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Business cycle accounting for the German fiscal stimulus program during the Great Recession

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Abstract

We take the neoclassical perspective and apply the business cycle accounting method as proposed by Chari, Kehoe, and McGrattan (2007, Econometrica) for the Great Recession and the associated stimulus program in Germany 2008-2009. We include wedges to the variables *government consumption, durables, investment, labor, net exports, and efficiency*. The results suggest: The crisis was mainly driven by the efficiency wedge, followed by the net exports and the investment wedge. The government consumption wedge and in particular the durables wedge acted counter-cyclical. We attribute the latter to an internationally incomparably large cash for clunkers program and conclude that this subsidy on durable goods was more effective than pure government consumption.

We introduce a strategy for likelihood maximization, which reliably and quickly locates the maximum; enables a detailed evaluation of the likelihood function and allows large robustness checks.

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1 Introduction

In response to the Great Recession in 2008 and 2009, the German government, like many others, launched an expansive fiscal stimulus program. This policy intervened on different markets by increasing transfers and government spending, decreasing tax rates and social contributions and expanding short-time work possibilities. Particularly noteworthy is the German cash for clunkers program, since this car subsidy affected one of Germany's core industries and was internationally incomparably large (5 Billion € or 0.2 % of Gross Domestic Product (GDP)). Altogether, the program amounted to 82 billion € or 3.2 % of GDP. These considerable expenditures raise the following questions: What are the consequences of these measures for macroeconomic markets and how effective was this program for aggregated output?

The debate on the effectiveness of economic stimulus packages induced by the current COVID-19 pandemic underlines the relevance of these questions. Unfortunately, such questions are difficult to answer, which is why fiscal stimuli might be the most controversially discussed anti-cyclical measures. To address them, there are basically two approaches (see e.g. Hebous (2010)): The first is to model a theoretical framework with deep structural equations, parameters, and shocks. An arbitrary number of shocks describes changes in fiscal policy, and impulse response functions as well as multipliers illustrate the consequences. Since the structure, the parametrization, and, at least partly, the parameter values ground on assumptions, the results are assumption-driven. The second approach bases on statistical models, in particular vector autoregressions (VARs). They are less theoretical and, in comparison to many of the former models, can be estimated with classical techniques. Unfortunately, in general it is impossible to distinguish between market distortions and the agent's responses to these distortions. This makes it rather impractical to study the effects of the various market interventions. Instead of selecting from these two approaches, we apply a third option, which we describe as kind of a middle course. By employing the Business Cycle Accounting (BCA) approach as proposed by Chari et al. (2007) and revisited by Brinca et al. (2016), we investigate the impact of the Great Recession during 2008 and 2009 in Germany, its aftermath, the impact of monetary policy, and in particular, the effects of the German stimulus program.

The BCA framework is based on the benchmark Real Business Cycle (RBC) model, which is extended by time-varying distortions in nearly every market, the so-called "prototype economy". Chari et al. (2007) interpret the origins of these market distortions as taxes, nominal and real frictions, changes in expectations, etc. and call them "wedges". In contrast to most medium or large scale Dynamic Stochastic General Equilibrium (DSGE) models, the mechanisms underlying these distortions are not structural. They are parameterized like taxes, technology, or government spending and are driven by a reduced-form Markov process.[1] Commonly this process is specified by a VAR(1). Using time series data one can estimate the parameters of the VAR process and measure the values of the wedges. These measured wedges are fed back into the model one by one, to assess the contribution of each wedge to the business cycle. In a nutshell, BCA is the fully developed "...through the lens of a neoclassical model"-approach.[2] The slim theoretical framework and the applicability of classical estimation techniques, in this instance maximum-likelihood estimation (MLE), minimizes the number of assumptions required and thus the results are less assumption-driven. Nevertheless, one can distinguish between market distortions and the agents responses.

To increase the practicability of BCA in general and make it more suitable for the study of the German stimulus program in particular, we differ from Chari et al. (2007) in our "prototype-economy", in our estimation methodology, and in our mapping strategy.

Prototype-economy: We extend the benchmark model for the following reasons in three ways. First, the wedges include a long- and a short-run component. This allows to differentiate between growth and business cycle accounting. Since the German reunification, subaggregates of demand grew at different rates. Without growth accounting, the underlying stochastic process is non-stationary. Chari et al. (2007) set a common growth rate unfoundedly for all countries equal to 1.6 %. Brinca et al. (2016) detrend in such a way that the average trend-adjusted log output of the economy under consideration is equal zero. The latter makes the estimation procedure more robust. Our approach can be seen as a further stage.[3] Second, we distinguish between government spending and net exports. This enables a government spending analysis and accounts for the fact that German industry is strongly depended on foreign trade. Third, we exclude durable consumption goods

[1]Note that Chari et al. (2009) argue that also some of the shocks in medium or large scale DSGE models, i.e. New Keynesian models, are rather reduced-form than structural.

[2]This long-lasting approach was established by Solow (1957). To name but a few more recent applications: Kehoe and Prescott (2002), Ohanian (2010), Lu (2012), Cho and Doblas-Madrid (2013), Karabarbounis (2014) or Hansen and Ohanian (2016).

[3]Note that growth accounting is implicitly applied whenever different time series are detrended by univariate filters, such as the HP-filter, the Hamilton filter or the Baxton-King filter. DeJong and Dave (2011, Chapter 6.1) suggest a general procedure to estimate a common linear trend. Even by applying this strategy, the estimated process lacks stationarity here.

from aggregated investment in order to consider the cash for clunkers program separately. After all, the model includes the following wedges: *government consumption, durables, investment, labor, net exports, and efficiency*. Previous work already extends the benchmark model in various ways, e.g. Šustek (2011) includes an asset market and a monetary policy wedge.

Estimation: We estimate two structural parameters and all parameters of the VAR process using MLE, in sum 59, and identify the wedges with Kalman-smoothing. MLE in this context is difficult, e.g. Gerth and Otsu (2018) report unsolved problems concerning likelihood optimization and BCA.[4] As many others, they avoid the problem by switching to Bayesian estimation. As we argue, Bayesian methods are impracticable for BCA, because the reduced-form process is highly abstract and thus, it seems impossible to make any a-priori assumptions. Furthermore, Brinca et al. (2018) argue that weak identification associated with parameters of the VAR process is negligible in the context of BCA. Unfortunately, this does not hold for structural parameters. We introduce a reliable and quick procedure to locate the maximum of the likelihood function. Using this procedure, it is a feasible exercise to apply tools that help overcome problems of weak identification, namely plotting the likelihood contour, detecting the global maximum, and executing robustness checks, all with respect to the uncertain structural parameters.

The procedure can be summarized as follows: In advance, we make sure that all uncertain parameters are locally strictly identified according to the strategy of Iskrev (2010). Then, we maximize the likelihood function, which we receive from a Kalman recursion, assuming that the initial states are fixed and known in their long-run equilibrium. As Huber (2020) shows, this initialization is in line with Chari et al. (2007) and provides two advantages, i) the computation of the likelihood function can be vectorized and ii) an analytical and unique solution exists for the maximizing conditional covariance matrix. Further, Huber (2020) proves that the average of this likelihood function converges pointwise towards the average of a likelihood function received from a Kalman recursion initialized with the unconditional first and second moments. Thus, we use the first parameter estimation only as a guess for the actual estimation based on the more common, unconditional likelihood function. As mentioned, we complete the process by determining the wedges with Kalman-smoothing.

Mapping: Chari et al. (2007) map different types of structural frictions towards the

[4] Gerth and Otsu (2018) do not account for growth, which potentially explains the problem.

reduced-form wedges, which they call "equivalent results". We map the particular measures of the fiscal stimulus program and monetary policy in a similar manner and analyze whether these interventions can explain counter-cyclical behavior. This follows Mulligan (2005) who initiates the study of policy interventions as reduced-form errors of RBC models, and Kersting (2008) who initiates the mapping of political measures, namely the 1980's U.K. labor market reforms, towards the wedges inside the BCA framework.

Our findings suggest that the crisis was mainly driven by the efficiency wedge, followed by the net exports and the investment wedge. The government consumption wedge and especially the durables wedge acted counter-cyclically. Furthermore, the labor wedge induced a fast recovery. The results are robust except for the investment wedge.

We attribute the counter-cyclicality of the durables wedge to the cash for clunkers program, which is equivalent to a durable good subsidy. Since the expenditures for government consumption were higher than for the cash for clunkers program and the effects were similar, subsidies for durable goods stimulated aggregated demand more efficiently. Mian and Sufi (2012) examine the U.S. cash for clunkers program as a representative of durables and investment subsidies using cross-section variation. They find that the program induced a large increase in car sales. Indeed, in their study, the positive effect vanishes within one year due to intertemporal substitution. In Germany, durable goods bust after the program, which suggest a similar substitution effect. However, our BCA analysis indicates that this is the transmission towards the trajectory of durables that would have occurred in the absence of the cash for clunkers program. In sum the program's effects are neither substituted entirely intra- nor intertemporally untill 2011-Q3. This is at odds with the results of a times-series analysis by Leuwer and Süssmuth (2018), who find large substitution effects. However, their work relies on the strong assumption that there were no substantial changes simultaneously to the car subsidy. Berger and Vavra (2015) investigates the households' responses to durables subsidies over the business cycle for the U.S. and find smaller effects in recessions, which is not at odds to our results, but make them more striking.

The labor market wedge induced recovery can be explained by expanded short-time work possibilities as they can decrease hiring frictions in the aftermath of recessions. Using the unemployment rate, Gehrke et al. (2019) argue that previous labor market reforms (so-called Hartz reforms) probably drove the labor market wedge induced recovery. Our method cannot distinguish between these explanations because both achieve equivalent results.

Similar interpretation problems concerning reduced-form shocks arise with measures of the stimulus program which we map towards the efficiency, investment, and net exports wedge. Since these wedges caused the crisis, pro-cyclical distortions exceed the effects of counter-cyclical fiscal stimulus and monetary policy measures in those markets. Hence, pro-cyclical wedges give no evidence for ineffective measures. Assuming that the fiscal stimulus program together with monetary policy were the only counter-cyclical distortions, counter-cyclical wedges give evidence for effective measures. Under this assumption, our results represent a lower bound for the impact of fiscal and monetary policy measures and the pro-cyclical distortions.

Existing BCA applications for the Great Recession in Germany by Brinca et al. (2016) and Gerth and Otsu (2018) suggest negligible effects of the investment wedge on the business cycle. Both treat durables and other investment goods as a composite. We get similar results, feeding back both wedges at the same time into the model. In detail, the pro-cyclicality of the investment wedge and the counter-cyclicality of the durables wedge offset each other, which is why previous work potentially underrate the importance of the investment wedge and, as a consequence, equivalent financial frictions.

Drygalla et al. (2018) as well as Gadatsch et al. (2016) investigate the German fiscal stimulus program in medium-scale New-Keynesian DSGE models using Bayesian inference. They find positive but small effects on GDP and the latter finds negative effects in the aftermath of the crisis. However, neither of these studies account for durable consumption goods separately.

