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Income Taxation and Job Creation

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Abstract

In this paper, I argue that there is an inefficiently high number of job creators in a model with labour market imperfections and an endogenous decision to become a job creator. I therefore augment the standard labour matching model developed by Mortensen and Pissarides by an endogenous job decision that is based on heterogeneous job creation abilities. In the decentralised market, job creators can appropriate large parts of the surplus from matches therefore making job creation too attractive relative to the first-best. It can hence be welfare enhancing to tax the profits from job creation. The introduction of a tax on the profits of job creators restores the first-best allocation by affecting the job decision. It drives rather unproductive job creators out of the market since the marginal job creator is affected and not the average one. Thus, the negative effects to job creation are small. Moreover, the tax does not distort vacancy posting and hiring choice of firms.

Keywords: optimal taxation, imperfect labour markets, job creation, entrepreneurship

JEL classification: H21, J21, J64, L26

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1 Introduction

When talking about taxation of entrepreneurs or job creators, a common fear is that higher taxation might lead to job cuts or less business creation. Moreover, if entrepreneurs are very innovative, taxation might lead to less innovation and is harmful for growth. If the trickle-down effect matters, it is beneficial for the whole economy to not tax job creators heavily, since they create jobs and help reduce unemployment.[1] The question that arises is whether job creators' profits suitably reflect their contribution to economic welfare or whether they might be too high. If the latter is the case, taxation of job creators' profits is justified even from an efficiency perspective. To satisfyingly answer the question whether higher tax rates for job creators can be welfare enhancing, it is important to make realistic assumptions about the labour market. I therefore use the labour matching model developed by Diamond, Mortensen and Pissarides (DMP-Model)[2] augmented with heterogeneous agents and endogenous job decisions. Individuals have an ability for job creation and can decide to become a regular worker or a job creator. Every worker in regular employment earns a wage independent of her job-creating ability, whereas job creators with a higher talent employ more workers and earn higher profits.

In the first section, I describe the model setup, the firms' optimal hiring decisions and vacancy posting and the job decision that each individual has to face. Next, the social planner's maximisation problem is outlined and I derive the first best allocation. I receive a condition stating how the planner sets the optimal threshold ability. Individuals with a lower job-creating ability become workers and individuals with an ability above the threshold become job creators. I then compare the outcome in the market with the social planner's solution and show that the market equilibrium is in general not efficient. Hereby and in the following sections, I focus on steady states to make the problem more tractable. Inefficiencies in the market can arise for two reasons: Firstly, job creators post too many vacancies if their private return of

[1]The term job creator in this paper does not only include entrepreneurs or innovators, but rather describes all people in a position that makes it possible for others to have a job. Therefore, it also incorporates individuals in management positions.

[2]See for example Diamond (1982), Pissarides (1985), Mortensen and Pissarides (1994) and Pissarides (2000). Yashiv (2007) gives an overview on how the labour search and matching model is growingly used in macroeconomics.

a match exceeds the social return and, secondly, there are too many job creators if they can acquire most of the surplus of a match. Therefore, when introducing taxation it is important to consider these problems simultaneously. The tax rate should affect the decision to become a job creator without distorting the vacancy posting choice of the individuals for whom it is still optimal to open up a firm. I show that there is a tax on the profits of job creators that allows to restore the first best result with fewer job creators than in the market equilibrium. I calculate the tax rate on job creators' profits by comparing the equilibrium condition for the optimal number of job creators in a social planner setting with the condition in the decentralized market equilibrium. The marginal tax rate is determined firstly for the case where just job creators' profits are taxed and secondly for the case where workers' incomes are taxed as well. I find some indication of a progressive tax system. Moreover, the taxation of profits from job creation is welfare enhancing since it crowds out only the least productive firms whereas job creators with a high job-creating ability are not affected in their decision to open up a firm. Overall welfare increases if the job creators who are unproductive relative to others become workers. This argument is in line with Jaimovich and Rebelo (2017) who state that an increase in capital income tax rates reduces the number of entrepreneurs or innovators. But the entrepreneurs that opt out are the marginal and not the average ones who are much more productive. Hence, the overall effect on growth is small. Furthermore, I demonstrate that the introduction of taxation does not distort the vacancy posting and hiring decisions of the remaining more productive job creators. To sum up, I find efficiency reasons that support higher taxation of incomes from job creation whereas most of the literature argues that job creation should be subsidized. My argument is that when thinking about the taxation of entrepreneurial income, it is also important to take into account that rather low productive job creators might start a firm when they have the possibility to extract large parts of the surplus of a match. This leads to an inefficiently large number of not so able job creators that compete with more productive firms for workers. My paper therefore can also be understood as an advise of caution not to exaggerate the support of job creation since this could have adverse effects on welfare.

There are two strands in the literature which are important when determining the effects of taxation on entrepreneurship and job-creation. The first one deals with innovation and top incomes. Aghion et al. (2018) find a positive correlation between innovation and top income inequality. They show in a Schumpeterian growth model that entrants' innovations have a positive impact on social mobility which can be dampened when lobbying activities by incumbents are strong. Quite similar is Jones and Kim (2018) who argue that the Pareto distribution of incomes is generated by existing entrepreneurs who exert effort to increase their profits and are eventually replaced by entrants with new ideas. In these models, taxation of entrepreneurial income from innovative activity would act as an entry barrier for new innovators. Bell et al. (2019) instead argue that exposure to innovation is much more important than financial incentives, as example top income tax reductions, to increase the supply of innovators. There are several other papers who deal with the effects of top tax rates on innovation activity and mobility of innovators (see e.g. Akcigit et al. (2016), Akcigit et al. (2018) and Akcigit and Stantcheva (2020)). The second strand considers the role of entrepreneurs in the top income distribution since entrepreneurs are highly represented in the group of top income earners. Quadrini (2000) includes entrepreneurial activity in a general equilibrium model and is able to replicate the empirically observed income and wealth distributions with high concentrations at the top. Cagetti and De Nardi (2006) study the effects of several tax reforms in an incomplete market model with heterogeneous agents and find small effects on the total number of households that engage in entrepreneurial activity whereas the effects on output and capital formation are larger. Brueggemann (2017) calculates a welfare maximizing top marginal tax rate of 60 percent in a Bewley-Huggett-Aiyagari model with entrepreneurship. She finds positive effects of a higher top marginal tax rate in the long run since entrepreneurs that are not in the highest tax bracket increase employment. In the cited papers, borrowing constraints limit the amount of entrepreneurs whereas labour markets are complete in the sense that wages equal marginal products. The contribution of my paper is the focus on an endogenous job decision in imperfect labour markets where wages deviate from marginal products which yields the possibility of rent-seeking and therefore an inefficient allocation of resources. There is some evidence in the literature that parts of the incomes of top earners are caused by rent-seeking behaviour. Bivens and Mishel (2013) for example argue that the increase in incomes of the top 1 percent since the 1980s is largely caused by the creation or redistribution of rents. In their opinion, very high incomes are not just efficient marginal returns to specific skills or high ability as is claimed as example by Kaplan and Rauh (2013). If this holds true, higher taxes on high income earners might be justified and potentially welfare enhancing since they reduce the returns to rent-seeking. Piketty et al. (2014) argue that compensation bargaining and tax avoidance behaviour speak in favour of very high taxes for top earners. Additionally, they empirically show that a decrease in top tax rates does not lead to higher economic growth, which is evidence against the trickle-down effect. Rothschild and Scheuer (2016) calculate Pareto optimal income taxes when agents can work either in traditional activities where private and social product coincide or in rent-seeking activities with negative externalities. In contrast to the above mentioned papers, I do not need to make the assumption that individuals behave in some way of rent-seeking. The appropriation of surpluses is included in the model through the way how firms and workers match and bargain about wages. Therefore, it provides some micro-foundation for rent-seeking.

In a similar model setup to mine with search and scarce entrepreneurial talent, Boone and Bovenberg (2002) analyse under which circumstances workers or entrepreneurs can reap surpluses and derive how different labour supply and labour demand elasticities influence the optimal tax system. They find that the labour market tightness should not be distorted by taxation. Hungerbühler et al. (2006) derive optimal tax rates in a matching model with directed search and workers with different skill levels but without endogenous job decision. They find positive marginal tax rates even at the top of the income distribution and larger marginal taxes compared to a Mirrlees setting.

The remainder of the paper is organized as follows. Section 2 describes the setup of the theoretical model, section 3 examines the social planner's allocation and 4 analyses the market equilibrium's inefficiency. Taxation and its effects on the efficient allocation is described in section 5. Finally, an extension of the wage bargaining process can be found in section 6 and section 7 provides a brief conclusion.

2 Model Setup

I consider a closed economy with a continuum of individuals who live forever. Their mass is normalized to one. Every individual has a talent for job creation which is denoted by a. The cumulative distribution function of talent is given by $\Phi(a)$, the density by $\phi(a)$.

Individuals can decide at the beginning of each period whether they want to become job creators or workers. If they decide to become a worker, they can either be employed or unemployed. An employed worker earns the wage w_t which is independent of the worker's talent for job creation and an unemployed worker receives unemployment benefits z which can also be interpreted as home production.

If an individual instead decides to become a job creator, she starts a firm and has to decide how many workers l_t to hire and how many vacancies v_t to open in every period. The cost of posting a vacancy γ is constant over time. There is a cut-off level $\bar{a}_t$ for which all individuals with talent $a \geq \bar{a}_t$ become job creators and the less talented ones become workers. A job creator with talent a posts $v_t(a)$ vacancies and hires $l_t(a)$ workers. The marginal job creator with entrepreneurial talent $\bar{a}_t$ then opens $v_t(\bar{a}_t)$ vacancies and employs $l_t(\bar{a}_t)$ workers. The number of all vacancies in the economy in period t therefore is $V_t = \int_{\bar{a}_t}^{\infty} v_t(a) d\Phi(a)$.

In aggregate, there are $\Phi(\bar{a}_t)$ workers which can be divided into employed workers L_t and unemployed workers N_t. $1 - \Phi(\bar{a}_t)$ then gives the number of job creators because the amount of individuals in the economy is normalised to one. There is a resource constraint on the supply of labour. The fraction engaged in creating jobs plus the fraction engaged as employees has to be smaller or equal to one in each period : $1 - \Phi(\bar{a}_t) + L_t \leq 1$. This can also be written as $L_t \leq \Phi(\bar{a}_t)$ which means that labour demand has to be smaller or equal to labour supply. Consequently, the aggregate number of employed workers in period t can be written as $L_t = \int_{\bar{a}_t}^{\infty} l_t(a) d\Phi(a) = \Phi(\bar{a}_t) - N_t$ and the unemployed are $N_t = \Phi(\bar{a}_t) - L_t$.

