BGPE Intensive Course: Contracts and Asymmetric Information

Adverse Selection, Signaling, and Screening in Markets

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Market Failure

Stylized Facts:

- used cars, even if they are like new, sell far below their dealership price
- laid-off workers experience longer spells of unemployment than workers for different reasons without a job (e.g. military)
- private health care for the elderly is essentially unavailable
- young drivers pay very expensive insurance premiums

What do these empirical regularities have in common?

These markets are characterized by informational asymmetries and suffer from adverse selection.
Productivity Uncertainty in the Labor Market

- 2 identical firms $F$, 1 representative worker $W$
- worker’s productivity is $\theta \in [\theta, \bar{\theta}]$, private information of $W$, c.d.f $F(\theta)$ is continuous and has full support
- utilities
  
  $$u_W = wq + r(\theta)(1 - q), \quad u_F = (\theta - w)q$$

  where $q = 1$ if $W$ works and $q = 0$ otherwise
  
  $w \in \mathbb{R}$ = wage paid by firm
  $r(\theta) = W’s$ reservation utility, $r' > 0$

- timing:
  
  - $t = 0$: $W$ observes $\theta$
  - $t = 1$: $F$ offer fixed-wage employment contracts
  - $t = 2$: $W$ accept/rejects $\rightarrow$ payoffs
Equilibrium Analysis

Equilibrium under Full Information

- both worker and firms know $\theta \rightarrow$ firms can offer productivity dependent wages $w(\theta)$
- competition between firms gives $w^*(\theta) = \theta$ in equilibrium
- $W$ accepts employment if $w^*(\theta) = \theta \geq r(\theta)$
- equilibrium is efficient

Equilibrium under Asymmetric Information

- only $W$ knows $\theta$, $F$’s only know $F(\theta)$
- assume $r(\theta) \leq \theta$ and $r(\theta) < \theta$, $\forall \theta > \theta$
  $\Rightarrow$ efficient employment has $q(\theta) = 1 \forall \theta$
Equilibrium under Asymmetric Information

• firms do not know $\theta \Rightarrow$ offer (same) fixed wage contract $w$ in equilibrium
• utility of worker of type $\theta$ who accepts wage offer of $w$ is $u_W = w$
• comparing this with outside utility $r(\theta)$ gives supply

$$\Theta(w) = \{\theta | r(\theta) \leq w\}$$

Note: average productivity $E\{\theta | \theta \in \Theta(w)\}$ depends on price
• expected profit of firm in the market is $E\{\theta | \theta \in \Theta(w)\} - w$
• competition gives zero profits; necessary equilibrium condition is

$$w^* = E\{\theta | \theta \in \Theta(w^*)\} = E\{\theta | r(\theta) \leq w^*\}$$
Market Equilibrium

• graphic illustration:
Properties of the Equilibrium

- equilibrium exists
- equilibrium is generically unique: if there are multiple solutions to equation $w^* = E\{\theta|\theta \in \Theta(w^*)\} = E\{\theta|r(\theta) \leq w^*\}$, equilibrium is highest $w$ satisfying condition
- welfare properties:
  - if $r(\bar{\theta}) \leq E(\theta)$, equilibrium is efficient
  - if $r(\bar{\theta}) > E(\theta)$, equilibrium is inefficient
  - if $r(\bar{\theta}) = \theta$, can have complete market breakdown
- intuition: $F$ cannot break even at wage $w = E(\theta) < r(\bar{\theta})$
  $w$ falls $\rightarrow$ even more high-productivity workers drop out of market $\rightarrow$ wage must drop further
  (alternatively: market participation of individual worker introduces externality)
Government Intervention

Can free-market outcome be improved upon?