The remainder of the paper reads as follows. The next section sketches the German fiscal stimulus program and the monetary policy of the European Central Bank (ECB). Furthermore, we provide long-term series with focus on the crisis from 2008 till 2011 for the reunified German economy. Thereafter, we describe our version of a prototype economy. We map the single measures of the program to the wedges. In a next step, we present our calibration exercises and the estimation strategy. We show the results with a robustness and discussion section and then the paper concludes. Our Appendix presents the entire model as well as the source of our data and the corresponding manipulation.

2 The German case

2.1 The fiscal stimuli packages I and II in detail

The German fiscal stimulus program was composed of two packages. The first became effective at the end of 2008 and the second at the beginning of 2009 (Bundesgesetzblatt, 2008, 2009).

As Rosenberger (2013) describes, the first package amounted to 32 Billion € plus a loan program of 15 Billion €. The fiscal stimulus consisted of a one year's tax exemption on new cars, higher tax deductions by permitting the reducing-balance method and increasing child allowance, a lower employment insurance tax, as well as higher transfers for students and retirees.

The second stimulus package amounted to 50 Billion € plus both a loan and guarantee program of 100 Billion € and an increase of the German export credit guarantee program (Hermes cover) of about 2 Billion €. The package consisted of investments in public infrastructure, financial support for local and state authority spending, a subsidy on new cars at the amount of 2500 € per car and in total 5 Billion €, subsidies for private innovations as well as lower income taxes and social contributions. Short-time work possibilities and benefits were expanded, further training was supported, and the Federal Employment Agency increased the number of job agents.

Table 1 presents following calculations by the OECD (2009) for the stimulus program. The size of the fiscal stimulus program was on equal terms by reducing taxes and increasing transfers and spending. Transfers to households amounted to 0.3 % of GDP, where the cash for clunkers composed two out of three. Extra government spending amounted to 0.8 % of GDP. The fiscal packages amounted to 3.2% of GDP, excluding all measures which did not affected the national budget directly, e.g. the loan and guarantee program.

Table 1: Composition of the fiscal program in % of GDP

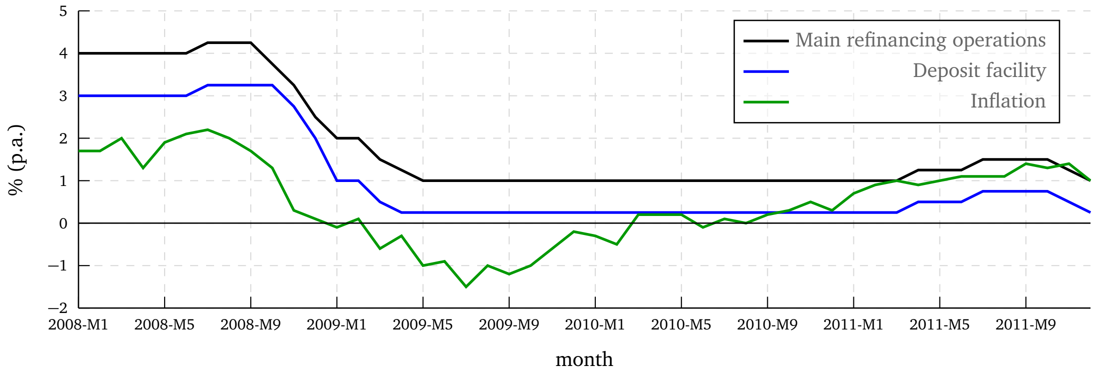
Tax	Individuals	Social Contribution	Business	Total*
	-0.6	-0.7	-0.3	-1.6
Spending	Transfers to households	Transfers to business	Government spending**	Total***
	0.3	0.3	0.8	1.6

Notes: * Including consumption tax measures. ** Final consumption + investment *** Including transfers to sub-national government. **Source:** OECD (2009).

2.2 Monetary policy in the Great Recession

The monetary policy of the ECB also reacted to the recession. Figure 1 shows the minimum bid rate on main refinancing operations and the interest rate on deposit facilities declined in the aftermath of the declined inflation rate. The former declined from 4.25% in mid 2008 to 1% by mid 2009. Both interest rates have persisted since then.

Figure 1: Monetary policy and usage of the deposit facility



Besides the conventional interest rate policy, the ECB applied further tools of monetary policy. Here we give a short overview of the detailed reports of the European Central Bank (2010, 2011). In October 2008 the ECB switched from a variable-rate to a fixed-rate tender, eased collateral requirements and enhanced the provision of liquidity. The ECB's Governing Council prolonged these measures several times. It decided to purchase bonds issued in the Euro area in May 2009 and launched the Security Markets Program in June 2009. This program conducted interventions on public and private debt securities markets in the Euro area. Then, in March and May 2010, the Governing Council decided to switch back and forth between a variable- and a fixed-rate tender and to intervene once again on the Euro area public and private debt securities markets. The Council determined long-term refinance operations to provide liquidity in August and October 2010.

2.3 Stylized facts for the German economy

Table 2 presents average long-run shares of subaggregates of the reunified German economy (1991–2018). Private consumption expenditures (PCE) account for 56%, whereby durables account for 6% and non-durables for the half of GDP. The share of investment is determined at 21% and of government consumption close to 19%. Net exports account for almost 4%.

Table 2: Long-run ratios in % of GDP (1991–2018)

Description	$\overline{x_t/\text{GDP}_t}$
Private consumption expenditures	56.05
Non-durables consumption	49.72
Durable consumption	06.33
Investment	21.32
Government consumption.	18.87
Net exports	03.76

Source: See Appendix C, own calculations.

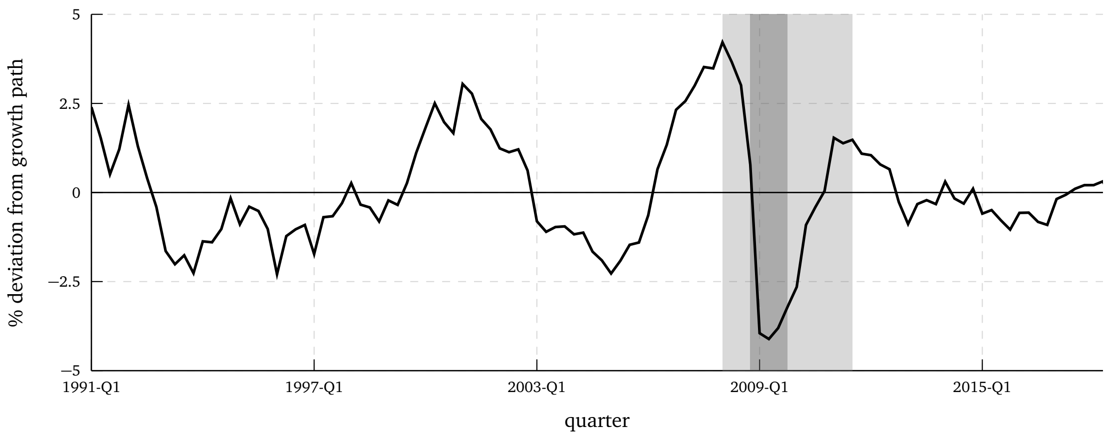
Figures 2 and 3 present the cyclical behavior of GDP, its subaggregates and hours worked. The time series are the relative deviations from the concerning linear trend. We choose a linear trend filter instead of the commonly used HP-filter to be consistent with our estimation strategy.[5]

We observe a boom-bust cycle in GDP at about the same time of the dot-com bubble. This cycle was followed by a recovery from 2005 till 2008, which ended in a heavy drop. This drop depicts the Great Recession. GDP recovered fast and has moved along the long-run trend since then.

Panel (a) of Figure 3 shows that investment has co-moved with GDP, but with a higher volatility. Panel (b) displays two heavy short boom-bust-cycles of durables. The first peaked at the end of 2006, shortly after the announcement of a value added tax (VAT) increase. This was followed by a bust at the beginning of 2007, when the increase took place. We observe the second peak at the same time as the German cash for clunkers program, which was also followed by a bust as the program expired. Government consumption was above its trend in the middle and late 1990's. It decreased at the beginning of the 2000's and increased from 2008 till 2010. Since 2010 it has fluctuated around its trend. Non-durable consumption was below its trend in the aftermath of the reunification, and was above the trend in the 2000's until the Great Recession and decreased slightly afterwards. Net exports relative to GDP decreased from 1997 till 2001 from their trend, and increased sharply afterwards till 2003. From then on until the crisis they moved above

[5] Flor (2014) presents an overview of HP-filtered second moments of similar data.

Figure 2: Cyclical behavior of GDP



Notes: The data is presented as relative deviations from linear trend. The light gray area indicates the crisis from 2008-Q1 – 2011-Q3, the dark gray area indicates the main effective period of the fiscal stimulus program 2008-Q4 – 2009-Q4. **Source:** see Appendix C, own calculations.

the trend. Since the crisis they have fluctuated around the trend. In the medium-run, hours worked declined after the German reunification till 2005 and from then on they have increased. Hours worked have co-moved with GDP from 2000 onwards.

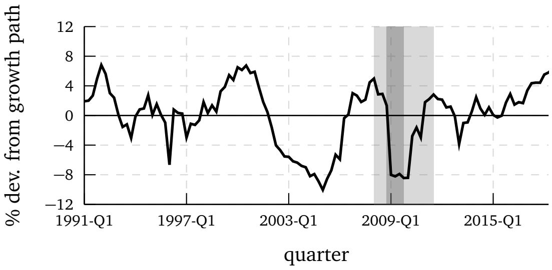
The light gray area in Figures 2 and 3 indicates the Great Recession. GDP, hours worked and investment decreased from the end of 2008 until the peak of the crisis in 2009-Q2 by 5%-points, 4%-points and 12%-points, respectively. Their recovery completed in 2011. Durables increased during the time of the car subsidy – indicated through the dark gray area – by 12%-points and decreased by 18%-points afterwards. Durables recovered at the end of 2010. Government consumption increased at the beginning of 2009 by 5%-points and remained till the end of 2011 by 4%-points above its trend. Non-durables were less than 2% below their trend at the end of 2009 and recovered fast.