How firms and workers come together and form a match is described by the match-

ing technology $m(N_t, V_t)$. It gives the number of aggregate contacts between the mass of vacancies and unemployed workers. The matching function is assumed to be homogeneous of degree one.
The tightness of the labour market is denoted with $\theta_t = \frac{V_t}{N_t}$. The probability to fill an open vacancy per unit of time can then be written as $\frac{m(N_t,V_t)}{V_t} = q(\theta_t)$ with $q'(\theta_t) < 0$. If the labour market tightness increases, it gets more difficult for job creators to fill their vacancies. Moreover, the probability of finding a new job is $\frac{m(N_t,V_t)}{N_t} = \theta_t q(\theta_t)$. It increases if the labour market gets less tight for the unemployed, so $\frac{d[\theta_t q(\theta_t)]}{d\theta_t} > 0$. When workers and firms are matched, production y_t is taking place according to a production function $y_t = af(l_t)$ with $f'(l_t) > 0$ and $f''(l_t) < 0$. A match between a worker and a firm does not have to last forever. There is an exogenous job destruction rate s which is assumed to be constant over time. Once a match is destroyed, the worker becomes unemployed and the job creator has to post a vacancy to hire a new worker.
The dynamics of unemployment therefore are described as follows: $N_{t+1} = (1 - \theta_t q(\theta_t))N_t + s(\Phi(\bar{a}_t) - N_t)$. In every period there are $\theta_t q(\theta_t)$ unemployed people who find a job and leave the unemployment pool and there are s employees who loose their job and join the unemployed.

2.1 The Regular Worker

It is assumed that there is no storage technology and that individuals are risk neutral. An employee receives a wage w_t and an unemployed worker receives unemployment benefits z. The rate of time preference is denoted by β.
One can now set up the value equations for the different types of individuals. The value equation for an employed worker is the following:

$$W_t^e = w_t + \beta \left[sW_{t+1}^n + (1-s)W_{t+1}^e \right].$$

It depends on the current wage and the future value of being a worker. The employed worker becomes unemployed with probability s and stays employed with probability $1 - s$.

The value of being an unemployed worker can be written as

$$W_t^n = z + \beta \left[\theta_t q(\theta_t) W_{t+1}^e + (1 - \theta_t q(\theta_t)) W_{t+1}^n \right].$$

She receives unemployment benefits in the current period and knows that with probability $\theta_t q(\theta_t)$ she finds a job and becomes employed in the next period. Otherwise she stays in unemployment.

2.2 The Firm

If an individual decides to become a job creator her value equation depends on the firm's profit that is maximised by choosing the optimal number of workers and vacancies. After having posted the vacancies and being matched with workers, wage bargaining is taking place. Since multiple workers are bargaining with a firm, the wage setting is more complex than in the standard DMP model. I assume that job creators do not try to decrease wages with their vacancy posting decisions and show that a common wage is paid across firm. The assumption is based on Westermark (2003) who shows that, if contracts are binding, a stationary subgame perfect equilibrium exists, where each worker receives a share of her marginal product and wages are in this sense competitive. Because we have a production function with decreasing returns to scale, a worker that bargains with the firm receives a fraction of her marginal product because all other workers that bargain with the firm as well are treated as employed. This is in contrast to the solution of Stole and Zwiebel (1996). They show that with non-binding contracts and continuous bargaining, workers receive a weighted average of the inframarginal contributions which leads to wage dispersion when firms are heterogeneous. Since I want to show the effects of an endogenous job decision and the potential inefficiencies resulting from it in the simplest and clearest form without introducing an additional channel to depress wages, I rely on the first approach when it comes to the wage bargaining process.[3]

[3]An extension with intra-firm wage bargaining is discussed in section 6.

The firm's maximisation problem for given w_t and $q(\theta_t)$ is the following:

$$W_t^j(a, l_t) = \max_{l_{t+1}, v_t} \left\{ af(l_t) - w_t l_t - \gamma v_t + \beta W_{t+1}^j(a, l_{t+1}) \right\}$$
$$\text{s.t. } l_{t+1} = (1-s)l_t + q(\theta_t)v_t$$

Solving the maximisation problem by using a Lagrange function,[4] one obtains the job creation condition:

$$\frac{\gamma}{q(\theta_t)} = \beta \left[af'(l_{t+1}) - w_{t+1} + (1-s)\frac{\gamma}{q(\theta_{t+1})} \right]. \tag{1}$$

It states that the expected costs of hiring a worker have to be equal to the value generated by having an additional worker. A hired worker increases the firm's production by the marginal product of labour multiplied with the job creator's ability minus the wage that is paid to him or her. With probability $1-s$ the worker stays at the firm in the next period as well and the continuation value has to be added. If one rearranges the job creation condition and defines the surplus of having an additional worker as P_t, so that $P_{t+1} = \frac{\gamma}{\beta q(\theta_t)}$ holds, it can be written as

$$P_t = af'(l_t) - w_t + \beta(1-s)P_{t+1}.$$

P_t is the value of having a match for the firm or the value of an occupied job. Since the first order conditions describing the job creator's optimal choice have to hold for every firm, P_t is the same for each firm no matter how high the entrepreneurial talent a is. There is no wage dispersion across firms. If there are no differences in wages and hiring costs across firms, more able job creators hire more workers so that the marginal product of labour is the same in each firm.

After having posted their vacancies according to the job creation condition, firms and workers are matched randomly according to the matching technology and they bargain about wages. Any wage setting process is consistent with the model as long as the wage is not too low, so that the present value of being unemployed is not higher than the present value of being employed, or too high which makes the

[4] For the derivation of the job creation condition see appendix A.1

match unprofitable for the job creator. It is assumed that wages are determined by the generalized Nash bargaining solution. Workers and job creators bargain about the surplus of the match. The worker's surplus of being employed consists of the difference between the value equations of being employed and unemployed. The job creator instead compares the value of having an additional worker P_t to the option of not employing that worker which is zero since there is free entry of firms. The wage is therefore determined as

$$w_t = \arg\max \ (W_t^e - W_t^n)^{\xi} P_t^{1-\xi}$$

with the worker's bargaining power $\xi \in [0, 1]$. The wage thus has to satisfy the first-order condition

$$\xi P_t = (1 - \xi)(W_t^e - W_t^n). \tag{2}$$

As a result, the solution to the Nash wage bargaining[5] is the wage curve

$$w_t = \xi \left[af'(l_t) + \gamma\theta_t \right] + (1 - \xi)z. \tag{3}$$

The wage consists of the fraction ξ of the worker's marginal product and the hiring costs plus a fraction of the unemployment benefits. Workers are therefore rewarded for the saving of vacancy posting costs since the firm does not have to pay it anymore after a match is formed.[6] I can show that the wage is constant across firms. The first order condition from wage bargaining can be written as

$$\frac{\xi}{1-\xi} \left[af'(l_t) - w_t + \beta(1 - s)P_{t+1} \right] = w_t - z + \beta(1 - s - \theta_t q(\theta_t)) \frac{\xi}{1-\xi} P_{t+1}.$$

The term on the LHS is the same for each firm as can be deducted from (1). Therefore, the term on the RHS also has to be constant. Since z, β, s, θ, ξ and $P_{t+1} = \frac{\gamma}{\beta q(\theta_t)}$ are the same for each firm, the wage has to be constant across firms as well for the above equation to hold.

[5]For derivation see appendix A.2.

[6]The average hiring costs for unemployed workers are $\gamma\theta_t = \frac{\gamma V_t}{N_t}$.

2.3 Decision to Become a Job Creator

The marginal job creator is indifferent between being a worker or becoming a job creator. The profit of the marginal job creator with talent $\bar{a}_t$ has to equal his outside opportunity of being a worker in every period: $W_t^j(l_t, \bar{a}_t) = W_t^e$.[7] Plugging in for the respective value equations leads to the indifference equation

$$\begin{aligned} &\bar{a}_t f(l_t(\bar{a}_t)) - w_t l_t(\bar{a}_t) - \gamma v_t(\bar{a}_t) + \beta W_{t+1}^j(\bar{a}_t, l_{t+1}(\bar{a}_t)) \\ = \; & w_t + \beta \left[s W_{t+1}^n + (1-s) W_{t+1}^e \right]. \end{aligned} \tag{4}$$

The profits of the marginal job creator plus the future value of being a job creator have to be equal to the wage that that individual would earn as an employed worker plus the future value of being a worker which comes with some uncertainty because she can lose her job with probability s.

If a is bounded above, it has to be made sure that at least one individual decides to become a job creator and employs workers. Therefore, the upper bound has to be sufficiently large so that it is more profitable for at least one individual to be a job creator instead of being a regular worker.

Proposition 1 *If a is bounded above, the upper bound a^{up} has to be sufficiently large so that at least one individual decides to become a job creator:*

$$\begin{aligned} &a_t^{up} f(l_t(a_t^{up})) - w_t l_t(a_t^{up}) - \gamma v_t(a_t^{up}) + \beta W_{t+1}^j(a_t^{up}, l_{t+1}(a_t^{up})) \\ \geq \; & w_t + \beta \left[s W_{t+1}^n + (1-s) W_{t+1}^e \right]. \end{aligned}$$

2.4 Market Equilibrium

After having described the model setup and the individuals' optimisation problems, I can define the market equilibrium.

Definition 1 $W_t^j(a, l_t)$, W_t^e, W_t^n, $v_t(a)$, θ_t *and* $\bar{a}_t$ *define a market equilibrium if the following conditions hold for all* t*:*

[7]Here, I assume that the job creator directly becomes an employed worker when the threshold shifts.

- *W_t^e, W_t^n and $W_t^j(a, l_t)$ fulfil the value equations stated above and satisfy* (2)*,*
- *optimal vacancy posting $v_t(a)$ takes place according to the job creation condition* (1)*,*
- *the threshold $\bar{a}_t$ is set in line with the indifference equation* (4)*,*
- *the labour market tightness θ_t is given with $\frac{V_t}{N_t}$.*

The next section introduces the social planner's optimisation problem to characterise the efficient allocation as a benchmark. The market outcome described in the former sections can then be compared to the social optimal allocation.