- assume: social planner/government does not know workers’ productivity \( \theta \) either (is subject to same informational constraints as firms)

  \[ \rightarrow \text{can only devise policies based on whether people choose to work or not (from IC’s)} \]

- let \( w_e \) (resp. \( w_u \)) be transfer to worker if \( q = 1 \) (resp. \( q = 0 \))

  \[
  w_u + r(\hat{\theta}) = w_e \quad (IC)
  \]

Claim. There is no (balanced) policy \( \{w_e, w_u\} \) that yields to a Pareto improvement over the market outcome where workers with \( \theta \leq \theta^* \) work and workers \( \theta > \theta^* \) do not.
Government Intervention

Proof.

- equilibrium is Pareto efficient if $r(\theta) \leq E(\theta)$
- if $r(\theta) > E(\theta)$, let $\hat{\theta}$ be cut-off type worker given $\{w_e, w_u\}$
- budget balance requires
  $$w_e F(\hat{\theta}) + w_u (1 - F(\hat{\theta})) = \int_{\theta}^{\hat{\theta}} \theta f(\theta)d\theta = F(\hat{\theta}) E(\theta | \theta \leq \hat{\theta})$$
- substituting for $w_u + r(\hat{\theta}) = w_e$ gives
  $$w_u(\hat{\theta}) = F(\hat{\theta}) [E(\theta | \theta \leq \hat{\theta}) - r(\hat{\theta})]$$
  $$w_e(\hat{\theta}) = F(\hat{\theta}) [E(\theta | \theta \leq \hat{\theta}) - r(\hat{\theta})] + r(\hat{\theta})$$
- note: $\hat{\theta} = \theta^*$ gives market outcome $w_u = 0$ and $w_e = r(\theta^*)$
- $\hat{\theta} < \theta^*$ decreases social surplus
- suppose $\theta^* < \hat{\theta} < \bar{\theta}$. Since $w > w^* \Rightarrow E(\theta | r(\theta) \leq w) < w$ and $w^* = r(\theta^*)$ with $r' > 0$, we get $E(\theta | r(\theta) \leq r(\hat{\theta})) = E(\theta | \theta \leq \hat{\theta}) < r(\hat{\theta})$ and, hence, $w_u < 0$ → unemployed are worse off
- $\hat{\theta} = \bar{\theta}$ implies $w_e = E(\theta) \rightarrow$ highest type workers are worse off since $r(\bar{\theta}) > E(\theta)$. □
Other Markets with Adverse Selection

• formal model translates into goods markets and insurance markets:

<table>
<thead>
<tr>
<th>labor</th>
<th>consumption good (cars)</th>
<th>insurance</th>
</tr>
</thead>
<tbody>
<tr>
<td>workers →</td>
<td>sellers</td>
<td>insuree</td>
</tr>
<tr>
<td>firms →</td>
<td>buyers</td>
<td>insurer</td>
</tr>
<tr>
<td>$\theta$ →</td>
<td>buyer’s value</td>
<td>-(exp. payments to insuree)</td>
</tr>
<tr>
<td>$r(\theta)$ →</td>
<td>seller’s value</td>
<td>-(inverse CE of risk)</td>
</tr>
<tr>
<td>$w$ →</td>
<td>price</td>
<td>-(insurance premium)</td>
</tr>
</tbody>
</table>

• other markets where adverse selection relevant
  • credit market ($\theta = \text{default risk of debtor}$)
  • dating and marriage market ($\theta = \text{attractiveness of partner}$)
  • stock market and corporate equity market (IPO’s) ($\theta = \text{firm value}$)
adverse selection can lead to total market failure – if trade occurs, it will be less than efficient

- in markets with adverse selection (asymmetric information)
- prices are correlated with quality
- prices serve dual role of info transmission and market clearing
- institutional/market responses against market failure caused by adverse selection
  - signaling and screening devices, e.g. warranties
  - reputation (brand names and chains)
  - experts, inspections, standards, licensing
  - mandatory insurance (health, automobile)
  - liability laws
Using a Signal

asymmetric information causes market failure → participants have incentives to develop ways to reduce informational asymmetries

- **signaling**: informed market participants move first to **convey** info
- **screening**: uninformed market participants move first to **elicit** info