3 Methods

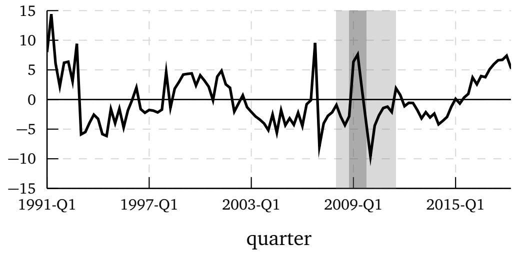
3.1 The prototype economy

The prototype economy consists of an infinitely-lived household, a firm facing perfect competition, and a government which finances its expenditures by levying taxes on labor, durables, and investment. The model of Chari et al. (2007) is extended in three ways. First, we distinguish between government spending and net exports and second, exclude

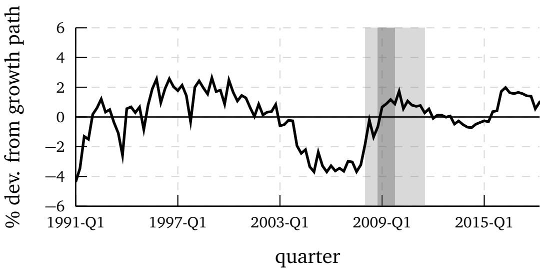
Figure 3: Cyclical behavior of different economic measures



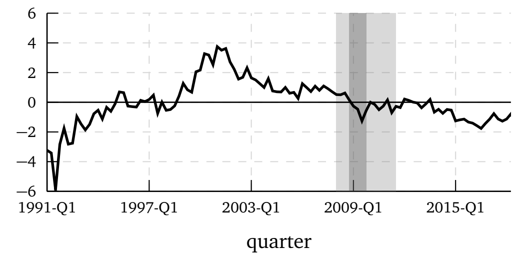
(a) Investment



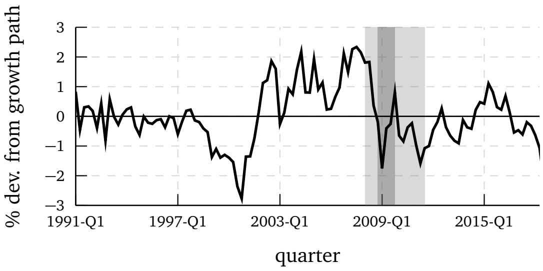
(b) Durables



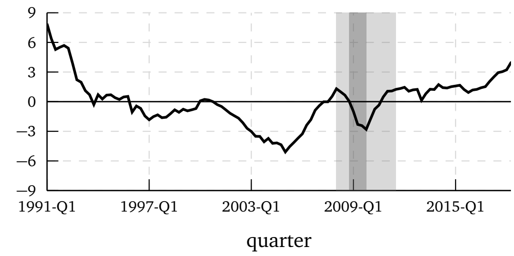
(c) Government Consumption



(d) Non-durable Consumption



(e) Net exports to GDP



(f) Hours

Notes: Despite hours worked, the data are presented as relative deviations from the corresponding linear trend. Hours worked is the relative deviation from the average. The light gray area indicates the crisis from 2008-Q1 – 2011-Q3, the dark gray area indicates the main effective period of the fiscal stimulus program 2008-Q4 – 2009-Q4. **Source:** see Appendix C, own calculations.

durables from aggregated investment goods. Both enable a deeper analysis of the stimulus program and the former allows to account for the strong export-dependency of the German economy. Third, wedges consist of a growth and a business cycle part. This allows separate procedures for growth and business cycle accounting and ensures stationarity of the stochastic process. The model also accounts for productive capital and durable consumption capital adjustment costs. Chang (2000) shows that adjustment costs for capital goods in the market and at home solves problems with excess volatility and negative comovements, because adjustment costs lower the substitutability, which is why we model this structural friction explicitly. The model is written in per capita terms.

3.1.1 Model

The per period utility of the representative household is parameterized as follows

$$u(C_t, D_t, N_t) = \begin{cases} \phi \ln(C_t) + (1-\phi)\ln(K_{Dt}) + \psi \ln(1-N_t) & \text{for } \eta = 1, \\ \frac{\left(C_t^{\phi} \cdot K_{Dt}^{1-\phi} \cdot (1-N_t)^{\psi}\right)^{1-\eta} - 1}{1-\eta} & \text{for } \eta \neq 1, \end{cases} \tag{1}$$

where C_t denotes consumption of non-durable goods and N_t is the household's labor supply. The stock of durable consumption goods K_{Dt} accumulates according to

$$\gamma_n K_{Dt+1} = (1-\delta_D)K_{Dt} + D_t - \Theta_{Dt}\left(\frac{D_t}{K_{Dt}}\right)K_{Dt}, \; \Theta_{Dt}\left(\frac{D_t}{K_{Dt}}\right) = \frac{a_D}{2}\left(\frac{D_t}{K_{Dt}} - b_D\right)^2, \tag{2}$$

where γ_n denotes the population growth factor, D_t are investments in durable consumption goods, and b_D is the ratio of investment in durables to the stock of durables in the long run. The household maximizes its expected life-time-utility

$$U_t = \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\gamma_n)^s u(C_{t+s}, K_{Dt+s}, N_{t+s}) \tag{3}$$

subject to the budget constraint

$$C_t + (1+\tau_{It})P_{It}I_t + (1+\tau_{Dt})P_{Dt}D_t \leq R_t K_{It} + (1-\tau_{Nt})W_t N_t + T_t - P_{Et}E_t, \tag{4}$$

where K_{It} denotes the productive capital stock (capital stock hereafter), I_t investment in capital, T_t lump-sum transfers, E_t net exports, R_t the rental rate on capital, and W_t the real wage. The tax rates τ_{Nt}, τ_{It} and τ_{Dt} are used to model wedges in the labor, investment and durables market. P_{Et}, P_{It} and P_{Dt} are the relative prices for net exports, investment, and durable goods and reflects the wedges' long-run element. The consumption good is the numeraire. The capital stock follows the law-of-motion

$$\gamma_n K_{It+1} = (1-\delta_I)K_{It} + I_t - \Theta_{It}\left(\frac{I_t}{K_{It}}\right)K_{It},\ \Theta_{It}\left(\frac{I_t}{K_{It}}\right) = \frac{a_I}{2}\left(\frac{I_t}{K_{It}} - b_I\right)^2, \tag{5}$$

with b_I as the investment-to-capital ratio in the long run.

The representative firm produces its output good Y_t with the Cobb-Douglas technology

$$Y_t = K_{It}^{\alpha}(\gamma_z^t Z_t N_t)^{1-\alpha} \tag{6}$$

and faces perfect competition. The parameter γ_z denotes the growth factor of labor augmenting technical progress and Z_t the efficiency wedge.

The government expenditures G_t are exogenous and the government chooses lump-sum transfers T_t, so that its budget constraint

$$P_{Gt}G_t + T_t \leq \tau_{Nt}W_tN_t + \tau_{It}P_{It}I_t + \tau_{Dt}P_{Dt}D_t \tag{7}$$

always binds. Thereby, the resource constraint of the economy is

$$Y_t = C_t + P_{It}I_t + P_{Dt}D_t + P_{Gt}G_t + P_{Et}E_t. \tag{8}$$

Growth component: As already mentioned, the population grows with γ_n and technical progress with γ_z. Furthermore, the wedges evolve differently. The relative prices reflect this. In the long run $P_{Xt} \in \{P_{It}, P_{Dt}, P_{Gt}, P_{Et}\}$ evolves with $P_{Xt} = g_{P_X} P_{Xt-1}$. The ensuing trend growth factors of different variables X_t are described in Table 3. These variables are scaled by $x_t = \frac{X_t}{g_X^t}$ and are thus stationary variables.

Table 3: Growth factors

X_t	Y_t	C_t	W_t	T_t	I_t	K_{It}	R_t	D_t	K_{Dt}	G_t	E_t	γ_z	N_t	P_{Xt}
g_X	g_Y	g_Y	g_Y	g_Y	g_I	g_I	g_Y/g_I	g_D	g_D	g_G	g_E	$g_Y^{\frac{1}{1-\alpha}} g_I^{\frac{\alpha}{\alpha-1}}$	1	$g_{P_X} = \frac{g_Y}{g_X}$

Business cycle component: The VAR(1)-process

$$\mathbf{s_{t+1}} = \Pi \mathbf{s_t} + \epsilon_{\mathbf{t+1}},\ \epsilon_{\mathbf{t}} \sim \mathcal{N}(0, \Sigma), \tag{9}$$

drives the fluctuation of the model, where $\mathbf{s_t} = \begin{bmatrix} \ln(s_{At}) & s_{Nt} & s_{It} & s_{Dt} & s_{Et} & \ln(s_{Gt}) \end{bmatrix}^{\mathbf{T}}$ and $\epsilon_{\mathbf{t}} = \begin{bmatrix} \epsilon_{At} & \epsilon_{Nt} & \epsilon_{It} & \epsilon_{Dt} & \epsilon_{Et} & \epsilon_{Gt} \end{bmatrix}^{\mathbf{T}}$. The stochastic process affects the wedges as follows

$$\begin{aligned} Z_t &= A^* \cdot s_{At}, & \tau_{Nt} &= \tau_N^* + s_{Nt}, & \tau_{It} &= \tau_I^* + s_{It}, \\ \tau_{Dt} &= \tau_D^* + s_{Dt}, & e_t &= e^* + s_{Et}, & g_t &= g^* \cdot s_{Gt}, \end{aligned}$$

where A^*, τ_N^*, τ_I^*, τ_D^*, e^* and g^* are the corresponding steady-state component of the different distortions. Similar to Chari et al. (2007), we define the six wedges as follows: The efficiency wedge Z_t, the net export wedge e_t, the government spending wedge g_t, the labor wedge $1-\tau_{Nt}$, the investment wedge $\frac{1}{1+\tau_{It}}$, and the durables wedge $\frac{1}{1+\tau_{Dt}}$. The latter two are defined so that, similar to the labor market wedge, increases act like subsidies and decreases like taxes in comparison to the steady-state value. Since the cyclical component includes the steady-state component, detrended prices p_{Et}, p_{Gt}, p_{It}, p_{Dt} are normed to one. We present in Appendix A the full dynamic equilibrium of the model with stationary variables.

Solution: To derive the model's decision rules, we use a linear perturbation method. In detail, we apply the method of undetermined coefficients as Uhlig (1999) and Christiano (2002) describe to solve the log-linearized model. The solved model then can be written as

$$\mathbf{y_t} = \mathbf{L_x^y} \cdot \mathbf{x_t} + \mathbf{L_s^y} \cdot \mathbf{s_t}, \tag{10}$$

$$\mathbf{c_t} = \mathbf{L_x^c} \cdot \mathbf{x_t} + \mathbf{L_s^c} \cdot \mathbf{s_t}, \tag{11}$$

$$\mathbf{x_{t+1}} = \mathbf{L_x^x} \cdot \mathbf{x_t} + \mathbf{L_s^x} \cdot \mathbf{s_t}, \tag{12}$$

where the matrices $\mathbf{L_x^{\cdot}}$characterize the policy function of the deterministic part of the model's solution, while $\mathbf{L_s^{\cdot}}$ describe the policy function of the stochastic part. With $\hat{x}_t = \ln(x_t) - \ln(x)$ as the approximation of the relative deviation of a variable x_t from its steady state value x, the vector of observables is $\mathbf{y_t} = \begin{bmatrix} \hat{y}_t & \hat{N}_t & \hat{i}_t & \hat{d}_t & \hat{g}_t & \widehat{\frac{e_t}{y_t}} \end{bmatrix}^{\mathbf{T}}$, while $\mathbf{c_t}$ denotes the vector of unobserved control variables and $\mathbf{x_t} = \begin{bmatrix} \hat{k}_{It} & \hat{k}_{Dt} \end{bmatrix}^{\mathbf{T}}$ the vector of endogenous states.[6]

3.1.2 Mapping

Chari et al. (2007), Brinca et al. (2016), and various other authors map structural models into their prototype economy. Nutahara and Inaba (2012) apply BCA for misspecified wedges and find they are able to approximate the true wedges and the corresponding response of the agents adequately. We show first how to map the stimulus program to the prototype economy. Since the wedges' drivers are modeled as taxes, this is straightforward for most of the measures. Secondly, we reflect monetary policy.