3 The Social Planner

The social planner wants to maximise the aggregate sum of the utilities of employees, unemployed workers and job creators. The utility functions only depend on consumption and are assumed to be linear. Therefore, they are of the form $u(c_t^i) = c_t^i$ for $i = e, n, j$. An individual consumes c_t^j if it becomes a job creator. Employed workers consume c_t^e and unemployed workers c_t^n. In aggregate, there are $\Phi(\bar{a}_t) - N_t$ employed workers, N_t unemployed workers and $1 - \Phi(\bar{a}_t)$ job creators. The social planner is constrained by a resource constraint, a labour supply constraint and the law of motion for unemployment. His maximisation problem therefore reads

$$\max_{c_t^e, c_t^n, c_t^j, V_t, N_{t+1}, \bar{a}_t, l_t(a)} \sum_{t=0}^{\infty} \beta^t \left[(\Phi(\bar{a}_t) - N_t) u(c_t^e) + N_t u(c_t^n) + \int_{\bar{a}_t}^{\infty} u(c_t^j(a)) d\Phi(a) \right]$$

$$\text{s.t. } (\Phi(\bar{a}_t) - N_t) c_t^e + N_t c_t^n + \int_{\bar{a}_t}^{\infty} c_t^j(a) d\Phi(a) + \gamma V_t = \int_{\bar{a}_t}^{\infty} a f(l(a)) d\Phi(a) + N_t z,$$

$$N_{t+1} = N_t - m(N_t, V_t) + s(\Phi(\bar{a}_t) - N_t),$$

$$\Phi(\bar{a}_t) - N_t = \int_{\bar{a}_t}^{\infty} l_t(a) d\Phi(a).$$

The matching function is assumed to be a Cobb-Douglas matching function:

$$m(N_t, V_t) = N_t^{\alpha} V_t^{1-\alpha}.$$

Since the social planner still faces the matching function and cannot directly match workers with firms without vacancy posting, the outcome of his maximisation problem is a second best outcome. Combining the first order conditions from the maximisation problem and simplifying them,[8] one obtains two important equations that describe the social planner's optimal choice. Firstly, it has to hold that

$$\frac{\gamma}{\beta q(\theta_t)} = (1-\alpha)\left[af'(l_{t+1}(a)) - z\right] - \alpha\gamma\theta_{t+1} + \beta(1-s)\frac{\gamma}{\beta q(\theta_{t+1})} \tag{5}$$

and, secondly,

$$\bar{a}_t f(l_t(\bar{a}_t)) + \frac{s\gamma}{(1-\alpha)q(\theta_t)} - \bar{a}_t f'(l_t(\bar{a}_t))(1 + l_t(\bar{a}_t)) = 0 \tag{6}$$

has to be fulfilled.

Equation (5) states that today's discounted costs of having an additional employed worker have to equal the social benefit of having that additional worker in the firm in the next period plus her future value.

Equation (6) instead describes the optimal choice of $\bar{a}_t$, the threshold above which every individual becomes a job creator. Rearranging (6) to

$$\bar{a}_t f(l_t(\bar{a}_t)) - \bar{a}_t f'(l_t(\bar{a}_t))l_t(\bar{a}_t) = \bar{a}_t f'(l_t(\bar{a}_t)) - \frac{s\gamma}{(1-\alpha)q(\theta_t)},$$

one can see that the threshold has to be set such that the value added to GDP that is attributable to the marginal job creator is equal to her contribution if she had been an employed worker. The contribution as a worker on the RHS of the equation consists of production attributable to that worker reduced by the hiring costs that have to be paid in case the worker loses the job. In constrast, an additional job creator means that there is one additional firm in the economy which opens vacancies, employs workers and contributes to aggregate production. Nevertheless, that job creator is not an employed worker anymore and is not available for production within a firm. Moreover, moving an individual from being a worker to being a job creator affects the tightness of the labour market. It gets harder for the firms to fill

[8] Detailed derivations can be found in the appendix A.3.

vacancies since there is increased competition for fewer workers.
In the following, I will focus on steady states. In steady state, (5) becomes

$$\frac{\gamma}{q(\theta)} = \frac{\beta(1-\alpha)}{1-\beta(1-s)+\alpha\beta\theta q(\theta)}\left[\bar{a}f'(l(\bar{a}))-z\right]. \tag{7}$$

The steady state version of (6) is

$$\bar{a}f(l(\bar{a})) - \bar{a}f'(l(\bar{a}))l(\bar{a}) = \ \bar{a}f'(l(\bar{a})) - \frac{s\gamma}{(1-\alpha)q(\theta)}.$$

Using the elasticity of the production function with respect to labour $\epsilon(\bar{a}) = \frac{\partial af(l)}{\partial l}\frac{l}{af(l)}$, this can be written as

$$\bar{a}f(l(\bar{a}))\left[1-\epsilon(\bar{a})\right] = \ \bar{a}f'(l(\bar{a})) - \frac{s\gamma}{(1-\alpha)q(\theta)}. \tag{8}$$

The RHS determines the value of a worker as already described above. The term on the LHS can be interpreted as the value of a job creator or her contribution to production. It is total production within a firm with a job creator of a certain ability minus the labour share of production because the job creator just provides his or her technology but no labour input. Equation (7) together with equation (8) describe the equilibrium resulting from the social planner's optimisation problem.
In the next section, I describe the steady state market equilibrium and analyse if it is efficient.

4 Efficiency in Market Equilibrium

The market equilibrium in steady state is characterised by the job creation condition, the wage curve, the Beveridge curve, the indifference equation and the value equations for employees, unemployed workers and job creators.
The job creation condition that is derived from the firm's maximisation problem determines labour demand in steady state:

$$af'(l(a)) = w + (1-\beta(1-s))\frac{\gamma\theta}{\beta\theta q(\theta)}.$$

The wage curve in steady state resulting from Nash wage bargaining after a match is formed is given with

$$w = \xi \left[af'(l(a)) + \gamma\theta\right] + (1-\xi)z.$$

Combining both leads to

$$\frac{\gamma}{q(\theta)} = \frac{\beta(1-\xi)}{1-\beta(1-s)+\xi\beta\theta q(\theta)}\left[af'(l(a)) - z\right]. \tag{9}$$

The costs of employing an additional worker have to be equal to the returns of employing that additional worker.
The Beveridge curve states that in steady state the aggregate flows into unemployment have to equal the aggregate flows out of unemployment:

$$s\int_{\bar{a}}^{\infty} l(a)d\Phi(a) = \theta q(\theta)\left[\Phi(\bar{a}) - \int_{\bar{a}}^{\infty} l(a)d\Phi(a)\right].$$

Therefore, unemployment in steady state can be derived as $N = \frac{s\Phi(\bar{a})}{s+\theta q(\theta)}$.
The indifference equation (4) in steady state takes the form

$$\begin{aligned}\bar{a}fl(\bar{a}) - \gamma v(\bar{a}) - wl(\bar{a}) &= w + \beta s(W_n - W_e) \\ &= (1-\beta)W_e.\end{aligned}$$

The marginal job creator is indifferent between being a job creator and receiving the immediate profits of the firm and being an employed worker that earns the wage w but might get unemployed in the future with probability s. Combining the wage curve and the job creation condition, the wage in steady state can be written as

$$w = \frac{\xi}{(1-\xi)}\frac{\gamma\theta}{\beta\theta q(\theta)}\left[1-\beta(1-s)+\beta\theta q(\theta)\right] + z. \tag{10}$$

Using the steady state wage, the value equation for being an employee is

$$(1-\beta)W_e = z + \frac{\xi}{(1-\xi)}\frac{\gamma\theta}{\beta\theta q(\theta)}\left[1-\beta+\beta\theta q(\theta)\right] \tag{11}$$

and the value equation for being an unemployed worker is

$$(1-\beta)W_n = z + \frac{\xi}{1-\xi}\gamma\theta. \qquad (12)$$

By inserting (10), (11) and (12) into the indifference equation in steady state, the marginal job creator's profit in market equilibrium is calculated[9]:

$$\begin{aligned}\bar{a}f(l(\bar{a})) - wl(\bar{a}) - \gamma v(\bar{a}) &= \frac{\xi}{(1-\xi)}\frac{\gamma\theta}{\beta\theta q(\theta)}\left[1-\beta+\beta\theta q(\theta)\right] + z \\ &= w - \beta s\frac{\xi}{(1-\xi)}\frac{\gamma\theta}{\beta\theta q(\theta)}.\end{aligned}$$

In the market equilibrium, the marginal job creator's profit is equal to the wage she would earn as an employed worker minus the discounted difference between the value of being employed and being unemployed[10] multiplied with the probability of loosing the job s. The marginal job creator's profits are lower than an employed worker's instantaneous income but as an employed worker there is always the danger of becoming unemployed in which case the worker would earn less than the marginal job creator. The following definition thus describes the steady state market equilibrium.

Definition 2 *The steady state market equilibrium is characterised by w, θ, N and $\bar{a}$ that fulfil*

- *the wage curve:* $w = \xi\left[af'(l(a)) + \gamma\theta\right] + (1-\xi)z$
- *the job creation condition:* $\frac{\gamma}{q(\theta)} = \frac{\beta}{1-\beta(1-s)}\left[af'(l) - w\right]$
- *the Beveridge curve:* $N = \frac{s\Phi(\bar{a})}{s+\theta q(\theta)}$
- *the indifference equation:*
  $\bar{a}f(l(\bar{a}))\left[1-\epsilon(\bar{a})\right] = \frac{(1-\beta)}{\beta(1-\xi)}\frac{\gamma}{q(\theta)}\left[\xi - (1-\xi)l(\bar{a})\right] + \frac{\xi}{1-\xi}\gamma\theta + z$

The wage curve and the job creation condition have a unique intersection in a (θ, w)-space and thus pin down the unique equilibrium for wages and labour market tightness. The wage increases linearly in tightness whereas the job creation con-

[9]For the derivation of the steady state wage, the value equations and the job creator's profits see appendix A.4

[10]Using (11) and (12), one can calculate $W_e - W_n = \frac{\xi}{(1-\xi)}\frac{\gamma\theta}{\beta\theta q(\theta)}$.

dition is convex and decreases in θ. If wages are higher, firms create fewer jobs and hence there are fewer vacancies per worker. The equilibrium for vacancies and unemployment is determined by the job creation condition and the Beveridge curve. In a (u, v)-space, the job creation condition is upward sloping and uniquely intersects with the Beveridge curve which is convex and downward sloping in u. As the number of posted vacancies increases, unemployment decreases because it is easier to be matched with a firm. The unique equilibrium threshold $\bar{a}$ is determined by the intersection of the value of the marginal job creator that is increasing in $\bar{a}$ with the value of a worker that decreases in $\bar{a}$. For a further discussion see chapter 4.1.
To compare the market equilibrium with the social planner's solution, it is convenient to reformulate the indifference equation[11] to

$$\bar{a}f(l(\bar{a}))\left[1-\epsilon(\bar{a})\right]+\frac{(1-\beta)}{\beta}\frac{\gamma(1+l(\bar{a}))}{q(\theta)}=\bar{a}f'(l(\bar{a}))-\frac{s\gamma}{(1-\xi)q(\theta)}. \tag{13}$$

Together with equation (9) it describes the equilibrium in the decentralised market. I now compare the conditions describing the steady state market equilibrium to the conditions for an optimal allocation in the social planner's setting.

First, if one compares (9) and (7) it is obvious that $\xi = \alpha$ for efficient job creation if we assume that the threshold $\bar{a}$ is the same in the market as in the efficient case. This result is the so called Hosios condition[12] which is a common outcome in the literature when analysing the efficiency of DMP-models. The market solution is efficient c.p. when the private returns of a match ξ are equal to the social returns α. If for example $\xi < \alpha$, job creators create too many vacancies compared to the efficient situation because their returns of a match exceed the socially optimal returns. Thus, equilibrium unemployment is too low. The job creators in decentralised markets do not take into account that their creation of jobs poses a negative externality on other job creators, since it gets harder for them to fill their own vacancies.
Secondly, if we assume that the Hosios conditions holds, we can determine whether the allocation of individuals into workers and job creators who open up a firm is effi-

[11] The derivation can be found in appendix A.4.1.
[12] See Hosios (1990).

cient. Comparing the indifference equation that conditions the threshold $\bar{a}$ in market equilibrium (13) to equation (8) which determines the optimal threshold, an extra term appears on the LHS of the equation for the market equilibrium. It suggests that production of the marginal job creator's firm is smaller in the decentralised market which indicates that the marginal job creator is of lower ability than what would be optimal. Moreover, a conjecture is that job creators in the decentralised market capture more of the production within their firm due to wage bargaining compared to the amount that the social planner would allocate to them. Therefore, too many individuals in the market decide to become job creators. To compare the threshold $\bar{a}$ in market equlibrium and social planner's solution more carefully, it is helpful to eliminate $\bar{a}f'(l(\bar{a}))$ from both equations. This is done in the next section.

