Inefficient lock-in and subsidy competition

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Abstract

This paper shows that subsidy competition may be efficiency enhancing. We model a subsidy game among two asymmetric regions in a new trade model, where capital can freely move among regions, but capital rewards are repatriated. We study subsidy competition, starting from an equilibrium where the industry core is inefficiently locked in to the smaller region. When regions weigh workers’ and capitalists’ welfare equally, the core region will set its subsidy low enough that the industry relocates to the larger region, restoring an efficient allocation. When workers’ welfare is weighted more heavily, the core may pay subsidies that are high enough to prevent a relocation of industry.

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1 Introduction

The many merits and drawbacks of capital tax competition and other forms of locational or jurisdictional competition have been established in a by now sizable literature.\(^1\) This paper advances a novel argument in favor of tax competition: inefficient lock-ins of industry can potentially be overcome, and a shift to a more efficient equilibrium be induced, through competition in capital subsidies.

Inefficient lock-in situations are well-known from the field of technology adoption (David, 1985; Arthur, 1989). Arguably the most famous example is the computer keyboard, which despite technologically superior systems today still has the same layout – a succession of letters beginning with QWERTY in the topmost row – as the old typewriter.

Decreasing unit costs and multiple equilibria are also a hallmark of the new trade theory and of economic geography. This research has unveiled that ‘history matters’ for national or regional specialization, and that it cannot be assured that the best equilibrium is chosen. Krugman and Obstfeld (2009) provide a simple textbook example that countries can get locked into undesirable specialization patterns when industries are competitive and there are external economies of scale at the country level: two countries, Switzerland and Thailand, are both (potentially) able to supply the world demand for watches at decreasing average costs. Although Thailand could (by assumption) do so more cheaply at any scale, the Swiss industry, has (historically) established its industry first. This head start and the associated scale of production implies that the Swiss industry has lower unit costs compared to a Thai watch firm which considers to enter the market, but realizes that it could not competitively produce the first unit in isolation (i.e. given that a watch industry is yet non-existing in Thailand). Path dependencies and hysteresis effects in location have similarly been shown to arise in the more recent economic geography models (see e.g. Fujita et al., 1999; Baldwin et al., 2003). Anecdotal evidence documenting that agglomeration patterns may persist even though the initial factors have vanished over time have been presented early on by Krugman (1991a,b). More recent econometric evidence documented in Redding et al. (2007) reinforces the hypothesis that history may matter: they find that the temporary shock of the division of Germany after World War II had a permanent effect on industry location in the sense that there are no signs that the associated shift of the German air hub from Berlin to Frankfurt is only temporary.

\(^1\)Recent surveys of this literature are provided in Wilson and Wildasin (2004) and in Wilson (1999).
These lock-in effects – in the fields of technology adoption, international trade and economic geography or other fields – have in common that a shift from (say) an inefficient equilibrium to a potentially more efficient equilibrium is prevented by a coordination failure among the agents. The starting point of our analysis is a situation of an inefficient lock-in of industry, where no single firm finds it profitable to shift location even though a coordinated move would make all of them better off. Following Martin and Rogers (1995), we develop a simple two region model of monopolistic competition. The commercial relations between regions consist of intra-industry trade based on love-of-variety on the part of consumers and mobility of physical capital. We make two key assumptions. First, as in Martin and Rogers (1995), regions may differ in size. Given the assumption that firms produce with internal increasing returns, and in the absence of other differences between regions, this has the well-known implication that the larger region attracts a more than proportionate share of firms (the ‘home market effect’). Second, there are localized intra-industry spillovers (e.g. knowledge spillovers) among monopolistic producers and also inter-industry spillovers from the modern sector to the other sector. Accordingly, local marginal production costs are lower, the more numerous local firms are. Taken together, these two key assumptions imply that, given a suitable set of parameters, the model has two stable equilibria which can unambiguously be welfare-ranked. One equilibrium has all firms concentrated in the larger region, exploiting both the advantages of the large market and the advantages associated with the external economies. However, quite intuitively, if the intra-industry spillovers are strong enough there also exists a second, inefficient equilibrium where all firms concentrate in the smaller region but are unable to coordinate on a shift to the more efficient equilibrium.

Our subsidy game starts from such an inefficient equilibrium, where all the industry is located in the smaller region (say region 2). Governments are assumed to dispose of one instrument, direct capital payments, which are financed through non-distortionary taxes, and which can be offered to the capital owners. Following a recent literature, we assume that the subsidy game is in three stages (e.g. Baldwin and Krugman, 2004; Borck and

\footnote{Localized external economies of scale have obtained strong empirical evidence. See the surveys by Audretsch and Feldman (2004) and Rosenthal and Strange (2004) and the recent paper by Badinger and Egger (2008), which finds strong empirical evidence in favor of intra-industry spillovers and also, though less strong, inter-industry spillovers for OECD manufacturing. Indirect evidence of intra-industry spillovers is provided by Devereux et al. (2007) who find that firms of a specific industry respond to subsidies only in the region which already hosts a critical share of the respective industry.}
in the first stage, the core region (the government in region 2) sets its subsidy, in the second stage, the government in the periphery (region 1) chooses its capital subsidy and the market allocation then unfolds in the third stage. The welfare functions of the regional governments are utilitarian with possibly different weights attached to workers and capital owners in their region.

Our main results are the following. If governments attach equal weight to capital owners and workers, then region 2 will never defend the core. Rather, it will accept that the more populous region 1 snatches the core by offering a capital subsidy which is just high enough to induce all capital to relocate. Intuitively, the larger region has an advantage in the competition game, because the agglomeration rent accruing to capital owners is larger when all capital is located in the larger region. Although residents of the smaller region benefit from a lower price index and higher wages when the core is located in their region, given that subsidies to capital accrue to capital owners in both regions, it becomes too costly for the government of the (smaller) core region to hold on to the core once the (larger) periphery actively bids for firms. Joint welfare as well as welfare in the two regions then increases. If, by contrast, governments assign a higher weight to workers’ than to capital owners’ welfare, there is a set of parameters where the smaller region defends the core, the inefficient lock-in persists, the periphery gains and the core loses in comparison with the situation before the start of this subsidy game. Intuitively, although allowing capital to relocate would allow capital owners to benefit from subsidies paid by the new core, this benefit would weigh less than the loss incurred in the form of lower wages and higher prices when the core region lets its industry go. Hence, in this case, the core will want to defend the core, even though global efficiency would rise if all industry were located in the larger region.

