $F(h)$ at $h=0$ and $h=1$, we obtain $F(0)=2\big[(1-u)\tilde{\alpha}^{1-\varepsilon}+u\gamma^{1-\varepsilon}-(1-u)\tilde{\alpha}-u\gamma\big]+\varepsilon u\gamma$ and $F(1)=\big\{2\big[(1-u)\tilde{\alpha}^{1-\varepsilon}+u\gamma^{1-\varepsilon}-(1-u)\tilde{\alpha}-u\gamma\big]+\varepsilon u\gamma\big\}(1-u)\tilde{\alpha}+\varepsilon u\gamma\big[(1-u)\tilde{\alpha}^{1-\varepsilon}+u\gamma^{1-\varepsilon}-(1-u)\tilde{\alpha}\big]$,

[15] To see this, one can note that $f'(1)$ is increasing in γ and takes a value of zero if $\gamma=1$.

so that $F(0) \geq 0$ is sufficient for $F(1) > 0$. Substituting $(1-u)$ and u from Eq. (12), we furthermore obtain

$$(\tilde{\alpha}^{\varepsilon} - \gamma^{\varepsilon})\, F(0) = 2\left[(1-\gamma^{\varepsilon})\left(\tilde{\alpha}^{1-\varepsilon} - \tilde{\alpha}\right) + (\tilde{\alpha}^{\varepsilon} - 1)\left(\gamma^{1-\varepsilon} - \gamma\right)\right] + \varepsilon(\tilde{\alpha}^{\varepsilon} - 1)\gamma \equiv G(\tilde{\alpha}). \quad \text{(A.10)}$$

Differentiation of $G(\tilde{\alpha})$ gives $G'(\tilde{\alpha}) = 2\big\{(1-\gamma^{\varepsilon})\big[(1-\varepsilon)\tilde{\alpha}^{-\varepsilon} - 1\big] + \varepsilon\tilde{\alpha}^{\varepsilon-1}\big(\gamma^{1-\varepsilon} - \gamma\big)\big\} + \varepsilon^2\tilde{\alpha}^{\varepsilon-1}\gamma$, $G'(1) = -2\varepsilon\big(1+\gamma-\gamma^{1-\varepsilon}-\gamma^{\varepsilon}\big)+\varepsilon^2\gamma$, $\lim_{\tilde{\alpha}\to\infty} G'(\tilde{\alpha}) = -2(1-\gamma^{\varepsilon})$, and $G''(\tilde{\alpha}) = -\varepsilon(1-\varepsilon)\big\{2\big[(1-\gamma^{\varepsilon})\tilde{\alpha}^{-\varepsilon-1}+(\gamma^{1-\varepsilon}-\gamma)\tilde{\alpha}^{\varepsilon-2}\big]+\varepsilon\tilde{\alpha}^{\varepsilon-2}\gamma\big\} < 0$. Two cases can be distinguished.[16] If $2\big(1+\gamma-\gamma^{1-\varepsilon}-\gamma^{\varepsilon}\big) \geq \varepsilon\gamma$, which is the case for sufficiently low values of γ, then $G'(1) \leq 0$, and hence $G'(\tilde{\alpha}) < 0$ holds for all possible $\tilde{\alpha} > 1$. In this case, $G(1) = 0$ is sufficient for $G(\tilde{\alpha}) < 0$ and thus $F(0) < 0$ hold for all $\tilde{\alpha} > 1$. We can therefore conclude that either $F(h) < 0$ for all h or there exists a critical h^0, such that $F(h) >,=,< 0$ if $h >,=,< h^0$. With these considerations, we cannot rule out that $\Gamma(h)$ has multiple extrema. However, $\Gamma(h)$ cannot have more than two interior extrema and if two extrema existed, the first one would have to be a maximum, while the second one would have to be a minimum. This is inconsistent with $\Gamma'(0) < 0$, $\Gamma'(1) < 0$, which requires in the case of two extrema that the first one must be a minimum and the second one must be a maximum. For the same reason, there cannot be a unique extremum, so that it must be true that $\Gamma'(h) < 0$ holds for all $h \in (0,1)$. This is sufficient for a unique interior solution of $\Gamma(h) = 0$. If $2\big(1+\gamma-\gamma^{1-\varepsilon}-\gamma^{\varepsilon}\big) < \varepsilon\gamma$, which is the case for high levels of γ, then $G'(1) > 0$ implies that $G(\tilde{\alpha})$ is positive for low levels of $\tilde{\alpha} > 1$ and negative for high levels of $\tilde{\alpha}$. From $\lim_{\tilde{\alpha}\to\infty} G(\tilde{\alpha}) = -\infty$ and the derivation properties of $G(\tilde{\alpha})$, it follows that there exists a unique $\tilde{\alpha}^1 > 1$, such that $G(\tilde{\alpha}) >,=,< 0$ if $\tilde{\alpha}^1 >,=,< \tilde{\alpha}$. The analysis above extends to the case $2\big(1+\gamma-\gamma^{1-\varepsilon}-\gamma^{\varepsilon}\big) < \varepsilon\gamma$ if $\tilde{\alpha} \geq \tilde{\alpha}^1$, which ensures that the solution of $\Gamma(h) = 0$ on the unit interval is unique. Things are different, however, if $\tilde{\alpha} < \tilde{\alpha}^1$ establishes $G(\tilde{\alpha}) > 0$ and thus $F(0) > 0$. However, using the monotonicity of $F(h)$ it follows from $F(1) > 0$ – due to our assumption of $(1-u)\tilde{\alpha}^{1-\varepsilon} + u\gamma^{1-\varepsilon} - (1-u)\tilde{\alpha} > 0$ – that $\Gamma''(h) > 0$ must hold. This implies that $\Gamma(h)$ has at most one extremum, which would have to be a unique minimum. However, a minimum is in contradiction to $\Gamma'(1) < 0$, so that we can safely conclude that $\Gamma'(h) < 0$ again holds for all $h \in (0,1)$, which is sufficient for the solution of $\Gamma(h) = 0$ to be unique. This completes the proof.