4.1 The Optimal Threshold $\bar{a}$

In this section, I will first describe how the optimal number of job creators is set by the social planner and then compare it to the amount of job creators in market equilibrium.

Solving equation (7) for $\bar{a}f'(l(\bar{a}))$ and plugging it into (8), one obtains the production of the marginal job creator's firm just depending on the market tightness and exogenously given parameters:

$$\bar{a}f(l(\bar{a}))\left[1-\epsilon(\bar{a})\right] = \frac{(1-\beta)}{\beta(1-\alpha)}\frac{\gamma}{q(\theta)} + \frac{\alpha}{(1-\alpha)}\gamma\theta + z. \tag{14}$$

The LHS of this condition can be interpreted as the value of a job creator whereas the RHS is the value of a worker. When an individual decides to be a job creator, he or she cannot be a worker anymore. As job creator an individual provides the technology a but does not supply any labour for production within the firm. Therefore, to calculate the job creator's contribution to production within a firm, the labour share of production $\epsilon(a)$ has to be deducted from the firm's overall production. As a worker instead, the individual would at least produce z and would have an influence on the labour market tightness which potentially saves vacancy posting costs. If more individuals decide to become workers, the labour market tightness decreases

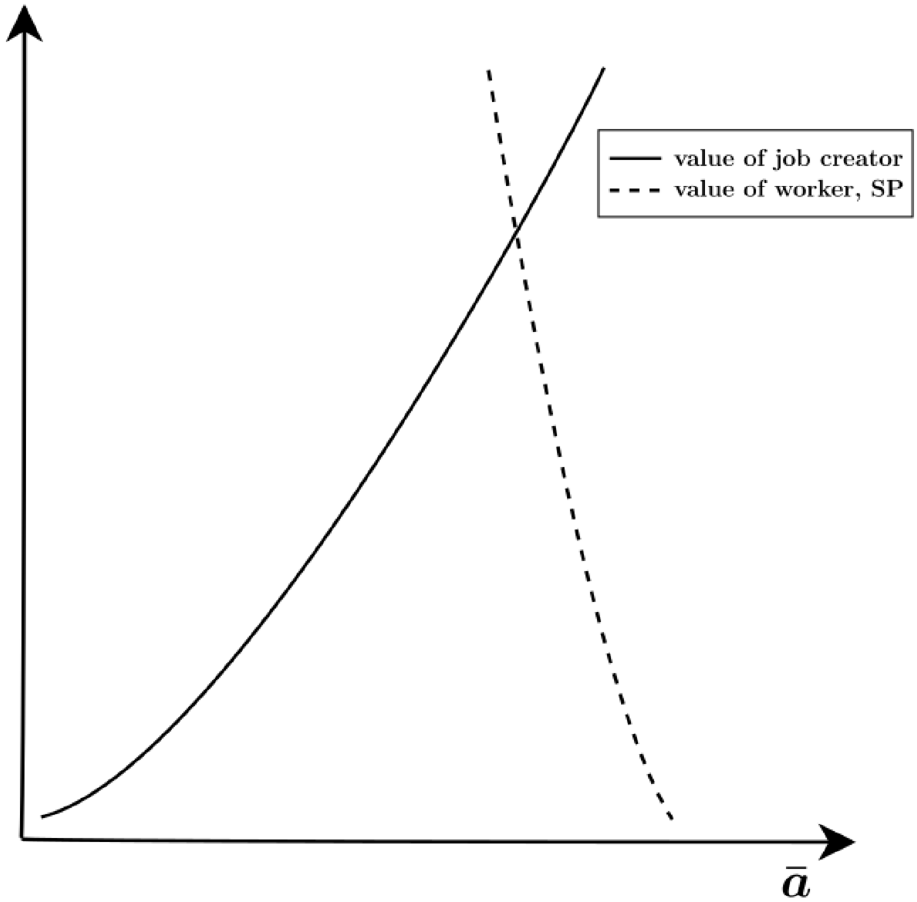

Figure 1: *Optimal threshold $\bar{a}$*

which makes it easier for job creators to fill their vacancies. Therefore, they do not have to post so many vacancies in excess just to make sure that some workers are hired and save costs. Moreover, if more individuals become workers there are fewer job creators per se which lowers the competition among them for workers. In equilibrium, the value of a job creator has to be equal to the value of a worker which determines the optimal number of job creators as can be seen in Figure 1.

Firstly, I focus on a partial equilibrium analysis. For given l and v, the LHS of the above equation increases in $\bar{a}$ whereas the RHS decreases. This is also depicted in figure 1, where the value of being a job creator is an increasing function of $\bar{a}$ and the costs of job creation decrease in $\bar{a}$. As $\bar{a}$ increases, the value of production increases as well. A higher $\bar{a}$ means that there is one less firm but one more worker available in the economy. Therefore, the hiring costs decrease ceteris paribus since it gets easier for the remaining firms to fill their vacancies. Put differently, labour supply increases if the former marginal job creator decides to become a worker. The market tightness decreases since there are more available workers and less firms looking for a worker. Hiring costs decrease. With lower hiring costs and unchanged policy functions, labour demand increases and a new equilibrium is established.

In general equilibrium, if $\bar{a}$ increases by a small amount, the equation above shows that θ has to increase since the LHS is clearly increasing in $\bar{a}$ and the RHS can just increase if θ gets larger. The tightness of the labour market is therefore increasing in $\bar{a}$. A higher $\bar{a}$ means that the marginal product $\bar{a}f'(l(\bar{a}))$ increases and with it the marginal product in all firms. Wages increase as well but not as much as productivity itself. At higher productivity, the profit from job creation is higher because the wages do not fully absorb the increase in productivity. Therefore, job creators post more vacancies which lowers unemployment. To sum up, the optimal allocation of individuals into workers and job creators has to ensure that the value of the marginal job creator for the economy has to equal the value of that individual being a worker.

Now, I determine how the threshold $\bar{a}$ is set in the decentralised market. Solving (9) for the marginal product of the marginal job creator's firm in the market equilibrium and inserting it into (13), one obtains

$$\bar{a}f(l(\bar{a}))\left[1-\epsilon(\bar{a})\right] = \frac{(1-\beta)}{\beta(1-\xi)}\frac{\gamma}{q(\theta)}\left[\xi-(1-\xi)l(\bar{a})\right] + \frac{\xi}{(1-\xi)}\gamma\theta + z. \tag{15}$$

The term on the left side is the value of the marginal job creator and the right side defines the value of a worker in the market equilibrium. This indifference equation differs from (14) just in the first term on the RHS which is multiplied by $[\xi-(1-\xi)l(\bar{a})]$ which is strictly smaller than one if $0<\xi<1$. Assuming that $\xi=\alpha$ holds, the threshold $\bar{a}$ and θ are smaller in the market equilibrium than in the social planner's equilibrium. Figure 2 shows that for given parameter values the threshold $\bar{a}$ in market equilibrium is smaller than in the social planning problem. There are too many firms compared to the efficient situation, since the value of being a worker is too low in the market equilibrium which makes being a job creator more profitable. Because of the wage bargaining process workers in the decentralized

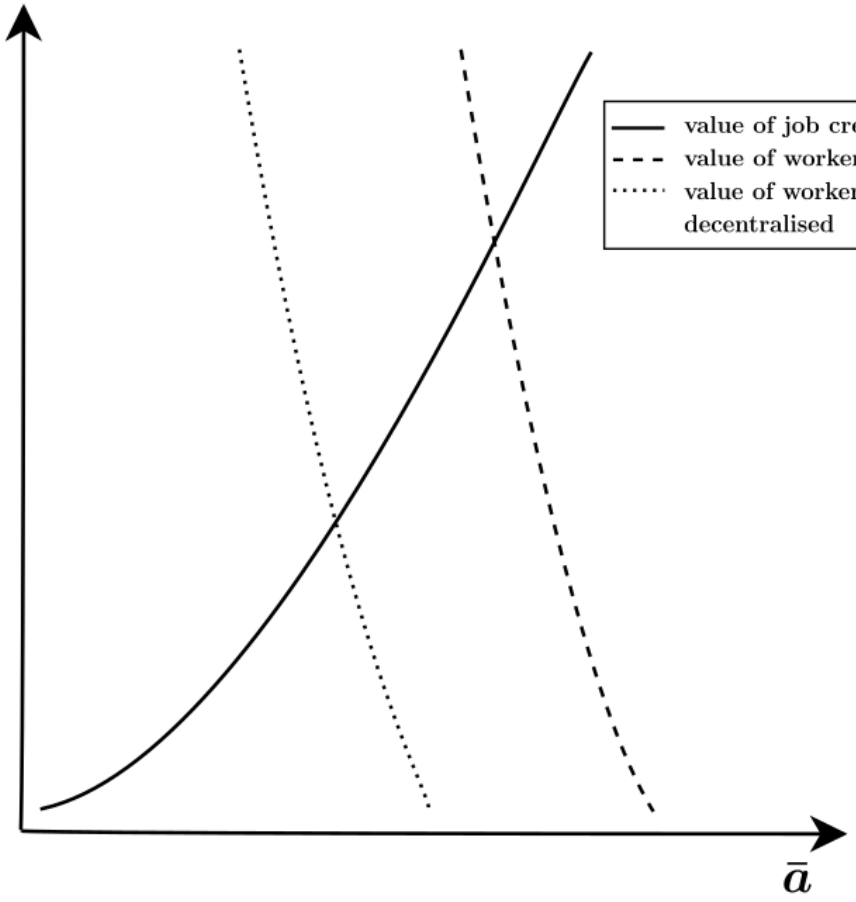

Figure 2: *Threshold $\bar{a}$ in market equilibrium compared to $\bar{a}$ in social planning solution*

market receive a smaller part of the surplus of a match than what is allocated to them by the social planner. Put differently, job creators can acquire an inefficiently large part of the surplus. Moreover, job creators with lower ability do not take into account the negative effect that they are having on the marginal product of more efficient firms. For an individual with a low a it might be optimal to become a job creator but it is inefficient for the whole economy since that job creator competes for workers with other firms. Within these other firms with a more talented job creator, an employed worker would contribute more to overall production. Job creators in the decentralised market are able to acquire a larger part of the worker's marginal product through wage bargaining and a job creator with a given ability for job creation makes higher profits in the market than in the social planner's setting. It is therefore profitable for an individual with low a to be a job creator in the market equilibrium whereas instead the social planner assigns her to become a worker.