**Signaling**

- some market participants may be worse off as a result of their privately held information (sellers in lemons market, consumers in insurance market) → would want to reveal this information to others
- problem: information revealed must be **credible** → use of **signals**
- examples: warranties, lineups, peacock tail
- **but**: for the signal to work (be credible), it must be **costly to fake**
Education as a Signal in Labor Markets (Spence)

- 2 identical firms \( F \), 1 representative worker \( W \)
- worker \( W \) with ability (=productivity) \( \theta \in \{\theta_L, \theta_H\} \), \( \theta_L < \theta_H \), private information of \( W \) with \( p = Prob\{\theta = \theta_H\} \)
- set \( r(\theta_H) = r(\theta_L) = 0 \) for simplicity
- worker can invest in education \( e \in \mathbb{R}_0^+ \)
  - marginal cost of education of \( \theta_i \)-type is \( c_i \) with \( 0 < c_H < c_L \)
  - education does not improve productivity \( \theta \)
- utilities:
  \[
  u_F = \theta_i - w \quad \text{and} \quad u_W = w - c_i e \quad i = H, L
  \]
- timing:
  \( t = 1 \): \( W \) learns \( \theta_i \), chooses \( e \)
  \( t = 2 \): \( F \)'s observe \( e \), form beliefs \( \mu(e) = Prop\{\theta = \theta_H|e\} \)
  \( t = 3 \): \( F \)'s offers (same) wage \( w \)
General Structure of Signaling (and Cheap Talk) Game

- dynamic game of incomplete information with sender $S$ and receiver $R$

- timing
  
  $t = 0$ nature draws type $\theta_i \in \Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$ for sender; with
  
  $$p_i = p(\theta_i) = Prob\{\theta = \theta_i\} > 0, \quad \forall \theta_i \in \Theta$$

  $t = 1$ sender observes own type $\theta_i$, chooses message
  
  $$m_j \in M = \{m_1, m_2, \ldots, m_N\}$$

  $t = 2$ receiver observes message $m_j$ (but not $\theta_i$) and chooses action $x \in X$

  $t = 3$ payoffs $u_S(x, m_j, \theta_i)$ and $u_R(x, m_j, \theta_i)$ realized

- signaling game: $u_S$ depends on $m_j$

- cheap talk game: $u_S$ (and $u_R$) is independent of $m_j$
Perfect Bayesian Nash Equilibrium

**Definition** A Perfect Bayesian Equilibrium (PBE) is a pair of strategies $m^*(\theta_i)$ and $x^*(m_j)$ and a belief $\mu^*(\theta_i|m_j)$ such that

a) $m^*(\theta_i)$ maximizes $S$’s utility given $R$’s strategy:

$$m^*(\theta_i) = \arg\max_{m_j \in M} u_S(x, m_j, \theta_i) \quad \forall \theta_i \in \Theta$$

b) $x^*(m_j)$ maximizes $R$’s utility given beliefs about $S$’s type:

$$x^*(m_j) = \arg\max_{x \in X} \sum_{\theta_i} \mu(\theta_i|m_j) u_R(x, m_j, \theta_i),$$

c) $R$ forms consistent beliefs that are calculated by Bayes Rule whenever possible:

$$\sum_{\theta_i \in \Theta} \mu(\theta_i|m_j) = 1, \forall m_j \quad \mu(\theta_i|m_j) = \frac{p(\theta_i)}{\sum_{\theta_i \in \Theta(m_j)} p(\theta_i)}$$

where $\Theta(m_j) \equiv \{\theta_i \in \Theta | m^*(\theta_i) = m_j\}$. 