Our paper is related to several strands of previous research, neither of which has come up with the argument in favor of subsidy competition advanced here, however.

First, our paper is related to the literature on tax competition. The traditional literature in this field is based on models with perfectly competitive markets and stresses that, as a result of fiscal externalities, taxes and government expenditures are bid down by benevolent governments to sub-optimal levels. There are circumstances, however, when tax competition may be favourable, notably when without such competition tax rates are inefficiently high. In this spirit, Edwards and Keen (1996) show that tax competition may help tame Leviathan governments, and Kehoe (1989) shows that tax competition may
alleviate excessive capital taxation in the absence of government commitment. However, lock-in situations do not arise in this traditional literature.

Second, a more recent literature reconsiders tax competition in the presence of market power on goods markets. Research in the tradition of the new economic geography (typically) uses models of monopolistic competition and shows that the government in the core region is able to maintain a higher tax on capital than the government in the periphery. A result similar in spirit has been obtained by Haufler and Wooton (1999). They show that in the competition to attract a foreign-owned monopolist, the government of the larger region is able to achieve this at a lower cost than the small region government. This result is based on the fact that the monopolist – similar to the firms in the differentiated goods sector in models of the new economic geography – has a locational preference for the larger market. Different market sizes are also studied by Ottaviano and van Ypersele (2005) who analyse monopolistic competition with mobile capital but without endogenous agglomeration, to show that, under certain conditions (notably when trade costs are low enough) tax competition is efficiency enhancing.

Even though our model has much in common with these studies, there are important differences, the most important one being that an inefficient lock-in – our starting point – has not been considered in this literature. The papers on tax competition and economic geography analyze symmetric-identical regions which are endogenously driven into a core-periphery constellation. Due to this fundamental symmetry, from a welfare perspective it is immaterial which region ends up being the core – hence there is no welfare improvement associated with a switch of the core. Haufler and Wooton (1999) and Ottaviano and van Ypersele (2005) allow for different market sizes, but they do not consider local external economies. Hence, the tension between local intra-industry spillovers and market size considerations, which gives rise to an inefficient lock-in is not present in their models.

Finally, there is a literature which addresses the coordination failure that emerges in models with decreasing average costs. In the context of city-industry equilibria considered in urban economics, the sustainability of inefficient lock-ins is contested by the idea of profit-seeking ‘land developers’. The idea, put forward by Henderson (1975), holds that

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3Important work in this area is by Janeba (2000). See also the surveys cited in footnote 1.
5Note, however, that this does not imply that the a core-periphery constellation is necessarily the welfare optimum. See e.g. Ottaviano and Thisse (2002) and Pflüger and Südekum (2008).
the existence of more efficient city sites can be exploited by forward-looking developers, who, by this efficiency differential, are able to profitably organize ‘city corporations’, and, hence to restore an overall efficient allocation. This idea has been revived by Rauch (1993) who shows that discriminatory pricing of land over time on the part of developers is key to the removal of such inefficiencies. Another mechanism to overcome multiple equilibria and coordination failures arising under external economies of scale has recently been worked out by Grossman and Rossi-Hansberg (2008). They analyze a model where production of final goods uses a continuum of tasks, each of which has a zero weight, and which can possibly be performed in two locations. They show that, by becoming external suppliers for these tasks, even ‘small’ agents can alleviate coordination problems.

Our analysis relates to these works insofar as we also address the coordination issue. In a non-technical paper, (Duranton, 2008, p.40) has recently put forward the intuitive notion that territorial competition can improve the spatial allocation of plants because “the places for which the external effects are the strongest are expected to bid the most”. We provide a formal analysis which is much in this spirit, but where the interaction of external economies and market size is key.

The remainder of the paper is organized as follows. The next section describes the model and the locational equilibria. A welfare analysis is conducted for symmetric and asymmetric region size. Section 3 analyzes the outcomes of subsidy competition between asymmetrically sized regions. The last section concludes.

2 The Model

2.1 Basic Set Up

The model builds on Martin and Rogers (1995). The world consists of two regions, indexed by \( i = 1, 2 \), which are symmetric in preferences and technology. There are two sectors. The modern sector \( (M) \), characterized by increasing returns, monopolistic competition and iceberg trade costs, produces a composite of industrial varieties. Spatial distance is modeled using iceberg trade costs. To consume one unit of a variety produced abroad, \( \tau > 1 \) units have to be shipped; the remainder melts away in transit.

The perfectly competitive traditional sector \( (A) \) produces a homogenous good under constant returns to scale. The \( A \)-good is taken as the numéraire good and hence, its price
is normalized to one, \( p_i^A = 1 \). We assume that the traditional good is produced in both regions and is traded without costs across regions.

There are two input factors, capital and labor. Each worker owns one unit of labor and each capitalist one unit of capital, which they both supply inelastically. The mass of workers and the mass of capitalists are both normalised to unity. Region 1 hosts the share \( s_l \) of workers and the share \( s_k \) of capital owners. Labor is immobile across regions and employed in both sectors. Capital is employed in the modern sector only, and each firm requires one unit of capital. Capital can be freely moved across the two regions, but capital owners are immobile. We assume perfect portfolio diversification: each capitalist owns an equal share of the international portfolio which delivers the return \( s_n r_1 + (1 - s_n) r_2 \), where \( r_i \) is the return to capital invested in region \( i \) and \( s_n \) is the share of capital (and, hence, firms) installed in region 1. The capital income of region 1 is therefore given by \( s_k (s_n r_1 + (1 - s_n) r_2) \).

