4 Adverse Selection (Screening) in the PA model

- Before (Anke): concerned with whether some given outcome (efficient, balanced) is implementable
  now: look for the constrained-optimal outcome from a certain individual’s point of view. We call this agent the ‘principal’, and seek to find the outcome that maximizes his utility. Problem: Principal doesn’t know the agent’s relevant characteristics, his type.

- Huge number of applications. Selection: monopolistic price discrimination (monopolistic producer doesn’t know customer preferences); optimal taxation (when deciding on income tax schedule, government doesn’t know individual opportunity costs of labor market participation); procurement (buyer doesn’t know seller’s production costs); financing and lending (bank doesn’t know client risk type) etc.

- Important: We need to assume that the uninformed party (the principal) makes the contract offer. Results do not extend to more general bargaining procedures.

The Two-Type Case

- consider example: monopolistic price competition

- two risk-neutral parties, a monopolistic seller (the principal $P$) and a single buyer (the agent $A$)

- Utility (profit) of seller is

$$U_P(t, x) = t - cx, \quad c \geq 0,$$

where $x \in [0, \infty] = \text{quantity of good produced by } P,$ and
utility of the consumer is 

\[ U_A(x; \theta) = \theta v(x) - t, \quad v' > 0, v'' < 0 \quad (4.2) \]

where \( \theta \in \{\theta_l, \theta_H\}, \theta_l < \theta_H \) and \( p = \text{Prob}\{\theta = \theta_H\} > 0 \)

Note: agent’s preferences satisfy the single crossing property:

\[ u(x, \theta_h) - u(x, \theta_l) = (\theta_h - \theta_l)v(x) \quad \text{strictly increases in } x \quad (\text{SCP}) \]

4.1 Perfect Information (First Best)

if \( \theta \) is commonly observable \( \rightarrow \) contracts can be made contingent on the type of consumer, \((t_i, x_i), i = l, h\)

\[ P \text{ solves } \max_{x_i, t_i} t_i - cx_i \]

subject to \( U_A = \theta_i v(x_i) - t_i \geq 0, \ i = l, h \quad (PC_i) \)

solution is characterized by (see Figure 1)

\[ \theta_i v'(x_i^{FB}) = c \quad i = l, h \]

and \( u_A(x_i^{FB}, t_i^{FB}) = 0 \) from \( (PC_i) \), so

\[ t_i^{FB} = \theta_i v(x_i^{FB}) \]

\( \rightarrow \) if the monopolist can perfectly price discriminate, she will offer the quantities \( x_h^{FB} > x_l^{FB} \) and extract the entire consumer surplus

\( \rightarrow \) Note: this outcome is efficient (Pareto optimal)
4.2 Imperfect Information

- $\theta$ is $A$’s private information. As a consequence: $\{(y_i^{FB}, t_i^{FB})\}_{i=l,h}$ no longer implementable or incentive compatible (see Figure 2). Formally:

$$u_A(x_h^{FB}, t_h^{FB}; \theta_h) = \theta_h v(x_h^{FB}) - t_h^{FB} = 0$$

$$< (\theta_h - \theta_l) v(x_l^{FB}) = \theta_h v(x_l^{FB}) - t_l^{FB} = u_A(x_l^{FB}, t_l^{FB}; \theta_h)$$

→ high-preference type would choose contract for low-demand agent

- Note: $P$ can still achieve efficient purchases by offering the menu $(x_l^{FB}, t_l^{FB})$ and $(x_h^{FB}, t_h^{FB} - \psi^{FB})$ where

$$\psi^{FB} = \psi(x_l^{FB}) = (\theta_h - \theta_l) v(x_l^{FB})$$

→ But: only if the high-type agent pays far less than $t_h^{FB}$ to the principal! Means: the $\theta_h$-type agent earns a rent (see Figure). $P$ doesn’t like this.