Mapping the stimulus program

Government Wedge: We assign total government spending to the government spending wedge. These are mainly investments in infrastructure and financial support for local and state authority spending. Hence, the stimulus program increases the government wedge directly.

Durables Wedge: The two measures concerning new cars affect the durables wedge. For a given producer price, both measures reduce the absolute tax or the relative price of durables from the households perspective. Hence, they increase the durables wedge.

Investment Wedge: The first part of the stimulus program which affects the investment wedge are subsidies for investments in innovations. The second are increased tax deductions by allowing for a reducing-balance method. For given producer prices, absolute taxes or the relative price of investment decreases and thus the investment wedge increases.

Chari et al. (2007) show how to map financial frictions in terms of a financial accelerator and Brinca et al. (2016) show how to map financial frictions in terms of collateral constraints into a prototype economy with an investment wedge. The loan and guarantee program lowers financial frictions, in particular they mitigate the banks' collateral constraints. Following this, the loan and guarantee program also raises the investment wedge.

Labor Wedge: The stimulus program loweres income tax and social contribution, this increases the labor wedge in general.

Brinca et al. (2016) show the link between a prototype economy with efficiency and labor wedges and an economy with search and matching frictions. The mentioned labor

[6] The use of $\frac{\widehat{e_t}}{y_t}$ instead of $\hat{e}_t$ is discussed in 3.2.2.

market actions, e.g. expanded short-time work, reduce such frictions and thus, increase the labor market wedge. The effects should be delayed in time due to lower hiring frictions in the aftermath of the crisis.

Efficiency Wedge: Due to the labor market actions in the previous paragraph, the efficiency wedge increases also due to a better matching. Further, the expanded short-time work possibilities reduce labor hoarding, since the firm can both retain employees to lower future hiring frictions and adjust hours worked. As a consequence, the efficiency wedge increases.

As shown by Chari et al. (2007), input-financing frictions are associated with efficiency wedges. These frictions appear when firms must borrow for an input good and some firms are financially more constrained than others. Such firms have to pay higher interest rates. The loan and guarantee program lowers financial constraints and thus increases the efficiency wedge.

Net exports: The increase in Hermes coverage advances the conditions for exports. Nevertheless, the effects are probably only rather small.

Mapping monetary policy

Government Wedge: Purchasing bonds lowers the bonds' interest rates and this lowers the costs of debt-financed government spending, which may indirectly increase the government wedge.

Durables Wedge: Since refinancing is cheaper, for a given real rate of return, investment increases. Hence, monetary policy changes the intertemporal decision of a household, which is reflected in a higher durables wedge. Furthermore, provision of liquidity also changes the intertemporal decisions of liquidity constrained households, which also reflects in a higher durables wedge.

Investment Wedge: Both mentioned effects of the durables wedge have the same effect on the investment wedge. The provision of liquidity and cheaper refinancing lowers frictions in the investment market.

As already mentioned, Brinca et al. (2016) show how to map an economy with a collateral constrained bank into a the prototype economy with an investment wedge. Lower collateral constraints lower frictions in the investment market. Thus, the slacked collateral requirements by the ECB increase the investment wedge.

Efficiency Wedge: As mentioned above, input-financing frictions are associated with efficiency wedges (see Chari et al., 2007). The friction appears when firms must borrow for input goods and some firms are financially more constrained than others. Those firms have to pay higher interest rates. The Security Markets Program can lower these frictions and thus, increases the efficiency.

3.1.3 Calibration

We estimate the elasticity, $\eta_I = \frac{I}{K_I}\Phi_I''$, of the price of capital with respect to the investment to capital ratio as well as the elasticity, $\eta_D = \frac{D}{K_D}\Phi_D''$, of the price of the stock of durables with respect to the new durables to stock of durables ratio in addition to the parameters that characterize the stochastic process $\mathbf{s_t}$. The remaining parameters are calibrated as follows:

The capital elasticity α is set to 0.34. Flor (2014) calculates this as the German capital share from 1991 to 2012. In line with Heer and Maussner (2009, Chapter 1.5), Flor (2014) also provides the discount parameter $\beta = 0.994$ for the German economy. We pin down the annual rate of capital depreciation at the average ratio of gross fixed capital formation and the net stock of fixed assets. The average quarterly capital depreciation rate arises from $\delta_I = 1-(1-\delta_{I,annual})^{\frac{1}{4}}$. In the same manner the rate of durables depreciation δ_D is computed.

The choice of ψ, ϕ and η, which characterize the household's preferences, is more problematic. For ψ and η we follow the baseline calibration from Chari et al. (2007) and fix ψ at 2.24 and η at 1. We calibrate the preference weight of durables ϕ by matching the durable to non-durable consumption ratio with the long-run marginal rate of substitution between consumption and durables. We do not estimate the steady-state values of the different wedges. Instead, we compute them from the model's static equilibrium equations in line with Lama (2011). We fix the steady-state values of output, government consumption, investment in capital as well as in durables to their average shares of output (see Table 2). The steady-state labor supply N is 0.122, which equals the average share of hours worked on the available time budget of a household.[7] Our calibration exercises are summarized in Table 4.

[7] Here we follow (Heer and Maussner, 2009, Chapter 1.5), who assume that the household's maximum working hours amount to $1,440 = 16$ hours per day$\times$90 days per quarter.

Table 4: Calibration of the model

Parameter	Description	Value
α	Capital share	0.34
β	Discount factor	0.994
δ_I	Rate of capital depreciation	0.017
δ_D	Rate of durables depreciation	0.045
ψ	Preference weight of labor	2.24
ϕ	Preference weight of consumption	0.879
η	Risk aversion	1

3.1.4 Identification

We check our prototype economy for strict local identification following Iskrev (2010), who shows that a linearized DSGE model with normally distributed shocks is locally identified for a given set of parameters, if the Jacobian matrix of theoretical first and second moments with respect to these parameters has full rank. To check the identifiability over a sufficiently large parameter space we draw 1,000,000 times from the following distributions for the elasticities of the adjustment costs η_D, η_I, for the the off-diagonals π_{ij}, $i \neq j$ of Π, for the diagonals π_{ii} of Π, and the elements b_{ij}, $i \leq j$ of the lower triangular matrix B with $\Sigma = BB^T$:

$$\eta_D, \eta_I \sim U(0,4), \qquad \pi_{ij} \sim \mathcal{N}(0,0.1), \qquad \pi_{ii} \sim \mathcal{N}(0.8,0.1), \qquad b_{ij} \sim U(-0.05,0.05).$$

The Jacobian of the first and second moments (up to two lags) has full rank at approximately 99.9 percent of the draws. Thus, the model is virtually identifiable in the chosen parameter space.[8]

Brinca et al. (2018) provide and apply strategies for identification strength. They show that weak identification of the stochastic process' parameters is secondary, but this does not hold for structural ones. To address this problem, we compute the likelihood surface of the uncertain deep parameters η_D and η_I to detect a global maximum as well as the

[8]In comparison, we proceed similarly for the benchmark economy of Chari et al. (2007) presented in Appendix B. The Jacobian of the first and second moments (up to two lags) has no full rank at 26 parameter draws from 1,000,000.

likelihood's curvature and execute robustness checks in section 4.

3.2 The business cycle accounting procedure

The BCA procedure is divided into three separate steps: The estimation of the parameters, the identification of the wedge states, and the assessment of the contribution of a single wedges towards the business cycle.

MLE determines the matrices Π and Σ that characterize the stochastic process $\mathbf{s_t}$ as well as the elasticities η_I and η_D that define the level of adjustment costs. Full-information estimation of DSGE models is typically done with Bayesian methods, although MLE involves less assumptions. Applying Bayesian estimation is usually meaningful, since the researcher has a structural parametrization in mind and, by association, an idea of probable parameter values. We would like to stress that the application of BCA requires MLE and any restrictions like the Bayesian approaches, such as Otsu (2010), Chakraborty and Otsu (2013) or Plotnikov (2017) are questionable. The wedges are superpositions and interactions of a variety of market distortions with an underlying reduced-form stochastic process, which complicates the interpretation of the Markov transitions. Furthermore, recall the findings of Nutahara and Inaba (2012) that the VAR(1) strips a potentially more sophisticated stochastic process down. Thus, the estimated parameters are only pseudo-true for the real model. As a consequence, in general the values of the process' parameters cannot be interpreted, and a-priori assumptions of them are meaningless, and even more seriously, may restrict the set of mappable models. Thus, we make a point for MLE and let the data speak through an unrestricted VAR.[9]

After all parameters are pinned down, either by calibration or MLE, we use a state-smoothing algorithm as described in Durbin and Koopman (2012, Chapter 4.4) to predict the wedge's states $\mathbf{s_t}$.

In a last step, in line with Chari et al. (2007), we feed the wedges separately back into the model, while others are set constant, to assess the contribution of each wedge to the quantities of interest.[10]

[9] We would like to point out two technical issues regarding Bayesian methods and BCA. First, to the best of our knowledge, there is no prior that includes all combination parameter values that generate eigenvalues of Π less than one and excludes all combinations that do not have these properties. Second, the posteriors of a VAR-driven DSGE model can be multi-modal. This makes the commonly used RWMH algorithms unsuitable. For a deeper discussion and solution for the latter issue, see Herbst and Schorfheide (2015, Chapter 5, 6.1)

[10] See the technical appendix by Chari et al. (2007) for more details.

3.2.1 MLE

To evaluate the likelihood function of the linear state-space model (9)–(12), most of the literature uses a Kalman-recursion initialized at the unconditional mean and variance of the state vector $[\mathbf{x_0^T}\ \mathbf{s_0^T}]^\mathbf{T}$ (see e.g. DeJong and Dave, 2011, Chapter 8.4)). However, for an asymptotic stable state-space model, the mean squared error (MSE) $\mathbf{P_{t|t}}$ of the point estimate for $[\mathbf{x_t^T}\ \mathbf{s_t^T}]^\mathbf{T}$ conditional on a observed set of data $\{\mathbf{y_1},\ldots,\mathbf{y_t}\}$ converges to a matrix $\mathbf{P}$, the steady-state MSE, as t goes to infinity.[11] Exploiting this property, Chari et al. (2007) use the steady-state MSE $\mathbf{P}$ instead of the unconditional variance to initialize their Kalman-recursion. As pointed out by Huber (2020), it can be shown that the steady-state MSE $\mathbf{P}$ is equal zero in standard DSGE models like the one presented here.[12] To get the intuition behind the result and for the sake of simplicity, let us consider the case without growth and with zero adjustment costs. In this case, equations (2) and (5) rewrite to

$$\begin{aligned} K_{Xt+1} &= X_t + (1-\delta_X)K_{Xt} \\ &= \sum_{i=0}^{t-1}(1-\delta_X)^i X_{t-i} + (1-\delta_X)^t K_{X1},\ X \in \{I, D\}. \end{aligned}$$

Imagine we observe the investment $X_i \in \{I, D\}$ in capital and in durables for all $i = 1,\ldots,t$. Assuming that K_{X1} is normally distributed with variance σ_X^2, the variance of K_{Xt+1} conditional on $\{X_1,\ldots,X_t\}$ yields $(1-\delta_X)^{2t}\sigma_X^2$. Since $\delta_I, \delta_D \in (0,1]$, it is straightforward that the uncertainty regarding the endogenous states $\mathbf{x_t}$ disappears as t goes to infinity. Furthermore, assuming $\mathbf{L_s^y}$ is non-singular,[13] it follows that

$$\mathbf{s_t} = \left[\mathbf{L_s^y}\right]^{-1}\left(\mathbf{y_t} - \mathbf{L_x^y}\cdot\mathbf{x_t}\right). \tag{13}$$

Thus, as the uncertainty of the endogenous states $\mathbf{x_t}$ disappears as t goes to infinity, the uncertainty over the exogenous states $\mathbf{s_t}$ disappears as well. Using a Kalman-recursion initialized at the steady-state, with the steady-state MSE $\mathbf{P}$ is therefore equivalent to the assumption that the initial state vector is fixed and known, $[\mathbf{x_0^T}\ \mathbf{s_0^T}]^\mathbf{T} = \mathbf{0}_{nx+ns\times 1}$. Huber (2020) elaborates two major advantages of a fixed and known initialization at the long-

[11] For a formal proof, see e.g. Hamilton (1994, Chapter 13).