### 2.2 Preferences and Demand

Households derive utility from consuming a range of differentiated modern goods and the traditional good. Preferences are represented by a two tier utility function, where the upper tier function is logarithmic quasi-linear and the lower tier utility function is CES. The utility function of a type-\( h \) individual (capitalist or worker) in region \( i \) is\(^6\)

\[
U_{ih}(A_{ih}, M_i) = \alpha \ln M_i + A_{ih} \quad \text{for } h = K, L.
\]  

A type-\( h \) individual in region \( i \) receives income \( y_{ih} \). We assume \( 0 < \alpha < y_{ih}, i = 1, 2, h = K, L, \) to assure that both types of goods are consumed by all individuals in each region. \( A_{ih} \) denotes consumption of the numéraire good and \( \alpha \) the amount of income spent on the composite good (see below). Consumption of the modern good \( M_i \) consists of all differentiated varieties \( v \):

\[
M_i = \left( \int_0^{n_i} m_{ii}(v)^{\frac{\sigma}{\sigma - 1}} dv + \int_{n_i}^{n_i + n_j} m_{ji}(v)^{\frac{\sigma}{\sigma - 1}} dv \right)^{\frac{\sigma - 1}{\sigma}}, \quad \sigma > 1, \quad i \neq j,
\]

where \( m_{ii} \) denotes consumption of a variety produced domestically and \( m_{ji} \) denotes consumption of a variety produced abroad. The constant elasticity of substitution between

\(^6\)To simplify notation, we use the fact that – due to quasilinear utility – all individuals consume the same amount of modern goods.
any two varieties is denoted by $\sigma$. The budget constraint of a representative household reads

$$\int_0^{n_i} p_i(v)m_{ii}(v)dv + \int_{n_i}^{n_i+n_j} \tau p_j(v)m_{ji}(v)dv + A_{ih} = y_{ih},$$

where $p_i$ and $p_j$ denote the producer prices of a respective variety. Solving the utility maximization problem yields the following demand functions, $m_{ii}(v)$, $m_{ji}(v)$, $M_i$ and $A_{ih}$ and indirect utility $V_{ih}$:

$$M_i = \alpha/P_i,$$
$$A_{ih} = y_{ih} - \alpha,$$
$$m_{ii} = \alpha p_i(v)^{-\sigma}P_i^{\sigma-1},$$
$$m_{ji} = \alpha(\tau p_j(v))^{-\sigma}P_i^{\sigma-1}.$$

$$P_1 \equiv [s_n p_1^{1-\sigma} + (1 - s_n)(\tau p_2)^{1-\sigma}]^{1/\sigma},$$

$$V_{ih} = y_{ih} - \alpha \ln P_i,$$

where $P_1$ denotes the CES price index in region 1 which already takes symmetry of producer prices into account. An analogous expression holds for the CES price index of region 2.

## 2.3 Production

We will henceforth derive all expressions for region 1 only. The corresponding expressions for region 2 are analogous.

### 2.3.1 Traditional sector

The A-good is produced using labor as the only input according to $q_1^A = (1+\mu s_n)L_1^A$, where $L_1^A$ is labor input and $q_1^A$ is output. The term $\mu s_n$ captures inter-industry spillovers, with $\mu > 0$. The larger the domestic share of firms, $s_n$, the higher is the marginal productivity of labor and the more units of the A-good can be produced with a given labor force. Due to perfect competition labor is paid its marginal product. Hence, we get $w_1 = 1 + \mu s_n$.\(^7\)

\(^7\)Note that contrary to previous economic geography models which assume that the immobile factor earns the same reward irrespective of whether employed in the concentrated or in the peripheral region we allow for a higher wage rate in the region where industry is agglomerated.
2.3.2 Modern sector

The representative firm in region 1 produces one variety using one unit of capital (the fixed input requirement) and \(1/(1 + \gamma s_n)\) units of labor according to the total cost function

\[ TC_1 = \left(1 + \frac{\mu s_n}{1 + \gamma s_n}\right)q_1 + r_1, \]  

(7)

where \(q_1\) is a firm’s output in region 1. Its fixed costs are given by \(r_1\) and its marginal costs are determined by the variable input requirement and by the wage as previously determined. Intra-industry spillovers \(\gamma\) have a positive effect on the productivity of a firm. The proximity to other producers in the same industry generates knowledge spillovers which lower firms’ variable costs. Inter-industry spillovers, on the other hand, drive up wages in the region and hence, the firm’s variable costs. In line with the empirical evidence we assume that spillovers are stronger within an industry than between different industries, i.e. spillovers increase industry specific skills of a worker more than general skills.\(^8\) The profit function of the representative firm in region 1 is given by

\[ \Pi_1 = \left(p_1 - \frac{1 +\mu s_n}{1 + \gamma s_n}\right)q_1 - r_1. \]  

(8)

Market clearing requires a firm’s supply \(q_1\) to be equal to aggregate demand, which consists of domestic and export demand, including the indirect demand associated with the iceberg trade costs:

\[ q_1 = m_{11}(s_l + s_k) + \tau m_{12}\left((1 - s_l) + (1 - s_k)\right). \]  

(9)

Equation (9) uses the familiar result that mill pricing is optimal in the Dixit Stiglitz model. Profit maximization yields optimal mill prices which are constant markups on marginal costs:

\[ p_1 = \frac{\sigma}{\sigma - 1}\left(1 + \frac{\mu s_n}{1 + \gamma s_n}\right). \]  

(10)

Using the zero pure profit condition and applying mill prices from (10) yields the break even output \(q_1\) of a firm:

\[ q_1 = r_1\left(\sigma - 1\right)\left(1 + \frac{\gamma s_n}{1 + \mu s_n}\right). \]  

(11)

2.3.3 Short run equilibrium

In the short run, the allocation of capital and hence the allocation of firms is exogenous. Eqs. (10) and (11) then immediately imply \( r_i = \frac{(p_i q_i)}{\sigma} \), i.e. the capital reward captures operating profits. Using this result as well as the mill prices from (10) and the market clearing condition (9), we find:

\[
\begin{align*}
    r_1 &= \frac{\alpha}{\sigma} \left( \frac{s_l + s_k}{s_n + (1 - s_n) \phi \chi} + \frac{\phi((1 - s_l) + (1 - s_k))}{\phi s_n + (1 - s_n) \chi} \right), \\
    r_2 &= \frac{\alpha}{\sigma} \left( \frac{\phi(s_l + s_k) \chi}{s_n + (1 - s_n) \phi \chi} + \frac{((1 - s_l) + (1 - s_k)) \chi}{s_n \phi + (1 - s_n) \chi} \right),
\end{align*}
\]

where

\[
\chi \equiv \left( \frac{p_2}{p_1} \right)^{1-\sigma} = \left( \frac{1 + \mu(1 - s_n)}{1 + \mu s_n} \right)^{1-\sigma}
\]

and \( \phi \equiv \tau^{1-\sigma} \) is the level of trade freeness, with \( 0 \leq \phi \leq 1 \).