Assuming that the Hosios condition holds, the threshold $\bar{a}$ in the market gets closer to the efficient threshold the larger ξ gets. If the worker's bargaining power increases, the job creator cannot acquire that much of the worker's marginal product and the saved vacancy posting costs.

It has to be determined whether there is a market equilibrium that is efficient. Comparing (7), (8), (9) and (13) it is not possible that the threshold $\bar{a}$ in the social planner problem equals the one in the market equilibrium and at the same time $\xi = \alpha$ holds.[13]

If $\xi = \alpha$ holds, the market equilibrium is still inefficient since too many individuals decide to become job creators. If instead the Hosios condition does not hold but the threshold $\bar{a}$ is the same in the market as in the social planner's allocation, the number of job creators in the market is efficient but these job creators inefficiently post too many or too few vacancies. Therefore, the market equilibrium is in general inefficient and taxation might be useful to restore efficiency.

5 Taxation

This chapter analyses how taxation can be used to restore the constrained first best allocation. Job creators' incomes are taxed with a marginal tax rate τ_t^j, workers' incomes with the marginal tax rate τ_t^e and the tax revenue is used for wasteful government spending.[14]

The firm's optimisation problem thus becomes

$$W_t^j(a, l_t) = \max_{l_{t+1}, v_t} \left\{ (1 - \tau_t^j) \left[af(l_t) - w_t l_t - \gamma v_t \right] + \beta W_{t+1}^j(a, l_{t+1}) \right\}$$
$$\text{s.t. } l_{t+1} = (1 - s) l_t + q(\theta_t) v_t.$$

[13] The market equilibrium is of course efficient when $\xi = \alpha = 1$. This is the trivial case when we have efficient matching e.g. when every unemployed worker directly finds a job. Then there are no labour market frictions and no unemployment.

[14] The job creator's taxable income is the net profit from the firm whereas the worker's taxable income is her labour income. I abstract from corporate taxation.

Setting up the Lagrange function, deriving the first order conditions and using the Envelope theorem, the condition for optimal job creation is

$$\frac{(1-\tau_t^j)\gamma}{\beta q(\theta_t)} = (1-\tau_{t+1}^j)\left[af'(l_{t+1}) - w_{t+1}\right] + (1-s)\frac{\beta(1-\tau_{t+1}^j)\gamma}{\beta q(\theta_{t+1})}.$$

Assuming that the tax rate is fixed over time, the job creation condition is the same as in the case without taxation because the net of tax rates cancel out. Job creation therefore is not distorted since wages and vacancy posting costs can be deducted from taxed gross profits. Defining $P_{t+1} := \frac{\gamma}{\beta q(\theta_t)}$ as above, the condition can be written as

$$P_t = af'(l_t) - w_t + \beta(1-s)P_{t+1}$$

which is the same formulation as in section 2.2. The workers' value equations instead change when we introduce the taxation of incomes. The value equation for an employed worker changes to

$$W_t^e = (1-\tau_t^e)w_t + \beta\left[sW_{t+1}^n + (1-s)W_{t+1}^e\right]$$

and the value of being an unemployed worker stays

$$W_t^n = z + \beta\left[\theta_t q(\theta_t)W_{t+1}^e + (1-\theta_t q(\theta_t))W_{t+1}^n\right]$$

because home production is not taxed. Using the above value equations and the surplus of a match for the job creator, the Nash wage bargaining result[15] is different compared to the case without taxes since vacancy creation is not distorted but employed and unemployed workers are affected differently by taxation. The wage curve therefore is

$$w_t = \xi\left[af'(l_t) + \gamma\theta_t\right] + (1-\xi)\frac{z}{(1-\tau^e)}$$

[15]See appendix A.5 for a derivation. I assume that workers and job creators bargain about gross wages.

and increases in τ^e. If employed workers' incomes are taxed heavier, the outside option becomes more attractive and therefore the wage has to increase as well. In the following, I again focus on steady states to calculate the marginal tax rate that restores the social planner's allocation. The indifference equation for the marginal job creator becomes

$$\begin{aligned}(1-\tau^j)\left[\bar{a}f(l(\bar{a}))-wl(\bar{a})-\gamma v(\bar{a})\right] &= (1-\tau^e)w+\beta\left[sW^n+(1-s)W^e\right]\\ &= (1-\tau^e)w-\frac{\beta s\left[(1-\tau^e)w-z\right]}{1-\beta(1-s)+\beta\theta q(\theta)}.\end{aligned}$$

Plugging in for w and using the job creation condition, it can be rearranged to

$$(1-\tau^j)\left[\bar{a}f(l(\bar{a}))-wl(\bar{a})-\gamma v(\bar{a})\right]=(1-\tau^e)\frac{\xi}{(1-\xi)}\frac{\gamma}{\beta q(\theta)}\left[1-\beta+\beta\theta q(\theta)\right]+z$$

and gives us the net income of the marginal job creator depending only on exogenously given parameters and labour market tightness. Substituting further for the wage on the LHS of the equation and substituting for $\bar{a}f'(l)$, the indifference equation can also be written as

$$\begin{aligned}&\frac{(1-\tau^j)}{(1-\tau^e)}\bar{a}f(l(\bar{a}))\left[1-\epsilon(\bar{a})\right]\\ &\quad=\frac{(1-\beta)}{(1-\xi)}\frac{\gamma}{\beta q(\theta)}\left[\xi-\frac{(1-\tau^j)}{(1-\tau^e)}(1-\xi)l(\bar{a})\right]+\frac{\xi}{(1-\xi)}\gamma\theta+\frac{z}{(1-\tau^e)}.\end{aligned}$$

Rearranging thus gives

$$\begin{aligned}&\bar{a}f(l(\bar{a}))\left[1-\epsilon(\bar{a})\right]\\ &\quad=\frac{(1-\beta)}{\beta(1-\xi)}\frac{\gamma}{q(\theta)}\left[\frac{\xi(1-\tau^e)}{1-\tau^j}-(1-\xi)l(\bar{a})\right]+\frac{\xi(1-\tau^e)}{(1-\xi)(1-\tau^j)}\gamma\theta+\frac{z}{1-\tau^j}\end{aligned} \tag{16}$$

which is easily comparable to (14) which gives us the efficient number of job creators. We firstly assume that $\tau^e=0$, so that just job creators' incomes are taxed. The τ^j that restores the first best allocation under the assumption that the Hosios condition holds therefore is

$$\tau^j = 1 - \left[\frac{\frac{\xi(1-\beta)}{(1-\xi)} \frac{\gamma}{\beta q(\theta^M)} + \frac{\xi}{(1-\xi)} \gamma \theta^M + z}{\frac{(1-\beta)}{(1-\xi)} \frac{\gamma}{\beta q(\theta^S)} + \frac{(1-\beta)}{\beta} \frac{\gamma}{q(\theta^M)} l(\bar{a}) + \frac{\xi}{(1-\xi)} \gamma \theta^S + z} \right].$$

θ^M denotes the labour market tightness in the market equilibrium whereas θ^S is the market tightness in the first best setting. If they are identical[16], one receives

$$\tau^j = \frac{(1-\beta)(1-\xi)\gamma\left[1 + l(\bar{a})\right]}{(1-\beta)\gamma\left[1 + (1-\xi)l(\bar{a})\right] + \beta q(\theta)\left[\xi\gamma\theta + (1-\xi)z\right]}. \tag{17}$$

The tax rate acts as a Pigouvian tax and restores the first best allocation. There are too many job creators in the market equilibrium without taxation. A tax that makes being a job creator less attractive relative to being a worker is therefore efficiency enhancing. Profit taxation makes it unprofitable for rather unproductive job creators to stay job creators. For them it is now better to become a worker and the number of job creators decreases. If there are fewer job creators and more workers, the labour market tightness decreases and it gets easier for the remaining firms with better technology to fill their vacant positions.

The above tax rate is clearly increasing in $l(\bar{a})$, so it is increasing in the number of workers that are hired by the marginal job creator. It also increases in θ if the unemployment benefit is sufficiently high. The tighter the labour market is, the higher the tax can be since it induces more job creators to become workers which in turn relaxes the labour market. Moreover, the tax rate decreases in ξ. A higher ξ means that workers have a higher bargaining power in the wage negotiations. Job creators therefore can just acquire a smaller part of the worker's marginal product and thus the tax rate on the firms' profits decreases.

The total tax payment that the marginal job creator has to make is given with

$$\begin{aligned} T(\bar{a}) &= \frac{\tau^j}{(1-\tau^j)} \left[\frac{\xi}{(1-\xi)} \frac{\gamma}{\beta q(\theta)} \left[1 - \beta + \beta\theta q(\theta)\right] + z \right] \\ &= \frac{1-\beta}{\beta} \frac{\gamma}{q(\theta)} (1 + l(\bar{a})). \end{aligned}$$

[16] If $\xi = \alpha$ and if $\bar{a}$ is the same in the social planner's allocation and the market equilibrium which is the aim of the introduction of taxation, conditions (7) and (9) tell us that $\theta^M = \theta^S = \theta$.

The gross income of the marginal job creator can be calculated as

$$\frac{(1-\beta)\gamma\left[1+(1-\xi)l(\bar{a})\right]}{\beta(1-\xi)q(\theta)}+\frac{\xi}{(1-\xi)}\gamma\theta+z.$$

It is increasing in $l(\bar{a})$, hence a more able job creator has a higher gross income. She also has to make a larger total tax payment which is increasing in the number of hired workers as well as can be seen above. Introducing taxation does not change the net income of the marginal job creator but reduces the number of job creators in the economy. The marginal job creator who is receiving the same net income as the marginal job creator in the case without taxation is now an individual with a higher talent for job creation.

Now, I assume that workers' incomes are taxed as well with a given fixed tax rate τ^e. Given any $\tau^e \in [0,1]$, the tax rate on job creators' profits that restores the first best allocation can be calculated[17] by comparing (14) to (16):

$$\tau^j=\frac{(1-\beta)(1-\xi)\gamma\left[1+l(\bar{a})\right]+\tau^e\xi\gamma\left[1-\beta+\beta\theta q(\theta)\right]}{(1-\beta)\gamma\left[1+(1-\xi)l(\bar{a})\right]+\beta q(\theta)\left[\xi\gamma\theta+(1-\xi)z\right]}. \tag{18}$$

This tax rate is clearly larger than in the case with $\tau^e = 0$ and increasing in τ^e. The introduction of taxation of workers' incomes leads to a heavier taxation of job creators' profits. If workers' incomes are taxed additionally, becoming a job creator gets more attractive and therefore τ^j has to increase to counteract this effect. The larger τ^e is, the larger the tax rate on profits has to be so that the relative attractiveness of being a worker instead of being a job creator is at its efficient level. Introducing the taxation of workers' incomes makes being a worker less attractive. Therefore, τ^j has to increase even more, so that more individuals decide to become a worker compared to the laissez-faire market equilibrium. The calculated τ^j also restores the first-best allocation for any given τ^e since the labour market and vacancy posting are not distorted. Since the above formula shows us that τ^j is larger than τ^e for not too high values of τ^e, it is evidence that progressive taxation is needed in this framework to establish efficiency.