[12] As long as $\mathbf{L_s^y}$ is non-singular and $\frac{1-\delta_D}{\gamma_n\cdot g_D}, \frac{1-\delta_I}{\gamma_n\cdot g_I} \in [0,1)$ our prototype economy satisfies the preconditions of Proposition 1 by Huber (2020).

[13] Huber (2020) discusses how to deal with cases where $\mathbf{L_s^y}$ is singular. However, this case never occurred in our analysis.

run equilibrium. First, the likelihood evaluation can be vectorized and more important, it provides an analytical solution of the MLE for Σ since we can observe the residuals ϵ_t independently of Σ.[14] The solution of the MLE for Σ for a given Π is

$$\hat{\Sigma} = \frac{\mathbf{1}}{\mathbf{N}} \sum_{\mathbf{t=1}}^{\mathbf{N}} \left[(\mathbf{s_t} - \Pi \cdot \mathbf{s_{t-1}}) \cdot (\mathbf{s_t} - \Pi \cdot \mathbf{s_{t-1}})^{\mathbf{T}} \right], \; \mathbf{s_0} = \mathbf{0}_{ns \times 1}. \tag{14}$$

The estimates of a standard Kalman-recursion, which is initialized at the unconditional first and second moments, are more natural, since the initial states are usually unknown. Huber (2020) however shows that the average likelihood of the steady-state Kalman-recursion converges pointwise to the average likelihood of the standardly initialized Kalman-recursion. Therefore, we choose the estimates of the steady state Kalman-recursion as the initial guess for a second optimization of the likelihood-function. This second estimation bases on Kalman-recursion initialized with the unconditional first and second moments of the states $[\mathbf{x}_0^{\mathbf{T}} \; \mathbf{s}_0^{\mathbf{T}}]^{\mathbf{T}}$.

3.2.2 Data manipulation

The observables are GDP, investment, durables, government expenditures, net exports to GDP, and hours worked. Regressions with the logarithm of the first four observables as dependent variable and time as independent variable provide necessary components. The coefficient estimates determine the growth rates and the residuals the relative deviation from the particular growth path. Negative values for net exports prevent logarithmization. A regression with net exports relative to GDP as dependent variable and time as independent variable provides auxiliary variables. The coefficient is the excess growth rate of net exports compared to GDP growth. The residuals are the deviation from the long-run net exports to GDP rate, which is computable in the model. The residuals of these regressions are used for business cycle accounting, the coefficients for growth accounting.

Since hours worked per capita do not include a trend, the relative deviations from the long-run average are used for business cycle accounting. Whereas growth accounting is of course not applicable in this manner.

For a detailed data source, see Appendix C.

[14] Huber (2020) presents a detailed and more general version, Monte Carlo studies and further applications of this approach.

4 Results

4.1 Growth accounting

Table 5 presents the growth rates of the observables. The GDP annual trend growth rate is 1.32%. The amount of durables and investment goods grows slower than GDP, while net exports grow faster. Government consumption grows similar to GDP.

Table 5: Growth accounting

Parameter	Description	Value
$\ln(\gamma_n^4)$	Annual growth rate of population	0.03%
$\ln(g_Y^4)$	Annual growth rate of GDP	1.32%
$\ln(g_I^4)$	Annual growth rate of investment	0.93%
$\ln(g_D^4)$	Annual growth rate of durables	0.35%
$\ln(g_G^4)$	Annual growth rate of gov. cons.	1.40%
$\ln(g_E^4)$	Annual growth rate of net exports	1.65%

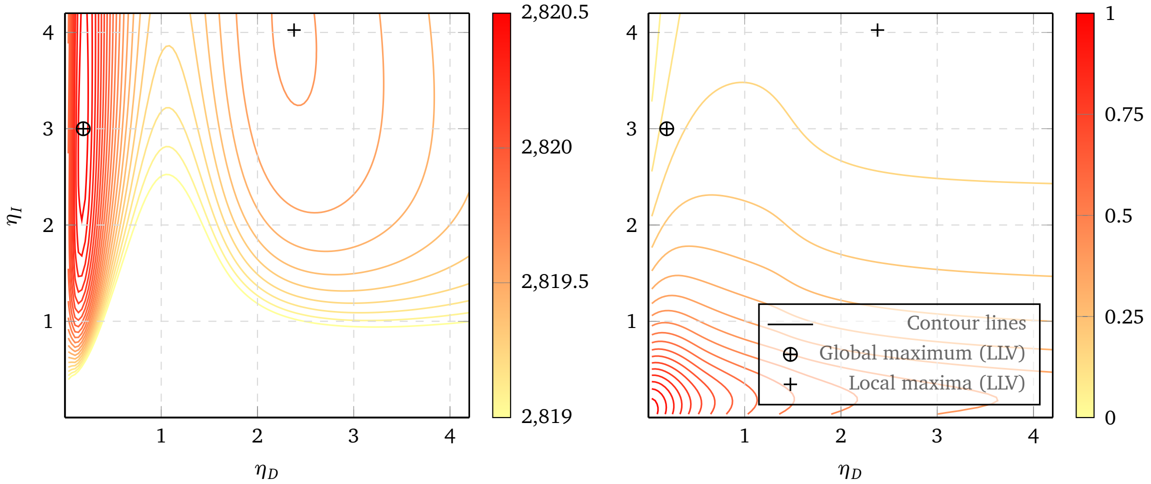
Similar to the shocks which drive the business cycle, the long-run components of the wedges P_{xt} and γ_z are reduced-form. Since we focus on the business cycle, we discuss only briefly potential causes for different growth rates. Differences in the long-run component of the durables and the investment wedge (P_{Dt}, P_{It}) may occur due to investment-specific technological change as described by Greenwood et al. (1997). The increase in German net exports since the launch of the Euro is investigated by in't Veld et al. (2014). The most important factors, summed up in P_{Et}, are: A higher German savings rate, positive supply shocks, especially due to labor market reforms, as well as a higher demand for German goods of non Euro area members.

4.2 Estimation

As already mentioned, the MLE includes Π, Σ, η_D and η_I. Panel 4(a) illustrates the likelihood function with respect to η_D and η_I, while Π and Σ are the argument maximum of the function for given η_I and η_D. The panel identifies two local maxima. The global is at $\eta_D = 0.19$ and $\eta_I = 3.00$.

Table 6 presents the estimates for the autoregressive matrix Π as well as second moments of the innovations ϵ_i. All wedges are highly autoregressive. The investment wedge

Figure 4: Maximum-Likelihood-Estimation



(a) Log-Likelihood (without constant) (b) Correlation between ϵ_{It} and ϵ_{Dt}

depends heavily on the other wedges with one lag. The innovations of the investment wedge have the highest volatility and are negatively correlated with the efficiency wedge. There is also a strong negative correlation between the innovations of the durables and the labor wedge. The net export wedge's innovation correlates with the labor wedge.

Panel 4(b) illustrates that the innovations of durables and investments are perfectly correlated in the absence of adjustment costs. Fehrle (2019) investigates different investment goods, vector-autoregressive processes and adjustment costs in detail and argues that adjustment costs can be viewed as a underpinning mechanism of reduced-form correlated shocks. Here, e.g. the mentioned high substitutability between durables and investments is prevented either by perfect correlated innovations, adjustment costs or a nest of them. Hence, it is useless to separate investments and durables without adjustment costs, since the corresponding wedges must co-move. Otherwise, as a result of Chang (2000), the high substitutability would lead to an excessive volatility of durables and investments and negative co-movements between them. However, this is contradicted by the data.

4.3 Business Cycle Accounting for the Great Recession and the German fiscal stimulus program, 2008-Q1 – 2011-Q3

The graphical analysis of our BCA exercise is reported in Figure 5. In Panels 5(a) to 5(e) we confront the observations of GDP, its subaggregates and hours worked with the model's prediction when only one wedge is allowed to fluctuate.