2.4 Long run equilibrium and welfare: the symmetric case

In the long run, capital is mobile and moves to the location where it earns the highest return. We assume that this movement is governed by the ad-hoc adjustment equation:

\[ s'_n = (r_1 - r_2)(1 - s_n)s_n. \]

A long run equilibrium is defined as a situation where capital no longer moves across regions. In this model, there are two types of locational long-run equilibria. Depending on the relative strength of centripetal and centrifugal forces industry will be either dispersed (symmetric interior equilibrium, where \( r_1 = r_2 \)) or agglomerated in one single region (a core-periphery equilibrium) at \( s_n = 0 \) (with \( r_1 > r_2 \)) or \( s_n = 1 \) (with \( r_1 < r_2 \)).

The different locational equilibria which emerge for different levels of trade costs are depicted in Figure 2 for the case where regions are equal sized. The parameters are \( \alpha = 0.3, \sigma = 4, \mu = 0.5, s_l = s_k = 0.5, \gamma = 1 \).

A symmetric equilibrium is stable for low trade freeness, e.g. \( \phi = 0.17 \). Starting from \( s_n = 1/2 \), increasing region 1’s industry share lowers the capital reward gap \( (r_1 - r_2) \) implying that firms will have an incentive to move back to region 2. A core-periphery
Figure 1: Locational equilibria

Figure 2: Bifurcation diagram
outcome is stable for high trade freeness ($\phi = 0.75$) but unstable for low trade freeness. For intermediate trade freeness ($\phi = 0.24$), all three allocations, the symmetric interior equilibrium and the two core-periphery equilibria are stable.

2.4.1 Locational forces

The market allocation is driven by different agglomeration and dispersion forces which can be identified by making use of (12) and (13).\(^\text{10}\)

Intra-industry spillovers are an agglomeration force. A higher local industry share lowers the variable input requirement and raises firms’ operating profits. Thus, more capital is attracted to that region.

The local competition effect (also termed crowding effect) and intra-industry spillovers act in favor of a dispersed outcome. The competition effect describes the tendency of firms to produce in regions with only few competitors. Starting from a symmetric allocation of industry, increasing the share of industry in one region (for given production costs) drives down operating profits in that region. This will in turn discourage capital owners to supply their capital there. The second dispersion force works through the worker’s wage rate. A higher number of firms lowers variable costs but, due to inter-industry spillovers, the wage paid to workers in the core exceeds the wage paid in the periphery. Higher production costs in turn lower firms’ operating profits which discourages a movement of capital into that region.

2.4.2 Symmetry Breaking

To assess the stability of the different long-run equilibria we derive the market break point, $\phi^B$, which is the threshold level of trade freeness above which the symmetric equilibrium becomes unstable.

Figure 2 depicts the stability of long run equilibria for symmetric region size. The model exhibits a subcritical pitchfork. As soon as $\phi$ exceeds the critical break point $\phi^B$, the only stable equilibrium is the core-periphery outcome. The expression for $\phi^B$ is given in Appendix A. The break point depends in intuitive ways on the parameters: when agglomeration forces become stronger, $\phi^B$ falls, so that the range of trade freeness levels at which the symmetric equilibrium is stable shrinks. This is the case when intra-industry

\(^{10}\)A formal exposition of the forces of the model can be found in Appendix C.
spillovers increase (higher $\gamma$), inter-industry spillovers decrease (lower $\mu$) or $\sigma$ decreases, which means higher economies of scale at the firm level.

2.4.3 Agglomeration rent and sustain point

Next, we assess the stability of the core-periphery equilibria and derive the level of trade freeness $\phi^S$ (the ‘sustain point’), up to which a core-periphery equilibrium can be sustained.

When all industry is agglomerated, say, in region 2, capital earns an agglomeration rent, $\Omega_2(\phi, \cdot) \equiv (r_2(\phi, \cdot) - r_1(\phi, \cdot)) \mid s_n = 0$:

$$\Omega_2(\phi, \cdot) = \frac{\alpha}{\sigma} \left[ 2 - \left( \frac{1 + \gamma}{1 + \mu} \right)^{1-\sigma} \left( \frac{s_k + s_l}{\phi} + [(1 - s_k) + (1 - s_l)]\phi \right) \right].$$

(14)

which is the loss that a firm would incur if it were to relocate from region 2, the core, to the periphery region 1, given that all other firms stay in the core.

The sustain point solves $\Omega_2(\phi, \cdot) = 0$. At this level of trade freeness, the agglomeration rent is zero so that full agglomeration is viable for $\phi > \phi^S$. The expression for $\phi^S$ is presented in Appendix B. Again, stronger agglomeration forces decrease the sustain point, which means full agglomeration can be sustained for smaller levels of trade freeness. This is the case when intra-industry spillovers increase, inter-industry spillovers decrease, or $\sigma$ decreases.

Moreover, the overlap between the sustain and market break point depicted in Figure 2 reflects the range of levels of trade freeness at which both types of equilibria, the symmetric as well as the core-periphery outcome are stable.

2.4.4 Welfare Analysis

To study the welfare effects of a reallocation of industry, we first derive the indirect utility functions of workers and capital owners in region $i$:

$$V_{K_i} = -\alpha \ln P_i + s_n r_1 + (1 - s_n)r_2, \quad V_{L_i} = -\alpha \ln P_i + w_i,$$

(15)

where $w_1 = (1 + \mu s_n)$ and $w_2 = (1 + \mu(1 - s_n))$. Regional welfare is assumed to be the weighted sum of indirect utilities of capital owners and workers residing in the respective region. We let the government attach a weight $\lambda$ to workers’ welfare. Then regional welfare is given by:

$$W_1 = \lambda s_l V_{L_1} + (1 - \lambda)s_k V_{K_1}, \quad W_2 = \lambda(1 - s_l) V_{L_2} + (1 - \lambda)(1 - s_k) V_{K_2}.$$

(16)
For weak $\gamma$ and low $\phi$, residents of any region unambiguously lose as the share of industry in their region declines, since to consumer prices rise and wage rates fall. Residents of the agglomerating region experience a welfare increase since they save on transport costs on imported varieties and workers earn a higher wage rate. By contrast, the effect of a reallocation of firms on regional welfare is ambiguous for strong intra-industry spillovers and high $\phi$. For instance, for high $\gamma$, at $s_n = 0$ even residents of region 1 may benefit from an agglomeration in region 2, since consumer prices are low due to strong spillovers. If at the same time $\phi$ is sufficiently high, the benefit from lower producer prices exceeds the cost of importing industrial goods. However, with an ongoing reallocation of industry towards region 1 the gains from intra-industry spillovers decline, increasing consumer prices, thereby hurting households in both regions.