[17]A detailed derivation can be found in appendix A.5.

The above derivation of τ^j was done under the assumption that the Hosios condition holds.[18] If the Hosios condition does not hold, we would of course need a second tax policy which aims at correcting the vacancy posting decisions of the single firms. If as example $\xi > \alpha$, so the private returns of a match for the worker are larger than the social returns, job creators post too few vacancies and unemployment is higher than in the efficient situation. Job creators do not take into account the negative externality that they exercise on the labour market. Subsidizing vacancy posting would then be a way to correct for that inefficiency. By taxing or subsidizing vacancy creation, it can be assured that the Hosios condition holds. If we have two channels that cause inefficiencies, we would need two distinct tax instruments to correct for them.

To sum up, I can calculate tax rates on job creators' profits that restore the first-best allocation with and without including the taxation of workers' incomes as well. Moreover, I find some indication for a progressive tax system since equation (18) shows that $\tau^j \geq \tau^e$ for reasonable parameter values and reasonable values of τ^e. A progressive tax system in the described model can be justified by pure efficiency arguments. It corrects for the private decision of too many individuals to become a job creator which causes a loss in welfare because of inefficiently high vacancy posting costs that are caused by a tight labour market. Moreover, a second tax policy can correct for inefficient vacancy posting by subsidizing or taxing the named and directly targeting the originator of the positive or negative externality.

6 Extension: Intra-firm Wage Bargaining

If the production function exhibits decreasing returns in labour productivity, job creators can exploit the diminishing returns to manipulate wages as is shown by Stole and Zwiebel (1996). In their model, wage setting is an ongoing process within the firm because contracts are non-binding and workers can quit the firm at any time.

[18] In appendix A.5.1, I calculate the marginal tax rate when the Hosios condition is violated. Whether the progressivity result still holds depends on α, ξ, the socially optimal labour market tightness and the labour market tightness in the market equilibrium. I argue, that a progressive tax system still arises when $\alpha > \xi$ and when z and τ^e are not too large.

Therefore, it is optimal for firms to overhire workers and to decrease the marginal product of labour to lower the wages of the incumbent workers and acquire larger rents. This section extends the described model with intra-firm wage bargaining. Including the notion that firms can directly influence wages with their hiring decision, the maximisation problem of a firm has the following form:

$$W_t^j(a, l_t) = \max_{l_{t+1}, v_t} \left\{ af(l_t) - w_t(a, l_t)l_t - \gamma v_t + \beta W_{t+1}^j(a, l_{t+1}) \right\}$$
$$\text{s.t. } l_{t+1} = (1 - s)l_t + q(\theta_t)v_t.$$

Solving the maximisation problem, one receives the job creation condition which is given with

$$\frac{\gamma}{q(\theta_t)} = \beta \left[af'(l_{t+1}) - \frac{\partial w_{t+1}(a, l_{t+1})}{\partial l_{t+1}} l_{t+1} - w_{t+1}(a, l_{t+1}) + (1 - s)\frac{\gamma}{q(\theta_{t+1})} \right].$$

Comparing it to (1), it know includes the derivative of the wage with respect to labour multiplied with labour input. One can show that the instantaneous marginal value of employing an additional worker is constant across firms because the average costs of employing an additional worker are the same for all job creators.[19] After firms and workers are matched randomly, they bargain about wages. Applying Nash wage bargaining, the wage curve is calculated as

$$w_t(a, l_t) = \xi \left[af'(l_t) - \frac{\partial w_t(a, l_t)}{\partial l_t} l_t + \gamma\theta_t \right] + (1 - \xi)z.$$

It now includes the effect that the hiring of an additional worker is having on wages. I assume that the production function exhibits decreasing returns to scale and is given with

$$f(l_t) = l_t^\eta.$$

[19] This is shown in appendix A.6.

The wage thus becomes

$$w_t(a, l_t) = \xi \left[\frac{\eta a l_t^{\eta-1}}{1 - \xi(1-\eta)} + \gamma \theta_t \right] + (1-\xi) z$$

and deviates from the wage in the model without intra-firm bargaining (if we use the same production function) just in the fraction $\frac{1}{1-\xi(1-\eta)} > 1$ that the marginal product is multiplied with. That means that for given employment wages are higher in the model with intra-firm wage bargaining than in the model without it because hiring an additional worker has a higher value for the firm since it decreases wages for the already employed workers within the firm. Nevertheless, since increasing employment reduces the wage bill, firms post more vacancies. The marginal product of workers decreases with increased hiring which in turn lowers the wages paid to them. Moreover, one can show that the wage is constant across all firms as in the model without intra-firm bargaining.[20] Krause and Lubik (2013) show that the aggregate effects of intra-firm wage bargaining are small in a matching framework with concave production functions and downward-sloping demand curves. Therefore, intra-firm wage bargaining is neglected in the previous sections. Considering it would complicate the analysis since another channel would be introduced that influences the individuals' job decisions by making being a job creator even more attractive because they can additionally depress wages and extract rents.

7 Conclusion

In this paper, I have shown that the market equilibrium features an inefficiently high number of job creators. Since job creators can acquire a large part of the surplus from a match with a worker, it is optimal for an individual to become a job creator in the decentralised market whereas the social planner would assign that individual to become a regular worker. In an economy with an imperfect labour market, job creators can acquire a disproportionately large part of the firms' revenue. When deciding about the job, individuals who become job creators do not consider their

[20] See appendix A.6.

effect on overall labour market tightness. If there is an additional job creator, there is one less worker available in the workforce making it more difficult for other job creators to hire the remaining workers. If an individual with a mediocre talent for job creation decides to become a job creator she competes with more productive firms for the available workers who would contribute more to production in a more productive firm with a better job creator. This overall effect on total production is not taken into account in individual utility maximisation. Inefficiencies in the market may also arise because job creators do not take into account the negative effect that their vacancy posting choice is exerting on other job creators. Throughout the paper, I assume that the Hosios condition holds and job creation thus is efficient. If the Hosios condition is violated, a second tax instrument is needed to correct for it. To sum up, it is shown that the value of being a job creator is too high in the market equilibrium compared to the efficient situation.

Having described the above problem, I calculate a Pigouvian tax on the job creators' profits that corrects for the externalities and increases the costs of engaging in job creation. It restores the first best allocation without distorting the labour demand and vacancy posting choice of individual firms. Moreover, the additional introduction of taxation of workers' incomes leads to a higher tax rate on profits. The marginal tax rate for job creators is increasing in the tax rate on workers' incomes since the taxation of workers' labour incomes makes it less attractive to become a worker relative to being a job creator. The tax rate on profits therefore has to increase to restore the efficient relative attractiveness between both professions. Additionally, I can show that the marginal tax rate on job creators' incomes is larger than the one on workers' incomes for not unrealistically high values of τ^e. This is a first hint on the effectiveness of a progressive tax system in the described setting. Since the analysis in this paper is limited on a steady state analysis, future work should concentrate on characterising complete policy functions and transition paths that occur from tax reforms. Then it must be possible to investigate the progressiveness of a complete tax schedule in more detail. Moreover, the model can be extended with various features influencing the taxation of job creators. One can argue that job creators might face a higher risk when opening up a firm than regular workers. Additionally, potential entrepreneurs might be exposed to financial frictions and are restricted by collateral constraints. Allowing for an open economy and the mobility of job creators might also be an interesting extension of the above model to include international tax competition. All these mentioned expansions rather speak in favour of lowering taxes on incomes from job creation. Therefore, it might be interesting to study the trade-offs of these different channels influencing the optimal taxation of job creators' incomes.

A Appendix

A.1 The Firm's Maximisation Problem

The Lagrangian function from the maximisation problem of the firm has the form

$$\mathcal{L} = af(l_t) - w_t l_t - \gamma v_t + \beta W^j_{t+1}(a, l_{t+1}) + \lambda \left[(1-s)l_t + q(\theta_t)v_t - l_{t+1}\right]$$

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial v_t} = -\gamma + \lambda q(\theta_t) = 0$$
$$\frac{\partial \mathcal{L}}{\partial l_{t+1}} = \beta \frac{\partial W^j_{t+1}(a, l_{t+1})}{\partial l_{t+1}} - \lambda = 0$$

Combining the first order conditions, one receives

$$\frac{\gamma}{q(\theta_t)} = \beta \frac{\partial W^j_{t+1}(a, l_{t+1})}{\partial l_{t+1}}.$$

Finally, using the Envelope condition

$$\frac{\partial W^j_t(a, l_t)}{\partial l_t} = af'(l_t) - w_t + \lambda(1-s)$$

one gets to the job creation condition.

A.2 Nash Wage Bargaining

The wage in a Nash wage bargaing process solves $w_t = \arg\max \ (W^e_t - W^n_t)^{\xi} P_t^{1-\xi}$. The FOC is

$$\xi P_t \frac{\partial (W^e_t - W^n_t)}{\partial w_t} + (1-\xi)(W^e_t - W^n_t)\frac{\partial P_t}{\partial w_t} = 0.$$

Calculating the derivatives and rearranging, the FOC becomes

$$\xi P_t = (1-\xi)(W^e_t - W^n_t) \tag{A.1}$$

The surplus of a match for the firm is

$$P_t = af'(l_t) - w_t + \beta(1-s)P_{t+1}$$

and for the worker it is

$$W_t^e - W_t^n = w_t - z + \beta(1 - s - \theta_t q(\theta_t))(W_{t+1}^e - W_{t+1}^n).$$

Using (A.1), we can write the surplus of a match for the worker also as

$$W_{t+1}^e - W_{t+1}^n = \frac{\xi}{1-\xi} P_{t+1}.$$

Plugging in the value equations, (A.1) becomes

$$\xi \left[af'(l_t) - w_t + \beta(1-s)P_{t+1}\right] = (1-\xi)\left[w_t - z + \beta(1 - s - \theta_t q(\theta_t))\frac{\xi}{(1-\xi)} P_{t+1}\right]$$

$$\Leftrightarrow \; \xi af'(l_t) + \xi\beta\theta_t q(\theta_t) P_{t+1} + (1-\xi)z = w_t.$$

Using $P_{t+1} = \frac{\gamma}{\beta q(\theta_t)}$ from the job creation condition, the wage curve can be derived:

$$w_t = \xi\left[af'(l_t) + \gamma\theta_t\right] + (1-\xi)z.$$

A.3 Social Planner's Maximisation Problem

The Lagrange function of the social planner's maximisation problem is

$$\begin{aligned}
\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Bigg\{ & \Phi(\bar{a}_t) - N_t)u(c_t^e) + N_t u(c_t^n) + \int_{\bar{a}_t}^{\infty} u(c_t^j(a))d\Phi(a) \\
& + \lambda_t \left[\int_{\bar{a}_t}^{\infty} af(l_t(a))d\Phi(a) + N_t z - (\Phi(\bar{a}_t) - N_t)c_t^e - N_t c_t^n - \int_{\bar{a}_t}^{\infty} c_t^j(a)d\Phi(a) - \gamma V_t\right] \\
& + \mu_t \left[N_{t+1} - N_t - s(\Phi(\bar{a}_t) - N_t) + m(N_t, V_t)\right] \\
& + \nu_t \left[\Phi(\bar{a}_t) - N_t - \int_{\bar{a}_t}^{\infty} l_t(a)d\Phi(a)\right] \Bigg\}
\end{aligned}$$

with λ_t being the Lagrange multiplier on the resource constraint, μ_t being the multiplier for the law of motion of unemployment and ν_t as multiplier on the labour supply constraint.