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Benchmark Cases

Equilibrium under Full Information

- **both** worker and firms observe ability $\theta_i$, $i = H, L$
- firms offer wage $w_i = \theta_i$ independent of $e$
  $\rightarrow$ worker chooses $e_H = e_L = 0 \Rightarrow$ efficient

Imperfect but Symmetric Information

- **neither** worker nor firms observe $\theta_i$, $i = H, L$
  $\rightarrow$ wage $w$ can no longer depend on $\theta$
- firms offer $w(e = 1) = w(e = 0) = p\theta_H + (1 - p)\theta_L$ independent of $e$
  $\rightarrow$ worker chooses $e_H = e_L = 0 \Rightarrow$ efficient
Imperfect and Asymmetric Information

- **only** worker **not** firms observe $\theta_i, i = H, L$
  - wage $w$ can no longer depend on $\theta$

Separating Equilibrium

- two types of workers choose different education levels $e_H \neq e_L$
  - firms have beliefs $\mu(e_H) = \theta_H$ and $\mu(e_L) = \theta_L$
  - firms offer wages $w(e_H) = \theta_H$ and $w(e_L) = \theta_L$

- $e_H \neq e_L$ optimal for $W$ requires (from (IC)'s) $e_L = 0$ and $e_H > 0$

where

$$c_H e_H < \theta_H - \theta_L \leq c_L e_H \quad (\star)$$

- off equilibrium beliefs? ... let $\mu(e) = 0$ for $e < e_H$, $\mu(e) = 1$ for $e \geq e_H$
Education as a Signal

- graphic illustration of a separating equilibrium:

\[
\{(w, e) | u(w, e; \theta_L) = u(w_H, e_H; \theta_L)\}
\]

\[
\{(w, e) | u(w, e; \theta_H) = u(w_H, e_H; \theta_H)\}
\]

- note: W’s preferences satisfy single crossing property:

\[
\frac{dw}{de} |_{\bar{u}} \uparrow \text{ in } \theta \quad \text{or} \quad u_e(w, e; \theta_L) - u_e(w, e; \theta_H) = c_L - c_H > 0 \quad (SC)
\]
Other Types of Equilibria

Pooling Equilibrium

• both types of workers choose same education level $e_H = e_L = \tilde{e}$
  → firms have beliefs $\mu(\tilde{e}) = p\theta_H + (1 - p)\theta_L$
  → firms offer wage $w(\tilde{e}) = \theta_H + (1 - p)\theta_L$

• off equilibrium beliefs? ... let $\mu(e) = 0$ for $e < \tilde{e}$, $\mu(e) = p$ for $e \geq \tilde{e}$
• note: both $\tilde{e} = 0$ and $\tilde{e} > 0$ may be possible

Semi-Separating Equilibrium

• some education levels are chosen by both types, and some by only one type

• no type chooses more than two levels of $e$ with positive probability

• three types of semi-separating equilibria, depending on who makes two different choices with positive probability
Properties of Signaling in Markets

- equilibrium with $e_H = e_L = 0$ always exists
- wasteful signaling in separating equilibria where $e_H > 0$
- wasteful education also possible in some pooling equilibria where $e_H = e_L > 0$
- multiple (types) of equilibria cannot be unambiguously ranked by the Pareto criterion
- out-of-equilibrium beliefs $\mu(e)$ are critical but not always plausible → equilibrium refinements

**Intuitive Criterion** (Cho-Kreps): if an out-of-equilibrium deviation is dominated for some type $\theta_i$, but not for all types, then out-of-equilibrium beliefs should put zero probability of that type action “dominated” = whatever the receiver’s action to the deviation, type $\theta_i$ is made worse off by the deviation

→ selects unique least-cost separating equilibrium

this equilibrium may be Pareto dominated, however (intuition = informational externality)
Conclusion

signaling can least to wasteful resource allocation and the market outcome may thus be inefficient

- in markets with signaling
  - privately informed individuals use signal to reveal their information
  - signal only works (is credible) if sending the same signal is too costly for other individuals
- other markets where wasteful signaling is relevant
  - consumer products (signal = warranty, advertisements, price)
  - corporate equity and start-ups (signal = equity/own money invested)
  - legal disputes (signal = pre-trial settlement demands)
  - bargaining (signal = rejection of offer/delay)
  - live entertainment and restaurants (signal = lineups)
  - marriage and dating (signal = fancy car)
  - poker (signal = stakes)
Competitive Screening

Screening = contractual arrangements originating from uniformed side of market to elicit information from informed market participants