Next, to check whether the arising location pattern is socially desirable (i.e. whether there is too much or too little agglomeration), we compare the social planner’s choice of industry allocation to the market outcome. Since conflicting interests among residents of different regions make the Pareto criterion unapplicable, we apply a utilitarian concept and assume the social welfare function to be the sum of household’s indirect utilities $W = W_1 + W_2$. We assume that the social planner takes market prices as given and only decides over the allocation of industry.\footnote{Pflüger and Südekum (2008) show that the resulting allocation is the same as when the planner can implement first-best welfare, which also corrects for the price distortion in the industrial sector stemming from imperfect competition.} Figure 3 depicts the social welfare function for different levels of trade freeness and symmetric region size.

While partial agglomeration is never optimal for the social planner, a symmetric allocation is chosen at low $\phi$ and a core periphery equilibrium at high $\phi$. We denote by $\phi^{SB}$ the level of trade freeness at which the social planner is just indifferent between implementing a symmetric allocation or a core periphery outcome. Formally $\phi^{SB}$ solves $W|_{s_n = \frac{1}{2}} = W|_{s_n = 1} = W|_{s_n = 0}$. Comparing $\phi^{SB}$ with $\phi^B$ allows us to detect whether the market outcome is socially desirable. It turns out that the social break point lies below the market breakpoint for our parameter restrictions,\footnote{The full expression for $\phi^{SB}$ is suppressed here but is available upon request.} which implies that for $\phi^{SB} < \phi < \phi^B$ the market exhibits under-agglomeration (see also Figure 2). Given that our model includes external economies, this is not really surprising.
2.5 Long run equilibrium and welfare: the asymmetric case

So far we have assumed regions to be equally endowed with the immobile factor. In this section, we generalise the model to allow for differences in regional workforces. In particular, we consider region 1 to host more workers than region 2, so that \( s_1 \geq \frac{1}{2} \).

2.5.1 Region size effect

Recall that capital moves in search of the highest nominal reward where the capital reward rates are given by (12) and (13). For simplicity we will assume that regions are equally rich in capital, i.e. each region owns half of the world capital stock \( s_k = 1/2 \) but they may differ in the number of workers. This gives rise to another agglomeration force, which we term region size effect. This describes the tendency of firms to produce in the larger market and to export to the smaller market.\(^\text{13}\) Formally, the market size effect is derived by differentiating the capital reward gap with respect to the share of immobile workers in region 1, \( s_1 \), evaluated at the symmetric equilibrium in the absence of inter-and intra-industry spillovers:

\[
\frac{\partial (r_1 - r_2)}{\partial s_1} \bigg|_{s_n=\frac{1}{2}, \mu_i=\gamma_i=0} = \frac{4\alpha}{\sigma} \frac{(1 - \phi)}{(1 + \phi)} \geq 0. \tag{17}
\]

\(^\text{13}\)The region size effect is actually made up of two effects: the market size effect described above, and the factor proportions effect: the larger region has larger relative supply of labour.
2.5.2 Bifurcation diagram and agglomeration rent

Once we allow regions to differ, the symmetric equilibrium can no longer be stable. The blue curve in the bifurcation diagram in Figure 4 identifies stable equilibria for different levels of $\phi$, assuming $s_l = 0.8$. For low levels of trade freeness a stable asymmetric interior equilibrium emerges, where the larger region (region 1) hosts more than half of the total industry. However, for high $\phi$, both the core in the large region as well as the core in the smaller region constitute stable equilibria.

![Bifurcation diagram for asymmetric region size](image)

Figure 4: Bifurcation diagram for asymmetric region size

Both core-periphery equilibria, $s_n = 1$ and $s_n = 0$ are stable, since all firms, once agglomerated in the region, earn a positive agglomeration rent. As Figure 5 shows, however, for $\phi < 1$, the agglomeration rent is clearly higher when all industry is in the larger region.

Our model then allows for the possibility that the entire industry is concentrated in the smaller region, despite the fact that firms could earn a higher agglomeration rent if all industry were located in the larger region.\textsuperscript{14} This new feature of the asymmetric model is

\textsuperscript{14} The literature typically assumes that there exists some coordination failure or absence of rational expectations (e.g. lack of information or costs that hinder firms to relocate) which makes firms unable or unwilling to commit to relocate (see Baldwin et al. (2003) or Krugman (1991c)). Without this assumption it becomes difficult to justify the existence of multiple equilibria. Krugman (1991a) argues that rational expectations are hard to justify since they call for a degree of information and sophistication that is
in contrast to the ‘footloose capital’ model described in Baldwin et al. (2003) and used by Ottaviano and van Ypersele (2005)\textsuperscript{15} where the larger region always hosts a larger share in industry irrespective of the underlying level of trade freeness.

2.5.3 Welfare

We stick to our definition of global welfare as the sum of regional welfare levels, where $W_1$ and $W_2$ are given by (16). Figure 6 depicts the social welfare function for asymmetric region size and different levels of trade freeness.

Note that for low $\phi$ (e.g. $\phi = 0.05$ in the Figure), partial agglomeration, with the larger region hosting a larger share in industry, is socially desirable. For sufficiently high $\phi$, global welfare is maximized when all industry is agglomerated in the large region:

Proposition 1 For $\phi > \phi^{SB}$, we have $W(1) > W(0)$ iff $s_l > \frac{1}{2}$.