The FOCs are:

$$\frac{\partial \mathcal{L}}{\partial c_t^e} = 1 - \lambda_t = 0$$
$$\frac{\partial \mathcal{L}}{\partial c_t^n} = 1 - \lambda_t = 0$$
$$\frac{\partial \mathcal{L}}{\partial c_t^j(a)} = 1 - \lambda_t = 0$$
$$\frac{\partial \mathcal{L}}{\partial V_t} = -\lambda_t \gamma + \mu_t \frac{\partial m(N_t, V_t)}{\partial V_t} = 0$$
$$\frac{\partial \mathcal{L}}{\partial l_t(a)} = \lambda_t a f'(l_t(a)) - \nu_t = 0$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial N_{t+1}} &= \mu_t + \beta \left[u(c_{t+1}^n) - u(c_{t+1}^e)\right] + \beta\lambda_{t+1}\left[z + c_{t+1}^e - c_{t+1}^n\right] \\ &- \beta\mu_{t+1}\left[1 - s - \frac{\partial m(N_{t+1}, V_{t+1})}{\partial N_{t+1}}\right] - \beta\nu_{t+1} = 0\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \bar{a}_t} &= \phi(\bar{a}_t)u(c_t^e) - \phi(\bar{a}_t)u(c_t^j(\bar{a}_t)) \\ &+ \lambda_t\left[-\phi(\bar{a}_t)\bar{a}_t f(l_t(\bar{a}_t)) - \phi(\bar{a}_t)c_t^e + \phi(\bar{a}_t)c_t^j(\bar{a}_t) + \phi(\bar{a}_t)\gamma v_t(\bar{a}_t)\right] \\ &+ \mu_t\left[-\phi(\bar{a}_t)\frac{\partial m(N_t, V_t)}{\partial V_t} v_t(\bar{a}_t) - \phi(\bar{a}_t)s\right] + \nu_t\left[\phi(\bar{a}_t) + \phi(\bar{a}_t)l_t(\bar{a}_t)\right] = 0\end{aligned}$$

The matching function is assumed to be of Cobb-Douglas form: $m(N_t, V_t) = N_t^\alpha V_t^{1-\alpha}$. The derivatives of the matching function with respect to N_t and V_t therefore are

$$\frac{\partial m(N_t, V_t)}{\partial V_t} = (1-\alpha)N_t^\alpha V_t^{-\alpha} = (1-\alpha)\left(\frac{m(N_t, V_t)}{V_t}\right) = (1-\alpha)q(\theta_t)$$

and

$$\frac{\partial m(N_t, V_t)}{\partial N_t} = \alpha N_t^{\alpha-1} V_t^{1-\alpha} = \alpha\left(\frac{m(N_t, V_t)}{N_t}\right) = \alpha\theta_t q(\theta_t).$$

The elasticity of the Cobb-Douglas matching function with respect to N_t is

$$\frac{\partial m(N_t, V_t)}{\partial N_t} \frac{N_t}{m(N_t, V_t)} = \alpha.$$

The utility function is assumed to be linear, therefore $u(c_t^i) = c_t^i$ for $i = e, j, n$.

In the following, the Lagrange multipliers are eliminated from the derivatives of the Lagrangian with respect to N_t and $\bar{a}$.

From $\frac{\partial \mathcal{L}}{\partial l_t(a)}$ it is clear that the marginal product of labour has to be the same for each individual, no matter how high a is. Therefore, instead of $af'(l_t(a))$ one can always use $\bar{a}_t f'(l_t(\bar{a}_t))$.

Derivative with respect to N_t:

First, $\frac{\partial \mathcal{L}}{\partial N_{t+1}}$ is rearranged to

$$\mu_t = \beta \left[u(c_{t+1}^e) - u(c_{t+1}^n)\right] + \beta\lambda_{t+1} \left[c_{t+1}^n - c_{t+1}^e - z\right] + \beta\mu_{t+1} \left[1 - s - \frac{\partial m(N_t, V_t)}{\partial N_t}\right] + \beta\nu_{t+1}.$$

Plugging in for μ_t, μ_{t+1}, λ_t, λ_{t+1} and ν_{t+1} one obtains

$$\frac{\gamma}{\beta q(\theta_t)} = (1 - \alpha) \left[af'(l_{t+1}(a)) - z\right] - \alpha\gamma\theta_{t+1} + \beta(1 - s)\frac{\gamma}{\beta q(\theta_{t+1})} \tag{A.2}$$

Derivative with respect to $\bar{a}_t$:

Divide the derivative $\frac{\partial \mathcal{L}}{\partial \bar{a}_t}$ by $\phi(\bar{a})$ and replace λ_t, μ_t and ν_t. One obtains

$$\bar{a}_t f(l_t(\bar{a}_t)) + \frac{s\gamma}{(1 - \alpha)q(\theta_t)} - \bar{a}_t f'(l_t(\bar{a}_t))(1 + l_t(\bar{a}_t)) = 0$$

Using the elasticity $\epsilon(\bar{a}) = \frac{\partial f(l(\bar{a}))}{\partial l(\bar{a})} \frac{l(\bar{a})}{f(l(\bar{a}))}$, the equation becomes

$$\bar{a}_t f(l_t(\bar{a})_t) \left[1 - \epsilon(\bar{a}_t)\right] - \bar{a}_t f'(l_t(\bar{a}_t)) + \frac{s\gamma}{(1 - \alpha)q(\theta)} = 0 \tag{A.3}$$

A.4 Efficiency in Market Equilibrium

The job creation condition in steady state is

$$af'(l(a)) = w + (1 - \beta(1-s))\frac{\gamma\theta}{\beta\theta q(\theta)}.$$

The wage curve in steady state is

$$w = \xi\left[af'(l(a)) + \gamma\theta\right] + (1-\xi)z.$$

Combining them, one obtains for the wage:

$$w = \frac{\xi}{(1-\xi)}\frac{\gamma\theta}{\beta\theta q(\theta)}\left[1 - \beta(1-s) + \beta\theta q(\theta)\right] + z. \tag{A.4}$$

The value equations for an employed and an unemployed worker in steady state are

$$W_e = w + \beta\left[sW_n + (1-s)W_e\right] \tag{A.5}$$

and

$$W_n = z + \beta\left[\theta q(\theta)W_e + (1-\theta q(\theta))W_n\right]. \tag{A.6}$$

Rearranging W_n gives

$$W_n = \frac{z + \beta\theta q(\theta)W_e}{1 - \beta + \beta\theta q(\theta)}.$$

Inserting this into (A.5), one can solve for W_e:

$$\begin{aligned} W_e &= w + \beta s\left(\frac{z + \beta\theta q(\theta)W_e}{1-\beta+\beta\theta q(\theta)}\right) + \beta(1-s)W_e \\ \Leftrightarrow \quad (1-\beta)W_e &= w - \frac{\beta s(w-z)}{1-\beta(1-s)+\beta\theta q(\theta)}. \end{aligned}$$

Analogously, rearranging (A.5) gives

$$W_e = \frac{w + \beta s W_n}{1 - \beta(1 - s)}. \tag{A.7}$$

By using (A.7), W_e can be eliminated from equation (A.6) and one obtains

$$(1 - \beta)W_n = z + \frac{\beta\theta q(\theta)(w - z)}{1 - \beta(1 - s) + \beta\theta q(\theta)}.$$

For w one can insert (A.4) into both reformulated value equations and receives

$$(1 - \beta)W_e = z + \frac{\xi}{(1 - \xi)} \frac{\gamma\theta}{\beta\theta q(\theta)} [1 - \beta + \beta\theta q(\theta)] \tag{A.8}$$

and

$$(1 - \beta)W_n = z + \frac{\xi}{1 - \xi}\gamma\theta. \tag{A.9}$$

A.4.1 Reformulating the Indifference Equation

The job creation condition in steady state can be rearranged to

$$w = \bar{a}f'(l(\bar{a})) - [1 - \beta(1 - s)] \frac{\gamma}{\beta q(\theta)}. \tag{A.10}$$

The labour demand of the marginal firm is given with

$$l(\bar{a}) = (1 - s)l(\bar{a}) + q(\theta)v(\bar{a}) \Leftrightarrow l(\bar{a}) = \frac{q(\theta)}{s}v(\bar{a}).$$

The indifference equation in steady state is given with

$$\begin{aligned} \bar{a}f(l(\bar{a})) - wl(\bar{a}) - \gamma v(\bar{a}) &= (1 - \beta)W_e \\ &= w - \frac{\beta s(w - z)}{1 - \beta(1 - s) + \beta\theta q(\theta)} \end{aligned}$$

Inserting (A.10), the equation becomes

$$\begin{aligned} &\bar{a}f(l(\bar{a})) - \bar{a}f'(l(\bar{a}))l(\bar{a}) + [1-\beta(1-s)]\frac{\gamma}{\beta q(\theta)}l(\bar{a}) - \gamma v(\bar{a}) \\ = \;& \bar{a}f'(l(\bar{a})) - [1-\beta(1-s)]\frac{\gamma\theta}{\beta\theta q(\theta)} - \frac{\beta s(w-z)}{1-\beta(1-s)+\beta\theta q(\theta)} \end{aligned}$$

and plugging in (A.4), one arrives at

$$\begin{aligned} &\bar{a}f(l(\bar{a})) - \bar{a}f'(l(\bar{a}))l(\bar{a}) + [1-\beta(1-s)]\frac{\gamma}{\beta q(\theta)}l(\bar{a}) - \gamma v(\bar{a}) \\ = \;& \bar{a}f'(l(\bar{a})) - \frac{s\gamma}{(1-\xi)q(\theta)} - (1-\beta)\frac{\gamma}{\beta q(\theta)}. \end{aligned}$$

Rearranging finally leads to

$$\bar{a}f(l(\bar{a}))\left[1-\epsilon(\bar{a})\right] + \frac{(1-\beta)}{\beta}\gamma\left[\frac{v(\bar{a})}{s} + \frac{1}{q(\theta)}\right] = \bar{a}f'(l(\bar{a})) - \frac{s\gamma}{(1-\xi)q(\theta)}$$

or

$$\bar{a}f(l(\bar{a}))\left[1-\epsilon(\bar{a})\right] + \frac{(1-\beta)}{\beta}\frac{\gamma\left[1+l(\bar{a})\right]}{q(\theta)} = \bar{a}f'(l(\bar{a})) - \frac{s\gamma}{(1-\xi)q(\theta)}.$$