Table 6: Estimation of exogenous shock process

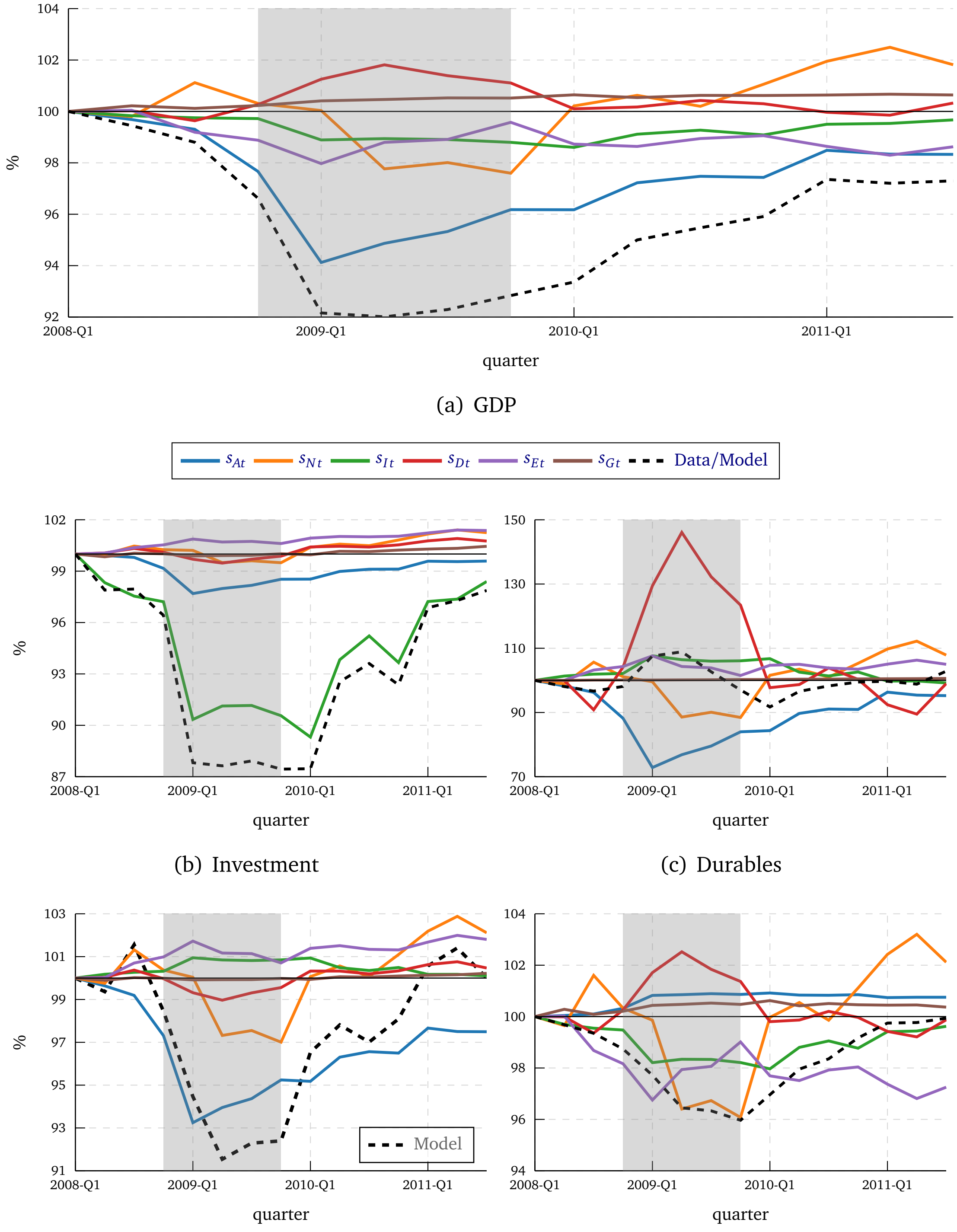
Autoregressive Matrix							
Π	$ln(s_A)$	s_N	s_I	s_D	s_E	$ln(s_G)$	
$ln(s_A)$	0.90	0.41	0.00	0.07	-0.21	-0.16	
s_N	0.01	0.83	0.01	-0.02	-0.12	-.01	
s_I	0.70	-1.71	0.96	-0.52	1.44	1.07	
s_D	0.27	-0.05	-0.00	0.66	0.16	-0.01	
s_E	0.06	-0.03	0.01	-0.05	0.62	-0.12	
$ln(s_G)$	-0.05	0.17	-0.01	-0.05	-0.22	0.80	
Correlation and standard errors							
$Corr(\epsilon_i, \epsilon_j)$	ϵ_A	ϵ_N	ϵ_I	ϵ_D	ϵ_E	ϵ_G	$100 \cdot StD(\epsilon_i)$
ϵ_A	1.00						0.94
ϵ_N	0.03	1.00					0.34
ϵ_I	-0.49	-0.06	1.00				7.12
ϵ_D	0.27	-0.83	0.13	1.00			1.44
ϵ_E	0.31	0.70	-0.02	-0.36	1.00		0.59
ϵ_G	-0.10	0.13	-0.19	-0.16	-0.13	1.00	0.80

Panel 5(a) illustrates that the crisis was mainly driven by the efficiency wedge. The investment and net exports wedge also contributed to the crisis. These three wedges together induced the decrease in GDP. The labor wedge contributed to the crisis from 2009-Q2 to 2009-Q4. Before, the wedge was counter-cyclical and afterwards it introduced the recovery. The durables wedge and government consumption were anti-cyclical. Panel 5(b) illustrates that the investment wedge drove the decline in investment mostly, while the efficiency wedge mattered little. The efficiency wedge influenced durables negatively as Panel 5(c) shows. The durables wedge on its own increased durables up to almost 50% in 2009. Afterwards, the wedge only had a slight impact. Panel 5(d) indicates that the efficiency wedge caused the decline in non-durable consumption mostly and the labor wedge partly. The durables and government consumption wedge had little impact on non-durable consumption. Panel 5(e) predicts the decline in net exports to GDP and the investment wedge introduced the decline in hours worked. The labor market wedge drove the decline between 2009-Q2 and 2009-Q4. Besides, the labor wedge was counter-cyclical. The other wedges were counter-cyclical.

Theory teaches us that the wedges of both investment goods D_t and I_t react similar to monetary policy and financial frictions in general.[15] Thus, Chari et al. (2007) and many

[15] Gertler and Gilchrist (2018) report for the U.S. financial frictions during the Great Recession a big negative

Figure 5: BCA - Results



(a) GDP

(b) Investment

(c) Durables

(d) Non-durable Consumption

(e) Hours

Notes: Dashed lines for GDP, investment, durables and hours are the data and the model's outcome. Here they are equivalent. The dashed lines for non-durable consumption is only the model's outcome. The gray area indicates the main effective period of the fiscal stimulus program 2008-Q4 – 2009-Q4.

others aggregate them. The business investment wedge drove the decline in business investment during the crisis. Financial frictions and other distortions dominated the fiscal and monetary policy measures. This is not true for durables. The only appreciable difference between the wedges during the crises were the car subsidies. Further, the positive impact of the durables wedge occurred simultaneously with the subsidies. The wedge began to stimulate the demand of durable goods with the introduction of the tax exemption for new cars in 2008-Q4. In 2009-Q1 the cash for clunkers program started, while the stimulating effect increased strongly. The stimulus disappeared between 2009-Q4 and 2010-Q1 while the last pay-off took place in 2009-Q4. Hence, we attribute the large increase due to the durables wedge to the car subsidies and can map changes due to the durables as well as government spending wedge to the fiscal stimulus program. The measures in other markets are dominated by frictions. Thus, it is unfortunately impossible to give statements about the measures with the chosen method.

With respect to GDP and hours, we find that the stimulus program due to the durables subsidies and government consumption had a positive effect during the crisis. The model predicts an approximately 2% bigger decline in GDP and an approximately 3.5% bigger decline in hours without changes in those wedges during the peak of the crisis (2009-Q2). Regarding non-durable consumption and investment the effect of the stimulus program is negative. Nevertheless, during the crisis the stimulus of durables and government consumption increased GDP and was not completely substituted by lower investments and non-durable consumption. Intertemporal substitution of durables investment in the aftermath of the program was small. The bust was driven by the efficiency wedge, which depressed durables over the whole period. The durables wedge virtually did not influence GDP negatively from 2008-Q1 till 2011-Q3.

The labor market wedge mitigated the crisis at the beginning and the end of the crisis. In particular at the end of the crisis, the model predicts an increase of more than 2% in GDP and more than 3% in hours worked.

The measurement ω_i quantifies the contribution of each wedge to GDP during the Great

impact on the durables market. Benmelech et al. (2017) explain one third of the decline in the U.S. car demand by frictions on the asset-backed commercial paper market. The decline in U.S. house prices weakens the household balance sheets, which also had a negative effect on the U.S. auto market, as shown by Mian et al. (2013).

Recession as

$$\omega_i = \frac{\sum_t (\hat{y}_t^{GDP} - \hat{y}_t^i)}{\sum_j \sum_t (\hat{y}_t^{GDP} - \hat{y}_t^j)} \text{ with } i, j \in \{s_A, s_N, s_I, s_D, s_G, s_E\},\ t \in [2008\text{-}Q1, ..., 2011\text{-}Q3],$$

where $\hat{y}_t^{GDP}$ is the GDP when all wedges are non-changing and $\hat{y}_t^i$ is the model outcome of wedge i alone. Thus, the contribution of all wedges together sums to 1, while the sign of ω_i points out if wedge i has mitigated $(-)$ or amplified $(+)$ a crisis.

The efficiency wedge accounts for 62% of the decline in GDP during this period, net exports for 26%, the investment wedge for 19%, and the labor market accounts for 3%. Government consumption accounts for -5% and the durables wedge for -4%. Since the effect of the durables wedge during the durables subsidies was at least twice as large as the effect of government consumption and effects throughout the whole crisis were similar but expenditures for these subsidies only made up for about 25% of the increase of government consumption, durables subsidies were more efficient to stimulate aggregated demand than government consumption.

With the identifying assumption that the fiscal stimulus program together with monetary policy were the only counter-cyclical distortions, our results represent a lower bound for the impact of fiscal and monetary policy measures as well as for the pro-cyclical distortions.

4.4 Robustness and discussion

Robustness in parameters. The results depend potentially on the values of adjustment costs η_I, η_D and on the intertemporal elasticity of substitution η. To evaluate the sensitivity, we calculate ω_i over a grid of the mentioned parameters. Therefore, we reestimate the (remaining) uncertain parameters at each node of the parameter grid.

Figure 6 illustrates the contribution of the concerning wedges for different amounts of adjustment costs. The efficiency wedge contributed the most to the decline in GDP, followed by net export for the whole set of adjustment costs. The results for the labor market wedge and government consumption are robust as well. The durables wedge mitigated the crisis for most of the parameter combinations. The contribution would have been pro-cyclical without adjustment costs. As mentioned above, in the absence of adjustment costs a separation of the durables and investment wedge is meaningless. The investment wedge's contribution to the crisis would have been negative for $\eta_I < 1/3$ where the likelihood is the lowest (see Panel 4(a)) and positive otherwise.

Subsidies in durables change the intertemporal rate of substitution. Hence, a robustness check to the elasticity of the substitution rate is relevant. Figure 7 presents the contribution to the decline in GDP over η. The contributions of the labor, investment, durables and the government consumption wedge are nearly constant. The contribution of net exports declines with a higher elasticity, nevertheless they contributed the second most over the whole domain. The contribution of the efficiency wedge increases with η.

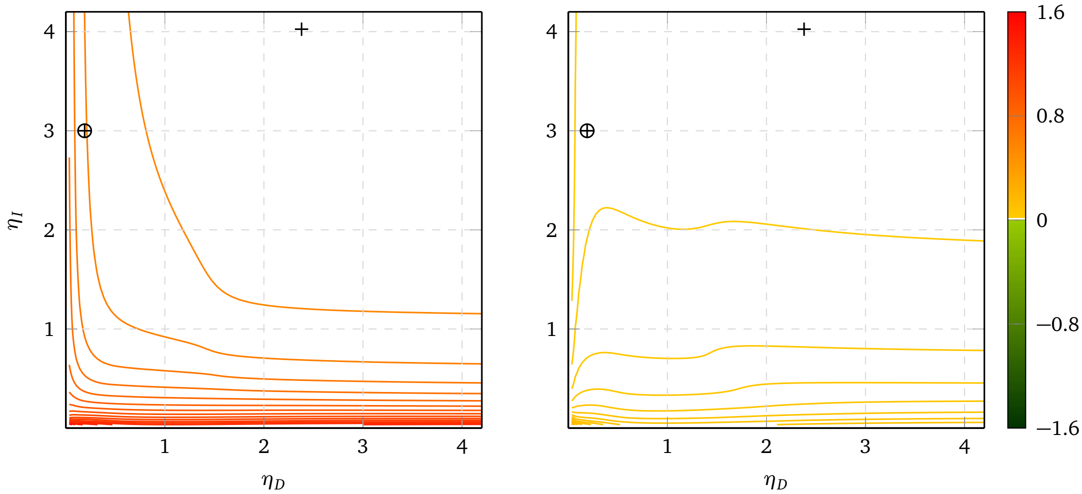
Robustness regarding the benchmark model. The assessment of the joint contribution of the investment and durables wedge as well as the joint contribution of government consumption and net exports maps our economy into the benchmark BCA economy ex post. The left panel of Figure 8 illustrates these effects. The right panel plots the impact of the investment and government spending wedge in the Chari et al. (2007) benchmark economy, where durables and investment as well as government spending and net exports are aggregated ex ante.[16] The results are similar, except in the more detailed economy the investment wedge was slightly counter-cyclical during the cash for clunkers program. Thus, the results of the detailed model are not counterfactual to the benchmark BCA model, but provide deeper insights.