Proof. See Appendix D. ■

\textsuperscript{15}In Ottaviano and van Ypersele (2005), for high trade costs there is a stable interior asymmetric equilibrium, where the larger region hosts a larger industry share, whereas for low trade costs all industry will be agglomerated in the larger region.
The intuition for the result is that when the core is in the larger region, the majority of households benefit from a lower cost-of-living index and higher wages.

However, as outlined above, our model allows for a stable core-periphery equilibrium in the smaller region. It therefore allows for an inefficient but stable allocation of industry. Figure 4 shows the welfare optimal allocation of industry as the red curves: The figure also shows that whenever there is an equilibrium with full agglomeration, this is also socially optimal.

3 Subsidy Competition

3.1 Basic Setup

We are interested in the outcome of subsidy competition in the presence of technological spillovers. Assume that the level of trade freeness is sufficiently high such that originally, industry is agglomerated in one region. Each regional government maximizes welfare of its residents by using subsidies to influence capital owners’ investment decision. The core region, say region 2, as well as the periphery benefit from retaining or attracting firms since hosting the industry core increases welfare of immobile factor owners residing in the core through lower transport cost (‘cost-of-living effect’) and a higher wage rate. In order to derive analytical expressions for the different subsidy levels we model subsidies $z_i$ in
their simplest form, namely as a direct lump-sum payment to capital owners. Firms move according to the highest post-subsidy capital reward rate, $r_{i}^* = r_{i} + z_{i}$. Laborers’ and capital owners’ endowment is taxed in a lump sum fashion to finance subsidy payments. The regional budget constraints are:

$$z_{1}s_{n} = T_{1}(s_{k} + s_{l}), \quad z_{2}(1 - s_{n}) = T_{2}((1 - s_{k}) + (1 - s_{l})). \quad (18)$$

For region 1, total subsidy payments are the subsidy times the share of firms $s_{n}$, while tax payments are lump-sum taxes paid by the $s_{k}$ capitalists and $s_{l}$ workers.

Government expenditure and tax revenue are zero once the region happens to become the periphery, since there are no firms to subsidize. Inserting the price indices from (5) as well as the post-subsidy capital reward rates, wage rates and tax payments into the indirect utility functions, using (16) allows us to derive regional welfare both for the case where region 1 hosts the industry core and for the case where region 1 is the periphery (the expressions for region 2 being analogous):

$$W^{C}_{1}(z_{1}) = W_{1}\bigg|_{s_{n}=1} = \lambda s_{l}(1 + \mu) + (1 - \lambda)s_{k}\left(z_{1} + \frac{2\alpha}{\sigma}\right) - (\lambda s_{l} + (1 - \lambda)s_{k})\left(\alpha \ln P^{C} + \frac{z_{1}}{s_{l} + s_{k}}\right) \quad (19)$$

$$W^{P}_{1}(z_{2}) = W_{1}\bigg|_{s_{n}=0} = \lambda s_{l} + (1 - \lambda)s_{k}\left(z_{2} + \frac{2\alpha}{\sigma}\right) - (\lambda s_{l} + (1 - \lambda)s_{k})\left(\alpha \ln P^{P}\right) \quad (20)$$

where $P^{C} \equiv \left(\frac{1 + \mu}{1 + \gamma}\right)$ and $P^{P} \equiv \phi^{\frac{1}{1 - \sigma}} (\frac{1 + \mu}{1 + \gamma})$ are the price indices for the core and periphery case, respectively. Whereas welfare of a peripheral region is increasing in the subsidy level offered in the core region, it decreases in its own subsidy level as soon as it hosts the industry core. This is due to the ownership structure of capital and the regional financing scheme. Since capital income is repatriated to the region of origin and subsidies are financed via regional taxes, each capital owner residing in the periphery benefits from a subsidy distributed in the core region. Welfare of the core is falling in its own subsidy level, since it is entirely financed by residents of the core, but part goes to capital owners residing in the periphery.

We adopt the same game structure as Baldwin and Krugman (2004) and apply a sequential move game. In the first stage the government of the core (Govt 2) sets its subsidy level, the periphery (Govt 1) then chooses its subsidy in the second stage. In the third stage
firms choose their location of production dependent on the gross capital reward rates. Production and consumption take place as described in the preceding sections. We continue to assume that $s_k = 1/2$ but allow for asymmetries in region size in terms of the number of workers and in particular allow for the possibility that the initial core region is smaller than the periphery. As before, we suppose that $s_l \geq \frac{1}{2}$, so that region 1 is larger, but region 2 is the core, so that the equilibrium without subsidies is inefficient since the core is in the smaller region. Hence, in contrast to the previous literature, we allow for a situation where the initial factors (e.g. market size) which caused this agglomeration have vanished over time but where locational hysteresis has led to a persisting inefficient agglomeration, where firms continue to produce in the smaller region. Differences in region size are only allowed to the extent to which welfare of the smaller core region, $W_C^2(z_2) \equiv W_2|_{s_n=0}$ still exceeds the welfare level in the periphery case, $W_P^2(z_1) \equiv W_2|_{s_n=1}$ such that the outcome of the subsidy competition game does not become trivial.\footnote{Otherwise the benefits of hosting the industry core in the form of lower living costs and higher wage rates would not suffice for the government of the core region to engage in a costly subsidy competition.}

3.1.1 Stage Two: Periphery’s Decision

In stage two Govt 1 (the periphery) decides whether to induce a relocation of the industry core or to stay out of the competition and leave the allocation of industry unchanged. However, due to the existence of agglomeration forces Govt 1 will not achieve any movement of capital if it sets its subsidy too low. In order to induce firms to relocate, the subsidy level has to be at least as high as the agglomeration rent accruing to firms in the core plus the core’s subsidy rate, i.e. $z_{1\text{ min}}(z_2) = \Omega_2 + z_2$. This would make a capital owner indifferent between staying in the core – realising the agglomeration rent $\Omega_2$ – and being paid a subsidy of $z_2$, or moving to the periphery and being paid $z_1$. Inserting $\Omega_2$ using (14) and $s_k = 1/2$ yields

$$z_{1\text{ min}}(z_2) = \frac{\alpha}{\sigma} \left[ 2 - \frac{(\frac{1+s_l}{1+s_l})^{1-\sigma}(1 + 2s_l - (2s_l - 3)\phi^2)}{2\phi} \right] + z_2. \quad (21)$$