A.5 Welfare effects of an increase in α in the closed economy

We first consider the case of homothetic (log-transformed Cobb-Douglas) preferences, so that welfare is given by $V_{CD}(\bar{e}, P_Y, P_X)$ in Eq. (22). Substituting $P_X = \frac{\sigma}{\sigma-1}\frac{w_Y}{\rho}M^{\frac{1}{1-\sigma}}$ and $\bar{e} = w_Y\lambda\big\{1 + h[(1-u)\tilde{\alpha}-1]\big\}$, and accounting for h and M from Eq. (20), we compute $V_{CD}(\cdot) = \ln\lambda + \frac{1-\beta}{\sigma-1}\ln\left(\frac{(1-\beta)H\lambda}{(\sigma-1)f}\right) + \ln[V_0(\alpha)]$, with

$$V_0(\alpha) = \left(\frac{\sigma}{\sigma-1}\frac{1}{\rho}\right)^{\beta}\left\{\frac{\sigma}{\sigma-1}\frac{1}{\rho} - \frac{1-\beta}{1-u}\left[(1-u)\tilde{\alpha}-1\right]\right\}^{-\frac{\sigma-\beta}{\sigma-1}}.$$

[16]From above, we know that $1-\gamma^{\varepsilon} < \gamma^{1-\varepsilon}$. However, this does not rule out one of these cases.

$dV_{CD}(\cdot)/d\lambda > 0$ is immediate. Furthermore, acknowledging $\rho = \frac{\sigma}{\sigma+\alpha(\sigma-1)}$, $\tilde{\alpha} = \alpha + \gamma$ and $1 - u = -\frac{\ln\gamma}{\ln\tilde{\alpha}-\ln\gamma}$, the derivative of $V_0(\alpha)$ can be computed according to

$$V_0'(\alpha) = V_0(\alpha)\left\{\frac{\beta}{\frac{\sigma}{\sigma-1}\frac{1}{\rho}} - \frac{\sigma-\beta}{\sigma-1}\frac{\beta-(1-\beta)\frac{1}{\tilde{\alpha}\ln\gamma}}{\frac{\sigma}{\sigma-1}\frac{1}{\rho} - \frac{1-\beta}{1-u}\left[(1-u)\tilde{\alpha}-1\right]}\right\}. \tag{A.11}$$

Evaluated at $\alpha = 1-\gamma$ (and thus $\tilde{\alpha} = 1$), we compute $V_0'(1-\gamma) < 0$. For higher levels of α, the marginal effect is however not clear. For instance, setting parameter values $\sigma = 2$, $\beta = 0.8$, and $\gamma = 0.98$, $V_0(\alpha)$ has a local minimum at $\alpha = 6.46$.