- 2 identical firms $F$, 1 representative worker $W$
- worker $W$ with ability (=productivity) $\theta \in \{\theta_L, \theta_H\}$, $\theta_L < \theta_H$, private information of $W$ with $p = Prob\{\theta = \theta_H\}$, $r(\theta) = 0$
- firm can set task difficulty level $t \in \mathbb{R}_0^+$
- utilities: $u_F = \theta_i - w$ and $u_W = w - c_it \quad i = H, L$
- task difficulty does not influence productivity and costs workers effort $c_i$ with $0 < c_H < c_L$
- timing:
  $t = 1$: $W$ learns $\theta_i$
  $t = 2$: $F$’s offer menu of contracts $\{(w, t)\}$
  $t = 3$: $W$ picks firm/contract
Equilibrium Analysis

- note: $W$'s preferences satisfy **single crossing property**: 

$$\frac{dw}{dt} \uparrow \text{in } \theta \quad \text{or} \quad u_t(w, t; \theta_L) - u_t(w, t; \theta_H) = c_L - c_H > 0 \quad (SC)$$

**Equilibrium under Full Information**

- **both** worker and firms observe ability $\theta_i, i = H, L$
- firms offer wage/task contracts $(w_i, t_i) = (\theta_i, 0)$
  $\rightarrow$ worker accepts employment $\Rightarrow$ SPE is **efficient**

**Imperfect but Symmetric Information**

- **neither** worker nor firms observe $\theta_i, i = H, L$
  $\rightarrow$ wage $w$ can no longer depend on $\theta$
- firms offer wage/task contract $(w, t) = (p\theta_H + (1 - p)\theta_L, 0)$
  $\rightarrow$ worker accepts $\Rightarrow$ **efficient**
Imperfect and Asymmetric Information

- **only** worker **not** firms observe $\theta_i, i = H, L$
  $\rightarrow$ wage $w$ can no longer depend on $\theta$

*find equilibrium in steps*

- let $(w_H, t_H)$ and $(w_L, t_L)$ be the lowest-wage contracts accepted by workers of type $i$ in equilibrium
- Step 1: in any SPE, firms earn zero profits
- Step 2: there is no SPE in which the high-type worker accepts a wage $w_H < \theta_H$ with positive probability
  $\rightarrow$ all low types must earn $w_L = \theta_L$ and the equilibrium must be separating
- in a separating SPE, must have $t_L = 0$ and $\theta_H - c_L t_H = \theta_L$, i.e., the only candidate for equilibrium is the least-cost separating one
Screening with Task Assignments

- graphic illustration of the separating equilibrium:

\[
E[\theta] \\
\theta_H \\
(\theta_L, t_L)
\]

- note: no deviation profitable; in particular, any contract attracting both types of workers lies above break even line
Screening with Task Assignments

- graphic illustration of the separating equilibrium:

- note: deviation profitable; a contract \((\tilde{w}, \tilde{t})\) attracting both types of workers lies below break even line

- if least-cost separating equilibrium is Pareto dominated by pooling, then no equilibrium exists
Properties of Market Screening

- equilibrium (in pure strategies) may not exist
- if equilibrium exist, is unique and identical to least-cost separating equilibrium
- equilibrium separation is necessary; otherwise, rival firms can do “cream skimming” and attract only high-types away from the firm
  → wasteful screening with $t_H > 0$ occurs in equilibrium → outcome is Pareto inefficient

- other screening devices in labor markets
  - probation periods
  - seniority wages
  - performance based compensation
Conclusion

Screening can lead to wasteful resource allocation and the market outcome may thus be inefficient.

- In markets with screening:
  - Uninformed individuals use screening devices to make informed individuals reveal their information by choice of (preferred) contract from menu.
  - Screening device only works (separates types) if accepting same contract is undesirable for other individuals.

- Other markets where wasteful screening is relevant:
  - Consumer products (screening device = warranty, price).
  - Credit (screening device = collateral, incomes).
  - Marriage and dating (screening device = household chores).
  - Poker (screening device = stakes).