Any subsidy level below $z_{1\text{ min}}(z_2)$ will fail to induce a relocation of firms. Clearly, whether Govt 1 decides to enforce a relocation by setting a subsidy level equal to $z_{1\text{ min}}$ depends on the subsidy level set by the core government in the first stage. Govt 1 chooses its subsidy...
level according to the following decision rule:

\[ z_1 = \begin{cases} 
z_1^{\text{min}}(z_2) & \text{if } W_1^C(z_1) > W_1^P(z_2), \\
0 & \text{otherwise.} 
\end{cases} \]

Intuitively, for Govt 1 to engage in the competition, welfare after having successfully attracted all industry \((W_1^C(z_1))\) has to exceed the welfare level for the case where region 1 remains the periphery \((W_1^P(z_2))\). Using this decision rule, we are able to derive the maximum subsidy level \(z_1^{\text{max}}\) that Govt 1 would be willing to incur. This subsidy level solves \(W_1^P(z_2) = W_1^C(z_1^{\text{max}})\). To enhance intuition we evaluate the resulting subsidy levels at \(\lambda = 1/2\) for the time being and turn later to the case of unequal welfare weights. Using (19) and (20) yields

\[ z_1^{\text{max}}(z_2)\big|_{\lambda=1/2} = 2\mu s_l + \frac{\alpha(1 + 2s_l)}{1 - \sigma} \ln \phi - z_2. \tag{22} \]

The first term in (22) captures the potential ‘wage effect’ for region 1’s workers that will occur if Govt 1 succeeds in attracting the industry core. The second term captures the ‘cost-of-living effect’ which enters through the price index prevailing in the respective region.\(^{17}\) This term is positive since \(\sigma > 1\) and \(\ln \phi < 0\). Finally, the last term expresses the ‘subsidy effect’ for each of region 1’s capital owners. The higher \(z_2\) set in the first stage, the lower will be \(z_1^{\text{max}}\), i.e. the lower will be the willingness of Govt 1 to attract the core. It follows that as soon as \(z_1^{\text{min}}(z_2) \geq z_1^{\text{max}}(z_2)\) Govt 1 will no longer be willing to attract the core, since the necessary subsidy is so high that the gain from attracting the core is lower than the cost.

### 3.1.2 Stage One: Core’s Decision

Turning to the first stage, Govt 2 acts as a Stackelberg leader, foreseeing the implications of its choice on the choice of Govt 1 in the following stage. Since Region 2 welfare falls in its own subsidy, Govt 2 will want to set the lowest subsidy level consistent with defending the core, if it wants to defend at all. This subsidy level, \(z_2^d\), is that at which the periphery in the second stage will no longer be willing to snatch the core. Formally, \(z_2^d\) solves \(z_1^{\text{min}}(z_2^d) = z_1^{\text{max}}(z_2^d)\). Using (21) and (22), we get:

\[ z_2^d\big|_{\lambda=1/2} = \frac{1}{2} \left\{ 2\mu s_l + \frac{\alpha(1 + 2s_l)}{1 - \sigma} \ln \phi - \frac{\alpha}{\sigma} \left[ 2 - \frac{(1 + \gamma)1 - \sigma}{2\phi} \left( (1 + 2s_l - (2s_l - 3)\phi^2) \right) \right] \right\} \tag{23} \]

\(^{17}\)Due to symmetric spillovers both regions benefit from high intra-industry spillovers through lower prices. Hence, any disparity in consumer prices between core and periphery stems from trade costs only.
Therefore, Govt 2 will set its subsidy at $z_2^d$ if its welfare when it defends the core exceeds the welfare it receives when becoming the periphery. Otherwise, it would set a subsidy of $z_2^d - \varepsilon$, where $\varepsilon$ is a small positive number. The reason is that by raising its subsidy, Govt 2 raises the subsidy which Govt 1 has to pay in order to attract industry. This benefits region 2’s capital owners via the repatriation externality. Hence, we have the following decision rule:

$$z_2^* = \begin{cases} z_2^d & \text{if } W^C_2(z_2^d) \geq W^P_2(z_1^{min}(z_2)), \\ z_2^d - \varepsilon & \text{otherwise.} \end{cases}$$

### 3.2 Equilibrium

Having derived the decision rules of the respective players and the according subsidy levels, this section identifies the outcomes of the game.

#### 3.2.1 Equilibrium 1: Relocation of industry

Whether Govt 2 decides to defend the industry core depends on how much Govt 2 values workers’ relative to capitalists’ welfare in region 2. We start with the case where workers and capitalists’ welfare is equally weighted.

**Proposition 2.** For equal welfare weights, $\lambda = 1/2$, Govt 2 will never defend the core for any $s_l \geq \frac{1}{2}$. The equilibrium subsidy levels are given by $z_2^* = z_2^d - \varepsilon$, $z_1^* = z_1^{min}(z_2^d - \varepsilon)$ with some small $\varepsilon > 0$.

**Proof.** See Appendix D. ■

By setting $z_2 = z_2^d - \varepsilon$, Govt 2 ensures that region 1 snatches the core offering $z_1^{min}(z_2^d - \varepsilon)$, thereby restoring an efficient allocation of industry. At the same time, since $z_2$ raises $z_1^{min}$, Govt 2 realizes the highest possible repatriation externality by setting $z_2 = z_2^d - \varepsilon$, which will benefit region 2’s capitalists via the repatriation of capital income. This result is rather intuitive, in the sense that the larger region has a ‘natural advantage’ in the subsidy game: when the core region is small, the agglomeration rent is small too. This implies that the periphery government has to offer capital owners a relatively small subsidy to induce a relocation. It also implies that the periphery government will be more willing to snatch the core, since the payoff to doing so increases with $s_l$. Hence, defending the core will be more costly for the core government. In fact, it becomes so costly that for a symmetric welfare
function, the core will only be defended if it is located in the larger region. In other words, subsidy competition restores an efficient allocation of industry. The next result states that welfare is then higher if it would be without subsidies and the core located in the smaller region.