A.5 Tax Rate

Workers and firms bargain about gross wages and the FOC from Nash wage bargaining is

$$\begin{aligned} &\xi P_t \frac{\partial(W_t^e - W_t^n)}{\partial w_t} + (1-\xi)(W_t^e - W_t^n)\frac{\partial P_t}{\partial w_t} = 0 \\ \Leftrightarrow\; & \xi P_t(1-\tau^e) = (1-\xi)(W_t^e - W_t^n). \end{aligned}$$

The surplus of the match for the worker is

$$W_t^e - W_t^n = (1-\tau^e)w_t - z + \beta(1-s-\theta_t q(\theta_t))(W_{t+1}^e - W_{t+1}^n)$$

and for the job creator it is

$$P_t = af'(l_t) - w_t + \beta(1-s)P_{t+1}.$$

Using these equations, plugging them into the FOC and substituting $W^e_{t+1} - W^n_{t+1} = \frac{\xi}{1-\xi}(1-\tau^e)P_{t+1}$, one receives

$$\begin{aligned} &\xi(1-\tau^e)\left[af'(l_t) - w_t\right] + \beta\xi(1-s)(1-\tau^e)P_{t+1} \\ = \ &(1-\xi)\left[(1-\tau^e)w_t - z + \beta(1-s-\theta_t q(\theta_t))\frac{\xi}{(1-\xi)}(1-\tau^e)P_{t+1}\right] \end{aligned}$$

Inserting $P_{t+1} = \frac{\gamma}{\beta q(\theta_t)}$ and rearranging then leads to

$$w_t = \xi\left[af'(l_t) + \gamma\theta_t\right] + (1-\xi)\frac{z}{(1-\tau^e)}.$$

The indifference equation for the marginal job creator is given with

$$\begin{aligned} &(1-\tau^j)\left[\bar{a}f(l_t(\bar{a}_t)) - w_t l_t(\bar{a}_t) - \gamma v_t(\bar{a}_t)\right] + \beta W^j_{t+1}(\bar{a}_t, l_{t+1}) \\ = \ &(1-\tau^e)w_t + \beta\left[sW^n_{t+1} + (1-s)W^e_{t+1}\right]. \end{aligned}$$

Inserting for W^e_t, W^n_t, w_t and $af'(l_t)$ in the indifference equation, it can be written (in steady state formulation) as

$$\frac{(1-\tau^j)}{(1-\tau^e)}\bar{a}f(l(\bar{a}))\left[1-\epsilon(\bar{a})\right] = \frac{(1-\beta)}{\beta(1-\xi)}\frac{\gamma}{q(\theta)}\left[\xi - \frac{(1-\tau^j)}{(1-\tau^e)}(1-\xi)l(\bar{a})\right] + \frac{\xi}{(1-\xi)}\gamma\theta + \frac{z}{(1-\tau^e)}$$

The marginal tax rate τ^j that restores the first-best allocation can be calculated in the following way: τ^j has to be set such that

$$\begin{aligned} \bar{a}f(l(\bar{a}))\left[1-\epsilon(\bar{a})\right] = &\frac{(1-\beta)\xi(1-\tau^e)}{\beta(1-\xi)(1-\tau^j)}\frac{\gamma}{q(\theta^M)} - \frac{(1-\beta)}{\beta}\frac{\gamma}{q(\theta^M)}l(\bar{a}) \\ &+ \frac{\xi(1-\tau^e)}{(1-\xi)(1-\tau^j)}\gamma\theta^M + \frac{z}{1-\tau^j} \end{aligned}$$

is equal to

$$\bar{a}f(l(\bar{a}))\left[1-\epsilon(\bar{a})\right] = \frac{(1-\beta)}{\beta(1-\alpha)}\frac{\gamma}{q(\theta^S)} + \frac{\alpha}{(1-\alpha)}\gamma\theta^S + z.$$

I assume that the Hosios condition holds, so $\xi = \alpha$.
One obtains

$$\begin{aligned}&\frac{1}{1-\tau^j}\left\{\frac{(1-\beta)\xi}{(1-\xi)}\frac{(1-\tau^e)\gamma}{\beta q(\theta^M)} + \frac{\xi}{(1-\xi)}(1-\tau^e)\gamma\theta^M + z\right\}\\ =\ &(1-\beta)\frac{\gamma l(\bar{a})}{\beta q(\theta^M)} + \frac{(1-\beta)}{(1-\xi)}\frac{\gamma}{\beta q(\theta^S)} + \frac{\xi}{(1-\xi)}\gamma\theta^S + z.\end{aligned}$$

Solving this for τ^j leads to

$$\tau^j = 1 - \left[\frac{(1-\beta)\xi\gamma(1-\tau^e) + \beta q(\theta^M)\left[\xi\gamma\theta^M(1-\tau^e) + (1-\xi)z\right]}{(1-\beta)\gamma\left[(1-\xi)l(\bar{a}) + \frac{q(\theta^M)}{q(\theta^S)}\right] + \beta q(\theta^M)\left[\xi\gamma\theta^S + (1-\xi)z\right]}\right].$$

If we use $\theta^S = \theta^M = \theta$ and expand the 1 such that we can write everything as one fraction and simplify, we receive expression (18).
The marginal tax rate τ^j exceeds τ^e until the point

$$\tau^j = \tau^e = \frac{(1-\beta)(1-\xi)\gamma\left[1+l(\bar{a})\right]}{(1-\beta)\gamma\left[1+(1-\xi)l(\bar{a})\right] + \beta(1-\xi)q(\theta)z - (1-\beta)\xi\gamma}$$

is reached.

A.5.1 If $\xi \neq \alpha$

If the Hosios condition does not hold, the tax rate τ^j that restores the first-best allocation is given with

$$\tau^j = \frac{\frac{(1-\beta)}{(1-\alpha)}\frac{\gamma}{\beta q(\theta^S)} + \frac{(1-\beta)\gamma l}{\beta q(\theta^M)} + \frac{\alpha}{(1-\alpha)}\gamma\theta^S - (1-\tau^e)\left[\frac{(1-\beta)\xi}{(1-\xi)}\frac{\gamma}{\beta q(\theta^M)} + \frac{\xi}{(1-\xi)}\gamma\theta^M\right]}{\frac{(1-\beta)}{(1-\alpha)}\frac{\gamma}{\beta q(\theta^S)} + \frac{(1-\beta)\gamma l}{\beta q(\theta^M)} + \frac{\alpha}{(1-\alpha)}\gamma\theta^S + z}.$$

Consider the case where $\alpha > \xi$, meaning that the social return of a match is larger than the private return of a match for the worker or lower than the private return for the job creator. For small values of τ^e, the calculated τ^j is larger than τ^e. If

$\tau^e = 1$, then $\tau^j < 1$. Both tax rates are equal at

$$\tau^j = \tau^e = \tau = \frac{\frac{(1-\beta)}{(1-\alpha)}\frac{\gamma}{\beta q(\theta^S)} + \frac{(1-\beta)\gamma l}{\beta q(\theta^M)} + \frac{\alpha}{(1-\alpha)}\gamma\theta^S - \frac{(1-\beta)\xi}{(1-\xi)}\frac{\gamma}{\beta q(\theta^M)} - \frac{\xi}{(1-\xi)}\gamma\theta^M}{\frac{(1-\beta)}{(1-\alpha)}\frac{\gamma}{\beta q(\theta^S)} + \frac{(1-\beta)\gamma l}{\beta q(\theta^M)} + \frac{\alpha}{(1-\alpha)}\gamma\theta^S + z - \frac{(1-\beta)\xi}{(1-\xi)}\frac{\gamma}{\beta q(\theta^M)} - \frac{\xi}{(1-\xi)}\gamma\theta^M}$$

which is smaller than one for $z > 0$. The larger z is, the lower is the point where τ^e becomes larger than τ^j.

A.6 Intra-Firm Wage Bargaining

The job creation condition with intra-firm wage bargaining becomes

$$\frac{\gamma}{q(\theta_t)} = \beta\left[af'(l_{t+1}) - \frac{\partial w_{t+1}(a, l_{t+1})}{\partial l_{t+1}}l_{t+1} - w_{t+1}(a, l_{t+1}) + (1-s)\frac{\gamma}{q(\theta_{t+1})}\right]$$

which can be rearranged to

$$\gamma\left[\frac{1}{\beta q(\theta_t)} - \frac{(1-s)}{q(\theta_{t+1})}\right] = af'(l_{t+1}) - \frac{\partial w_{t+1}(a, l_{t+1})}{\partial l_{t+1}}l_{t+1} - w_{t+1}(a, l_{t+1}).$$

The term on the LHS does not depend on any of the choice variables of the firm, and therefore, the RHS has to be equal across all firms in equilibrium. It holds, that

$$af'(l_{t+1}(a)) - \frac{\partial w_{t+1}(a)}{\partial l_{t+1}}l_{t+1}(a) - w_{t+1}(a) = \bar{a}f'(l_{t+1}(\bar{a})) - \frac{\partial w_{t+1}(\bar{a})}{\partial l_{t+1}}l_{t+1}(\bar{a}) - w_{t+1}(\bar{a}).$$

The instantaneous marginal value of a match for the firm is the same for all firms. The surplus of a match for the firm is given with

$$P_t = af'(l_t) - \frac{\partial w_t(a, l_t)}{\partial l_t}l_t - w_t(a, l_t) + \beta(1-s)P_{t+1}$$

and the surplus of a match for the worker with

$$W_t^e - W_t^n = w_t - z + \beta\left[1 - s - \theta_t q(\theta_t)\right](W_{t+1}^e - W_{t+1}^n).$$

The wage given by the Nash wage bargaining solution fulfils

$$w_t(a, l_t) = \arg\max \ (W_t^e - W_t^n)^{\xi} P_t^{1-\xi},$$

which gives the following FOC:

$$\xi P_t = (1-\xi)(W_t^e - W_t^n).$$

If we plug in the surplus of a match for the job creator and the worker, it becomes

$$\begin{aligned}
&\frac{\xi}{1-\xi}\left[af'(l_t) - \frac{\partial w_t(a,l_t)}{\partial l_t} l_t - w_t(a,l_t) + \beta(1-s)P_{t+1}\right] \\
=\ & w_t(a,l_t) - z + \beta\left[1 - s - \theta_t q(\theta_t)\right]\frac{\xi}{1-\xi}P_{t+1} \\
\Leftrightarrow\ & \frac{\xi}{1-\xi}\left[af'(l_t) - \frac{\partial w_t(a,l_t)}{\partial l_t} l_t - w_t(a,l_t)\right] = w_t(a,l_t) - z - \beta\theta_t q(\theta_t)\frac{\xi}{1-\xi}P_{t+1} \\
\Leftrightarrow\ & \frac{\xi}{1-\xi}\left[af'(l_t) - \frac{\partial w_t(a,l_t)}{\partial l_t} l_t - w_t(a,l_t)\right] = w_t(a,l_t) - z - \theta_t\frac{\xi}{1-\xi}\gamma.
\end{aligned}$$

The LHS from the above equation needs to be constant across all firms. So has to be the RHS, which implies that $w_t(a, l_t)$ is the same for all firms.

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