Although the impact of the composed investment wedge was negligible during the Great Recession, our results suggest that the decomposed wedges were not. The pro-cyclical effect of the investment wedge and the policy-driven counter-cyclical effect of durables wedge offset each other. Hence, without our decomposition the importance of the investment wedge and, by association, the importance of financial frictions during the Great Recession is underrated. For example, the financial frictions of Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997), Bernanke et al. (1999), or Gertler and Kiyotaki (2010) are equivalent to the investment wedge.

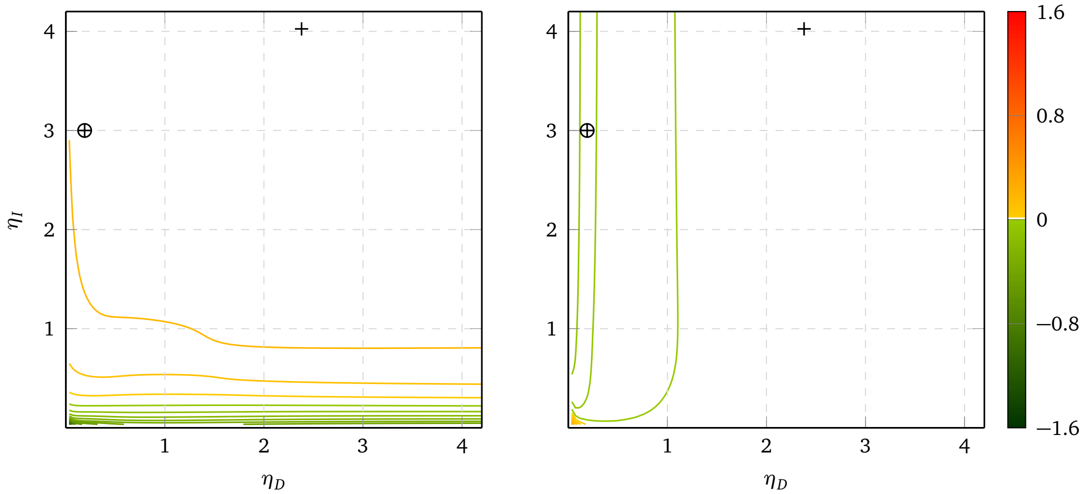
Comparing two durables boom-bust cycles. As mentioned, there were two boom-bust cycles in the durables market. We compare them in Figure 9. Panels 9(a) and 9(b) show the data and the impact of the durables wedge on durables from 2008-Q1 to 2010-Q4 and from 2006-Q1 to 2007-Q4. The durables wedge accounts during the car subsidies programs for the boom, but only marginally for the bust afterwards. During 2006 the VAT increase announcement passed the institutions and at this time durables investments increased. The introduction of the increase was in 2007-Q1, when the bust took place. The durables

[16]Appendix B sketches the model and provides our estimation strategy and results for the Chari et al. (2007) benchmark economy of the presented time series.

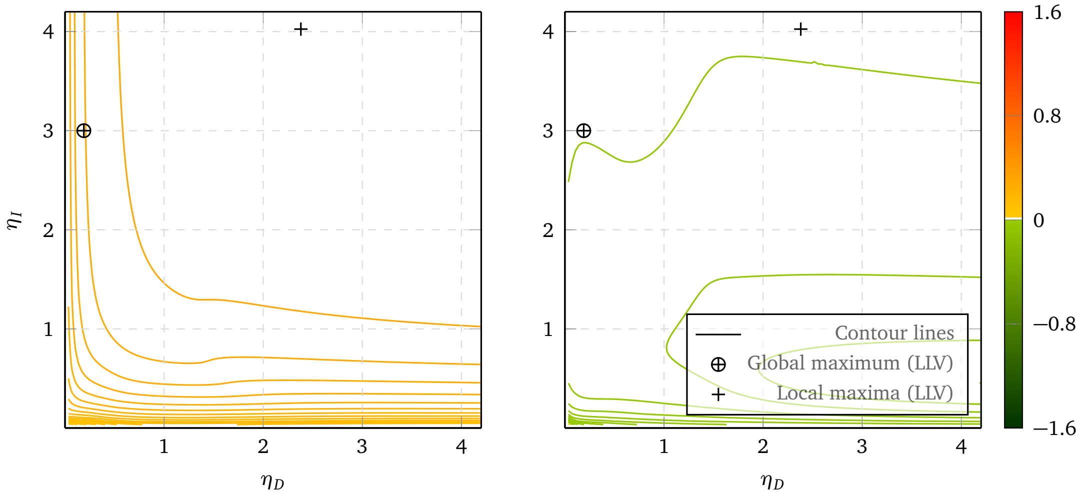
Figure 6: Adjustment costs specific wedge contribution



(a) Contribution of s_{At} to GDP ($\omega_{s_{At}}$) (b) Contribution of s_{Nt} to GDP ($\omega_{s_{Nt}}$)



(c) Contribution of s_{It} to GDP ($\omega_{s_{It}}$) (d) Contribution of s_{Dt} to GDP ($\omega_{s_{Dt}}$)



(e) Contribution of s_{Et} to GDP ($\omega_{s_{Et}}$) (f) Contribution of s_{Gt} to GDP ($\omega_{s_{Gt}}$)

Figure 7: Inverse elasticity of intertemporal substitution specific wedge contribution

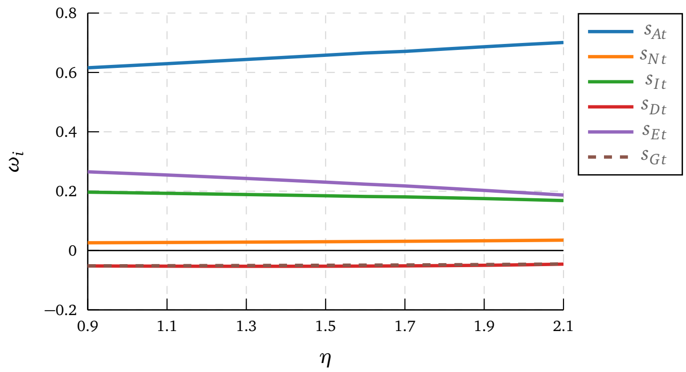
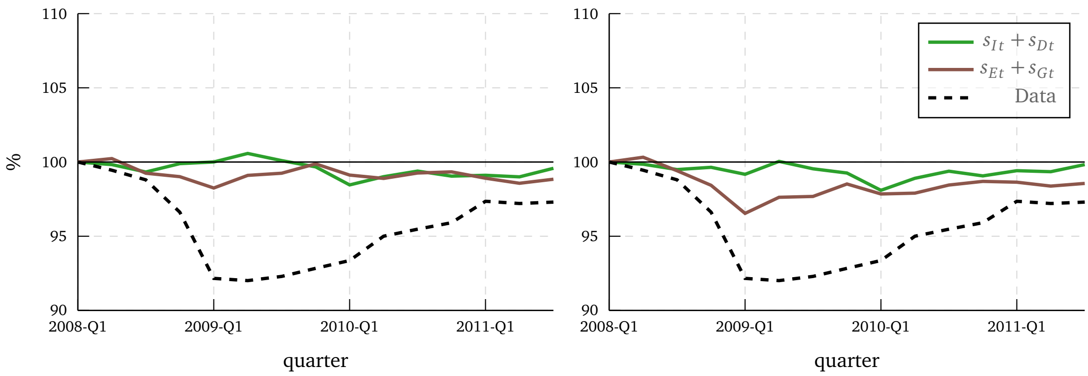


Figure 8: Robustness to the Chari et al. (2007) benchmark economy



(a) Detailed Economy

(b) Chari et al. (2007) - Benchmark Economy

wedge caused the whole boom-bust cycle and illustrates intratemporal substitution.[17]

5 Conclusion

We use the BCA analysis to investigate the impact of the German stimulus program during the Great Recession from 2008-Q1 to 2011-Q3. We extended the prototype economy by two wedges. Wedges correspond to the following variables: *government consumption, durables, investment, labor, net exports, and efficiency*. To account for the fiscal stimulus we map fiscal and monetary policy towards these wedges, thus enabling a policy evaluation.

We introduce two procedures that enable a fast and reliable MLE and the application

[17]Currently the German government adopted a temporary reduction of the VAT in the second half of 2020 to stimulate demand. This has the same intertemporal substitution effect as the policy under investigation. However, the temporary reduction of the VAT comes with a positive income effect.

Figure 9: The durables boom-bust cycles 2008-2010 and 2006-2007 in comparison

(a) Cash for clunkers program (b) VAT increase with announcement

of tools which help to overcome problems of weak identification. The first procedure separates between growth and business cycle accounting which ensures the stationarity of the underlying stochastic process. The second procedure is a new strategy to find a good guess for the argument maximum of the likelihood function. The applicability of MLE is crucial for, and one of the major advantages of BCA at the same time. Since MLE is difficult, and so Bayesian methods or other restrictions towards the stochastic process are used for BCA, we hope to give new impetus to the use of MLE and BCA with both procedures.

In our BCA analysis we find that the Great Recession in Germany was mainly driven by the efficiency wedge, net exports, and the investment wedge. In contrast, the durables and the government spending wedge acted counter-cyclical. We argue that the latter two collect parts of the German stimulus. The labor market wedge was pro-cyclical between 2009Q2 and 2009-Q4, besides it mitigated the crisis and especially induced the recovery. Due to higher expenditures for government consumption and a similar impact compared to the cash for clunkers program, subsidies for durable goods stimulated aggregated demand more efficiently. We check the robustness of our results to different choices of parameters that determine the elasticity of intertemporal substitution as well as capital and durables adjustment costs. We find that our results are robust for all wedges except the investment wedge. However, the results indicate that previous studies underrate the negative impact of the investment wedge and, as a consequence, the role of investment wedge equivalent financial frictions. We have to mention that BCA is only a first but useful step for the identification of market distortions, and thus we aim to motivate further research on the efficiency of durable goods' subsidies, the role of financial frictions during the Great

Recession and the labor market driven recovery in Germany.