Let us now turn to the limiting case of $\varepsilon = 1$. Accounting for h and M from Eq. (21), we can express welfare by $V_{QL}(\cdot) = (\lambda-\beta)^{\frac{\sigma}{\sigma-1}}\left[\frac{H}{(\sigma-1)f}\right]^{\frac{1}{\sigma-1}}\hat{V}_0(\alpha)^{\frac{\sigma}{\sigma-1}} - 1 + \beta$, with

$$\hat{V}_0(\alpha) = \left\{\frac{\sigma}{\sigma-1}\frac{1}{\rho} - \frac{1}{1-u}\left[(1-u)\tilde{\alpha}-1\right]\right\}^{-1} = \left(\frac{\sigma}{\sigma-1} - \gamma + \frac{\alpha}{1-\gamma}\right)^{-1}. \tag{A.12}$$

Thereby, the second equality sign makes use of the definition of ρ and $1 - u = \frac{1-\gamma}{\alpha}$ from Eq. (12). From these computations, we can conclude that $V_{QL}(\cdot)$ increases in λ and decreases in α. This completes the proof.

A.6 Proof of Proposition 2

Let us first consider the limiting case of homothetic (log-transformed Cobb-Douglas) preferences, with welfare given by $V_{CD}(\overline{e}, P_Y, P_X)$ in Eq. (22). Substituting h and M from Eq. (31) into $\overline{e} = w_Y\lambda\{1 + h[(1-u)\tilde{\alpha}-1]\}$ and $P_X = \frac{\sigma}{\sigma-1}\frac{w_Y}{\rho}\left(M\frac{\mu+t}{1+t}\frac{1+t}{t}\right)^{\frac{1}{1-\sigma}}$, we can compute

$$\overline{e} = w_Y\lambda\frac{\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t}}{\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t} - \frac{1-\beta}{1-u}[(1-u)\tilde{\alpha}-1]}, \tag{A.13}$$

$$P_X = \frac{\sigma}{\sigma-1}\frac{w_Y}{\rho}\left(\frac{(1-\beta)H\lambda\rho}{\sigma f}\right)^{\frac{1}{1-\sigma}}\left(\frac{\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t}}{\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t} - \frac{1-\beta}{1-u}[(1-u)\tilde{\alpha}-1]}\frac{1+t}{t}\right)^{\frac{1}{1-\sigma}}. \tag{A.14}$$

Substituting into $V_{CD}(\overline{e}, P_Y, P_X)$, then gives $V_{CD}(\cdot) = -(1-\beta)\ln\left(\frac{\sigma}{\sigma-1}\frac{1}{\rho}\right) + \ln\lambda + \frac{1-\beta}{\sigma-1}\ln\left(\frac{(1-\beta)H\lambda\rho}{\sigma f}\right) + \ln\left[V_1(t)\right]$,

$$V_1(t) = \left(\frac{\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t}}{\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t} - \frac{1-\beta}{1-u}[(1-u)\tilde{\alpha}-1]}\right)^{\frac{\sigma-\beta}{\sigma-1}}\left(\frac{1+t}{t}\right)^{\frac{1-\beta}{\sigma-1}}, \tag{A.15}$$

where μ is given by Eq. (29). Differentiating $f(t) \equiv \frac{\mu+t}{1+t}$ establishes

$$f'(t) = \frac{1-\mu}{(1+t)^2} + \frac{d\mu}{dt}\frac{1}{1+t} = \frac{1}{1+t}\left[\frac{1-\mu}{1+t} + \frac{1-\eta^2}{[\delta(t)-\eta]^2}\delta'(t)\right]. \tag{A.16}$$

Noting that $\mu >, =, < 1$ if $\eta >, =, < 1$ from Eq. (29) and that $\delta'(t) > 0$ from Eq. (30), we can safely conclude that $f'(t) >, =, < 0$ if $1 >, =, < \eta$. Furthermore, differentiating $V_1(t)$ gives

$$V_1'(t) = V_1(t)\left[-\frac{\sigma-\beta}{\sigma-1}\frac{\frac{1-\beta}{1-u}[(1-u)\tilde{\alpha}-1]\frac{1+t}{\mu+t}f'(t)}{\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t} - \frac{1-\beta}{1-u}[(1-u)\tilde{\alpha}-1]} - \frac{1-\beta}{\sigma-1}\frac{1}{t(1+t)}\right]. \tag{A.17}$$

This derivative is unambiguously negative if either $1 > \eta$ (home net-exporting differentiated goods) and $(1-u)\tilde{\alpha} > 1$ or $1 < \eta$ (home net-importing differentiated goods) and $(1-u)\tilde{\alpha} < 1$. In contrast,

$$\lim_{\sigma\to\infty} V_1'(t) = -\frac{(1+\alpha)\frac{1-\beta}{1-u}[(1-u)\tilde{\alpha}-1]f'(t)}{\left\{(1+\alpha)\frac{\mu+t}{1+t} - \frac{1-\beta}{1-u}[(1-u)\tilde{\alpha}-1]\right\}^2} \tag{A.18}$$

is positive if $1 > \eta$ (home net-exporting differentiated goods) and $(1-u)\tilde{\alpha} < 1$ or if $1 < \eta$ (home net-importing differentiated goods) and $(1-u)\tilde{\alpha} > 1$. This completes the proof of Proposition 2 for the limiting case of $\varepsilon = 0$.