4.3 The Second Best Optimum

- Q: What is the optimal (second-best) allocation and the corresponding mechanism?

- Revelation Principle (see Anke) → restrict ourselves w.l.o.g. to direct mechanism/ menu of contracts $\{(x(\hat{\theta}), t(\hat{\theta}))\}$ which is incentive compatible, i.e. $\hat{\theta} = \theta$

- the principal chooses $\{(x_i, t_i)_{i=l,h}\}$ so as to

$$\max_{(x_i, t_i)} E(U_P) = (1 - p)[t_l - cx_l] + p[t_h - cx_h]$$

subject to $U_A(\cdot, \theta_i) = \theta_i v(x_i) - t_i \geq 0, \quad i = l, h \quad (PC_i)$
\[
\begin{align*}
\theta_l v(x_l) - t_l &\geq \theta_l v(x_h) - t_h \quad (IC_l) \\
\theta_h v(x_h) - t_h &\geq \theta_h v(y_l) - t_l. \quad (IC_h)
\end{align*}
\]

Claim: the optimal contract has the following properties

1. \((PC_l)\) is binding \(\rightarrow t_l = \theta_l v(x_l)\)
2. \((IC_h)\) is binding \(\rightarrow t_h = t_l + \theta_h [v(x_l) - v(x_l)]\)
3. \(x_h \geq y_l\)
4. \((PC_h)\) and \((IC_l)\) are slack (and, hence, can be ignored)

\(-\) using properties 1–4, \(P\)'s max program reduces to

\[
\max_{(x_h, x_l)} (1 - p)[\theta_l v(x_l) - cx_l] + p[\theta_h v(x_h) - cx_h - (\theta_h - \theta_l)v(x_l)]
\]

\(-\) the FOC are

\[
\begin{align*}
\theta_h v'(x_h^{SB}) &= c & \Rightarrow & & x_h^{SB} = x_h^{FB} \\
\theta_l v'(x_l^{SB}) &= \frac{c}{1 - p\frac{\theta_h - \theta_l}{\theta_l}} & > c & \Rightarrow & & x_l^{SB} < x_l^{FB}.
\end{align*}
\]

\(-\) we get the optimal transfers from \((PC_l)\) and \((IC_h)\),

\[
\begin{align*}
t_l^{SB} &= \theta_l v(x_l^{SB}) \quad \Rightarrow & & u_A(x_l^{SB}, t_l^{SB}; \theta_l) = 0 \\
t_h^{SB} &= \theta_h v(x_h^{SB}) - (\theta_h - \theta_l)v(x_l^{SB}) \quad \Rightarrow & & u_A(x_h^{SB}, t_h^{SB}; \theta_h) = (\theta_h - \theta_l)v(x_l^{SB}) = \psi(x_l^{SB}) > 0.
\end{align*}
\]

4.4 Interpretation

\(-\) in order to ensure self-selection, \(P\) has to give a rent of \(\psi(x_l) = (\theta_h - \theta_l)x_l\) to the \(\theta_h\)-type agent, with \(\psi' > 0\)
• it is the necessity to pay this rent that makes it optimal for $P$ to distort the quantity bought by the $\theta_l$ type agent downwards (see Figure)

• size of distortion depends on relative probability of a low-demand agent: $x_i^{SB} \rightarrow x_i^{FB}$ as $p \rightarrow 0$ and $x_i^{SB} = 0$ for sufficiently high $p$

• Output of $h$-type agent is same than in FB (‘no distortion at the top’-property)

• $P$ has perfect information *ex post* → further gains from trade to be realized in state $\theta_l$ since $x_i^{SB} < x_i^{FB}$

→ self selection requires commitment (no renegotiation)

• there is a non-linear tariff $t(x)$ that is equivalent to the second-best optimal contracts:

\[
t = t_i^{SB} \text{ if } x = x_i^{SB} \quad \text{and} \quad t = \infty \quad \text{otherwise.}
\]