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APPENDIX

Business cycle accounting for the German fiscal stimulus program

A Model

The following equations determine the model with stationary variables

$$y_t = k_{It}^{\alpha} (Z_t N_t)^{1-\alpha}, \tag{15}$$

$$r_t = \alpha \frac{y_t}{k_{It}}, \tag{16}$$

$$w_t = (1-\alpha) \frac{y_t}{N_t}, \tag{17}$$

$$\lambda_t = \phi c_t^{\phi(1-\eta)-1} k_{Dt}^{(1-\phi)(1-\eta)} (1-N_t)^{\psi(1-\eta)}, \tag{18}$$

$$(1-\tau_{Nt}) = \frac{\psi}{\phi} \frac{c_t}{(1-N_t) w_t}, \tag{19}$$

$$y_t = c_t + i_t + d_t + g_t + e_t, \tag{20}$$

$$\mu_{It} = \lambda_t \frac{1+\tau_{It}}{1-\Theta'_{It}}, \tag{21}$$

$$\mu_{Dt} = \lambda_t \frac{1+\tau_{Dt}}{1-\Theta'_{Dt}}, \tag{22}$$

$$g_I \cdot \gamma_n k_{It+1} = (1-\delta_I) k_{It} + i_t - \Theta_{It} \cdot k_{It}, \tag{23}$$

$$g_D \cdot \gamma_n k_{Dt+1} = (1-\delta_D) k_{Dt} + d_t - \Theta_{Dt} \cdot k_{Dt}, \tag{24}$$

$$\mu_{It} = \beta g_{M_I} \mathbb{E}_t \left[\mu_{It+1} \left(1 - \delta_I - \Theta_{It+1} + \frac{i_{t+1}}{k_{It+1}} \Theta'_{It+1} \right) + \lambda_{t+1} r_{t+1} \right], \tag{25}$$

$$\mu_{Dt} = \beta g_{M_D} \mathbb{E}_t \left[\mu_{Dt+1} \left(1 - \delta_D - \Theta_{Dt+1} + \frac{d_{t+1}}{k_{Dt+1}} \Theta'_{Dt+1} \right) + \lambda_{t+1} \frac{1-\phi}{\phi} \frac{c_{t+1}}{k_{Dt+1}} \right], \tag{26}$$

with

$$g_{M_I} = g_Y^{\phi(1-\eta)} \cdot g_D^{(1-\phi)(1-\eta)} \cdot g_I^{-1}, \tag{27}$$

$$g_{M_D} = g_Y^{\phi(1-\eta)} \cdot g_D^{(1-\phi)(1-\eta)-1}, \tag{28}$$

$$\Theta_{Xt} = \frac{a_X}{2} \left(\frac{x_t}{k_{Xt}} - b_X \right)^2, \tag{29}$$

$$\Theta'_{Xt} = a_X \left(\frac{x_t}{k_{Xt}} - b_X \right), \tag{30}$$

$$b_X = x^* / k_X^*, \tag{31}$$

with $X \in \{I, D\}$, $x \in \{i, d\}$ and where * indicates the steady-state value. The fluctuation in the model is driven by the VAR(1)-process

$$\underbrace{\begin{bmatrix} \ln(s_{At+1}) \\ s_{Nt+1} \\ s_{It+1} \\ s_{Dt+1} \\ s_{Et+1} \\ \ln(s_{Gt+1}) \end{bmatrix}}_{:=\mathbf{s_{t+1}}} = \Pi \underbrace{\begin{bmatrix} \ln(s_{At}) \\ s_{Nt} \\ s_{It} \\ s_{Dt} \\ s_{Et} \\ \ln(s_{Gt}) \end{bmatrix}}_{:=\mathbf{s_t}} + \underbrace{\begin{bmatrix} \epsilon_{At+1} \\ \epsilon_{Nt+1} \\ \epsilon_{It+1} \\ \epsilon_{Dt+1} \\ \epsilon_{Et+1} \\ \epsilon_{Gt+1} \end{bmatrix}}_{:=\epsilon_{\mathbf{t+1}}}, \qquad \epsilon_t \sim \mathcal{N}(0, \Sigma). \tag{32}$$

The stochastic process affects the wedges as follows

$$Z_t = A^* \cdot s_{At}, \tag{33}$$

$$\tau_{Nt} = \tau_N^* + s_{Nt}, \tag{34}$$

$$\tau_{It} = \tau_I^* + s_{It}, \tag{35}$$

$$\tau_{Dt} = \tau_D^* + s_{Dt}, \tag{36}$$

$$e_t = e^* + s_{Et}, \tag{37}$$

$$g_t = g^* \cdot s_{Gt}. \tag{38}$$

B Chari et al. (2007) benchmark

B.1 Model

$$\begin{aligned}
y_t &= k_t^{\alpha}(Z_t N_t)^{1-\alpha}, && (39)\\
r_t &= \alpha \frac{y_t}{k_t}, && (40)\\
w_t &= (1-\alpha)\frac{y_t}{N_t}, && (41)\\
\lambda_t &= c_t^{(1-\eta)-1}(1-N_t)^{\psi(1-\eta)}, && (42)\\
(1-\tau_{Nt}) &= \psi \frac{c_t}{(1-N_t)w_t}, && (43)\\
y_t &= c_t + i_t + g_t, && (44)\\
\mu_{It} &= \lambda_t \frac{1+\tau_{It}}{1-\Theta'_{It}}, && (45)\\
g_I \cdot \gamma_n k_{t+1} &= (1-\delta_I)k_t + i_t - \Theta_{It}\cdot k_t, && (46)\\
\mu_{It} &= \beta g_{M_I}\mathbb{E}_t\left[\mu_{It+1}\left(1-\delta_I-\Theta_{It+1}+\frac{i_{t+1}}{k_{t+1}}\Theta'_{It+1}\right)+\lambda_{t+1}r_{t+1}\right], && (47)
\end{aligned}$$

with

$$\begin{aligned}
g_{M_I} &= g_Y^{1-\eta}\cdot g_I^{-1}, && (48)\\
\Theta_{It} &= \frac{a_I}{2}\left(\frac{i_t}{k_t}-b_I\right)^2, && (49)\\
\Theta'_{It} &= a_I\left(\frac{i_t}{k_t}-b_I\right), && (50)\\
b_I &= i^*/k^*, && (51)
\end{aligned}$$

where * indicates the steady-state value.

The fluctuation in the model is driven by the VAR(1)-process

$$\underbrace{\begin{bmatrix}\ln(s_{At+1})\\ s_{Nt+1}\\ s_{It+1}\\ \ln(s_{Gt+1})\end{bmatrix}}_{:=\mathbf{s}_{t+1}} = \Pi \underbrace{\begin{bmatrix}\ln(s_{At})\\ s_{Nt}\\ s_{It}\\ \ln(s_{Gt})\end{bmatrix}}_{:=\mathbf{s}_t} + \underbrace{\begin{bmatrix}\epsilon_{At+1}\\ \epsilon_{Nt+1}\\ \epsilon_{It+1}\\ \epsilon_{Gt+1}\end{bmatrix}}_{:=\epsilon_{t+1}}, \qquad \epsilon_t \sim \mathcal{N}(0,\Sigma). \tag{52}$$

The stochastic process affects the wedges as follows

$$Z_t = A^* \cdot s_{At}, \tag{53}$$

$$\tau_{Nt} = \tau_N^* + s_{Nt}, \tag{54}$$

$$\tau_{It} = \tau_I^* + s_{It}, \tag{55}$$

$$g_t = g^* \cdot s_{Gt}. \tag{56}$$

B.2 Observables and data manipulation

The vector of observables reads as follows $\mathbf{y_t} = \begin{bmatrix} \hat{y}_t & \hat{N}_t & \hat{i}_t & \hat{g}_t \end{bmatrix}^{\mathbf{T}}$. In contrast to our modified model government consumption is the sum of government consumption and net exports and investments are the sum of durables and investments.

B.3 Calibration and estimation

The calibration and estimation strategy is similar to our modified model. We estimate the elasticity of the price of capital η_I as well as the parameters of the stochastic process. All other parameters are calibrated and the long-run ratios are pined down to their long-run averages. Tables 7 and 8 present all relevant parameters.

Table 7: Calibration and growth accounting for the Chari et al. (2007) economy

Parameter	Description	Value
α	Capital share	0.34
β	Discount factor	0.994
δ_I	Rate of capital depreciation	0.0203
ψ	Preference weight of labor	2.24
η	Risk aversion	1
η_I	Elasticity of the price of capital	0.86
$\ln(\gamma_n^4)$	Annual growth rate of population	0.03%
$\ln(g_Y^4)$	Annual growth rate of GDP	1.32%
$\ln(g_I^4)$	Annual growth rate of investment	0.79%

Table 8: Estimation of exogenous shock process of the Chari et al. (2007) economy

Autoregressive Matrix					
Π	$ln(s_A)$	s_N	s_I	$ln(s_G)$	
$ln(s_A)$	0.93	0.09	0.05	-0.03	
s_N	-0.01	0.73	0.04	-0.00	
s_I	0.03	2.03	0.67	-0.02	
$ln(s_G)$	0.09	-1.17	0.08	0.84	

Correlation and standard errors					
$Corr(\epsilon_i, \epsilon_j)$	ϵ_A	ϵ_N	ϵ_I	ϵ_G	$100 \cdot StD(\epsilon_i)$
ϵ_A	1.00				0.94
ϵ_N	0.21	1.00			0.29
ϵ_I	-0.27	-0.61	1.00		1.77
ϵ_G	0.43	0.77	-0.34	1.00	2.71

C Data

The data is taken from the Fachserie 18: National accounts, domestic product from the German Federal Statistical Office.

- **Pop:** Total Population 1991:I-2018:I

  Source: 2.1.7 Population and labour force participation 1; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

- **Hours:** Hours worked by persons in employment 1991:I-2018:I

  Source: 2.1.8 Persons in employment, employees and hours worked (domestic concept) 2; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

- **GDP:** 1991:I-2018:I

  Nominal source: 2.3.1 Use of gross domestic product at current prices 2; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

  Real source: 2.3.2 Use of gross domestic product, price-adjusted 2; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

- **PCE:** Private Consumption Expenditures of households 1991:I-2018:I

  Nominal source: 2.3.3 Final consumption expenditure at current prices 3; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

  Real source: 2.3.4 Final consumption expenditure at , price-adjusted; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

- **Govern. Consumption:** Government final consumption expenditure (domestic use) 1991:I-2018:I

  Nominal source: 2.3.3 Final consumption expenditure at current prices 3; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

  Real source: 2.3.4 Final consumption expenditure at , price-adjusted; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

- **Investment:** Gross fixed capital formation 1991:I-2018:I

  Nominal source: 2.3.1 gross fixed capital formation at current prices 2; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

  Real source: 2.3.2 gross fixed capital formation, price-adjusted 2; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

- **Net Exports:** Balance of exports and imports 1991:I-2018:I

  Nominal source: 2.3.1 Balance of exports and imports at current prices 2; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

  Real source: 2.3.2 Balance of exports and imports, price-adjusted 2; Seasonally adjusted quarterly results using Census X-12-ARIMA and BV4.1 - Fachserie 18 Reihe 1.3 - 1st Quarter 2018

- **Durables:** Langlebige Güter (Durable Goods) 1991:I-2018:I

  Nominal source: 2.14 Konsumausgaben der privaten Haushalte im Inland nach Dauerhaftigkeit der Güter, Saison- und kalenderbereinigt in jeweiligen Preisen 4; Private Konsumausgaben und Verfügbares Einkommen - 1. Vierteljahr 2018

  Real source: 2.14 Konsumausgaben der privaten Haushalte im Inland nach Dauerhaftigkeit der Güter, Saison- und kalenderbereinigt - preisbereinigt 4; Private Konsumausgaben und Verfügbares Einkommen - 1. Vierteljahr 2018

  (available in German only: Domestic consumer spending on durable goods, seasonally and calendar adjusted 4; Private consumption expenditure and disposable income - 1st quarter of 2018)