If preferences are quasilinear, welfare is given by $V_{QL}(\overline{e}, P_Y, P_X)$ in Eq. (22). Substituting h and M from Eq. (32) into $\overline{e} = w_Y\lambda\{1 + h[(1-u)\tilde{\alpha}-1]\}$ and $P_X = \frac{\sigma}{\sigma-1}\frac{w_Y}{\rho}\left(M\frac{\mu+t}{1+t}\frac{1+t}{t}\right)^{\frac{1}{1-\sigma}}$, we can compute

$$\frac{\overline{e}}{P_Y} - \beta = \frac{(\lambda-\beta)\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t}}{\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t} - \frac{1}{1-u}[(1-u)\tilde{\alpha}-1]}, \tag{A.19}$$

$$P_X = \frac{\sigma}{\sigma-1}\frac{w_Y}{\rho}\left(\frac{H\rho}{\sigma f}\right)^{\frac{1}{1-\sigma}}\left(\frac{(\lambda-\beta)\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t}}{\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t} - \frac{1}{1-u}[(1-u)\tilde{\alpha}-1]}\frac{1+t}{t}\right)^{\frac{1}{1-\sigma}}. \tag{A.20}$$

This allows us to determine $V_{QL}(\cdot) = \left(\frac{\sigma}{\sigma-1}\frac{1}{\rho}\right)^{-1}\left(\frac{H\rho}{\sigma f}\right)^{\frac{1}{\sigma-1}}\hat{V}_1(t) - 1 + \beta$, with

$$\hat{V}_1(t) = \left(\frac{(\lambda-\beta)\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t}}{\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t} - \frac{1}{1-u}[(1-u)\tilde{\alpha}-1]}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{1+t}{t}\right)^{\frac{1}{\sigma-1}}. \tag{A.21}$$

Differentiation with respect to t gives

$$\hat{V}_1'(t) = \hat{V}_1(t)\left[-\frac{\sigma}{\sigma-1}\frac{\frac{1}{1-u}[(1-u)\tilde{\alpha}-1]\frac{1+t}{\mu+t}\hat{f}'(t)}{\frac{\sigma}{\sigma-1}\frac{1}{\rho}\frac{\mu+t}{1+t} - \frac{1}{1-u}[(1-u)\tilde{\alpha}-1]} - \frac{1}{\sigma-1}\frac{1}{t(1+t)}\right], \tag{A.22}$$

where $\hat{f}(t) \equiv \frac{\mu+t}{1+t}$ and $\mu = \frac{\eta\hat{\delta}(t)-1}{\hat{\delta}(t)-\eta}$ have been considered. In analogy to the case of homothetic preferences, we find that this derivative is unambiguously negative if $1 < \eta$ (home net-importing differentiated goods) and $(1-u)\tilde{\alpha} < 1$. In contrast, we find that $\lim_{\sigma\to\infty} \hat{V}_1'(t)$ is positive if $1 > \eta$ (home net-exporting differentiated goods) and $(1-u)\tilde{\alpha} < 1$. This completes the proof of Proposition 2 for the limiting case of $\varepsilon = 1$.

A.7 Formal details for the analysis in Section 5.2

Let us consider the limiting case of $\varepsilon = 1$ and focus on an interior solution with $h, h^* \in (0,1)$. Then, accounting for the definition of $\hat{\delta}(t)$ in Eq. (30), we can follow the steps from the main text to compute

$$\mu = \frac{\hat{\eta}\hat{\delta}(t) - 1}{\hat{\delta}(t) - \hat{\eta}}, \qquad \hat{\eta} \equiv \frac{\lambda^* - \beta}{\lambda^*}\frac{\lambda}{\lambda - \beta}. \tag{A.23}$$

Thereby, $\mu >, =, < 1$ if $\lambda^* >, =, < \lambda$ and thus $\hat{\eta} >, =, < 1$. Noting that h and M are given by (32) and following the derivation details from Appendix A.6, we can compute $V_{QL}(\cdot) = \left(\frac{\sigma}{\sigma-1}\frac{1}{\rho}\right)^{-1}\left(\frac{H\rho}{\sigma f}\right)^{\frac{1}{\sigma-1}}\hat{V}_1(t) - 1 + \beta$, with $\hat{V}_1(t)$ given by Eq. (A.21). The welfare effects of trade discussed in Section 5.2 then follow from the proof of Proposition 2.