**Proposition 3.** For $z^d_2 > 0$, 

(i) overall welfare is higher in the equilibrium with than without subsidies, 

$$W(z^{\min}_1(z^d_2 - \varepsilon), z^d_2 - \varepsilon) > W(0, 0),$$

(ii) region 1’s residents experience a welfare gain after having successfully attracted all industry compared to the initial regional welfare level:

$$W^C_1(z^{\min}_1(z^d_2 - \varepsilon)) > W^P_1(0).$$

**Proof.** See Appendix D.  ■

What cannot be unambiguously determined is whether the new periphery region (region 2) will be worse or better off after the relocation of industry compared to the initial welfare. On the one hand, a relocation of industry induced by a positive subsidy level set by Govt 1 imposes a positive externality on capital owners’ income in the new periphery. Half of the subsidy payment promised to industrial firms by Govt 1 accrues to capital owners of region 2. On the other hand, region 2 loses all industry thereby suffering from a lower wage rate and a higher cost-of-living index. Overall welfare however, will be higher after the relocation of industry towards an efficient industry allocation. This is an important result, since it shows that fiscal competition can help redress an inefficiency stemming from increasing returns to scale.

### 3.2.2 Equilibrium 2: Persistent inefficient industry allocation

In this subsection, we look at the case where the welfare function assigns a higher weight to workers than to capitalists. We may think of a government which leans towards representing worker interests, for distributional or political reasons.

Once we allow for $\lambda > 1/2$, region 2’s welfare differential $W^C_2(z^d_2) - W^P_2(z^{\min}_1(z^d_2 - \varepsilon))$ is no longer unambiguously negative for $s_t > 1/2$. This opens up the possibility that the core region will defend the core even if it is smaller and efficiency would require locating
all industry in the larger region. Intuitively, for \( \lambda = \frac{1}{2} \) and \( s_l > \frac{1}{2} \), we have just shown that the benefit capitalists incur through the repatriation of subsidies when the core moves to region 1 more than outweighs the loss to workers and capitalists through lower wages and a higher price index. When \( \lambda > \frac{1}{2} \), then, the core government weighs the loss to workers from falling wages and rising consumer prices after industry relocation more heavily than the gain to capitalists from the subsidies paid by the foreign government. In particular, we can show the following:

**Proposition 4.** There exists a region size \( \tilde{s}_l = s_l(\gamma) \) such that region 2 defends the core if and only if \( s_l < \tilde{s}_l \). Further, \( \tilde{s}_l \) satisfies

\[
(i) \tilde{s}_l = \frac{1}{2} \quad \text{for } \lambda = \frac{1}{2} \tag{24}
\]

\[
(ii) \frac{d\tilde{s}_l}{d\gamma} > 0 \quad \text{for } \lambda > \frac{1}{2} \tag{25}
\]

**Proof.** See appendix D. ■

Figure 7 plots \( \tilde{s}_l \) for \( \lambda = 0.8 \) in order illustrate the effect of region size and localization economies on core’s decision. For all \( s_l, \gamma \)-combinations above \( \tilde{s}_l \), the core government will not defend the core and industry will relocate towards the larger region 1; for all \( s_l, \gamma \)-combinations on and below \( \tilde{s}_l \), the core government defends the core and the allocation of industry remains inefficient. Most importantly, note that there are \( s_l, \gamma \)-combinations for which Govt 2 decides to defend the industry core against region 1 despite region 1 being larger in terms of workers (the shaded region in Figure 7). Hence, the disadvantage from becoming the periphery which predominantly affects workers via reduced real wage income exceeds the benefit of a relocation (the subsidy effect) for governments acting in workers’ interests. Figure 7 also shows \( \tilde{s}_l \) for \( \lambda = \frac{1}{2} \), which is horizontal at \( s_l = \frac{1}{2} \); in this case, the core defends if and only if it is the larger region.

Intuitively, the figure shows that an inefficient industry allocation can persist only if the difference in region sizes is small and if spillovers are relatively large. On the one hand, for given \( \gamma \), a larger \( s_l \) implies that it will be more and more difficult for the (smaller) core to keep the industry from leaving. Larger spillovers imply that the core will be more willing to hang on to the core. On the one hand, the costs of retaining the core are reduced, since the agglomeration rent increases and Govt 2 therefore has to pay higher subsidies to snatch the core. On the other hand, this means that if Govt 2 defends, capital owners do not
benefit from the higher subsidy paid by region 1. But this second effect is dominated by the first (see the Proof of Proposition 4), so that the core government will be more willing to defend the core when spillovers increase.

\[ \frac{W_2^C}{W_2^P} < \frac{W_1^C}{W_1^P} \]

\[ \gamma \]

Figure 7: Govt 2’s Decision \( (z_2^d > 0) \)

**Proposition 5.** If Govt 2 defends the core by setting \( z_2 = z_2^d \), compared to a situation without subsidies,

(i) aggregate welfare falls,

(ii) region 2 welfare decreases for \( z_2^d > 0 \), and

(iii) region 1 welfare increases.

**Proof.** See appendix D. 

This is intuitive, since the allocation of industry is not changed by subsidies. The only effect relevant for welfare is the payment of subsidies. Since these are paid by region 2 residents but part of the subsidy accrues to residents of region 1, subsidies redistribute from region 2 to region 1. Overall welfare falls since the subsidy redistributes from workers to capitalists, and this reduces welfare for \( \lambda > \frac{1}{2} \).
4 Conclusion

The paper studies subsidy competition among asymmetric regions in a model with mobile capital and agglomeration forces. We start from a situation where industry is agglomerated in the smaller region for historic reasons, and ask whether subsidy competition can lure industry to the larger region. When governments maximize a weighted welfare function, we find the answer is yes when the welfare weights of workers and capital owners are equal. In this instance, the smaller region does not prevent the larger region from paying subsidies which lures all capital to that region. However, when workers’ welfare is weighted more heavily, the smaller region might pay subsidies to capital owners that are just large enough to prevent them from shifting their capital to the other region. In this case, if the size difference between the regions is not too large, an inefficient industry location prevails. Our paper thus provides a formalization of the intuitive argument that, when external economies are prevalent, jurisdictional competition can improve the spatial allocation of economic activity (e.g. Duranton, 2008). Unless territorial welfare functions are skewed towards immobile workers and size differences between regions are small, this notion is shown to be correct.

Appendix

A Break point

Solving

\[ \frac{d(r_1 - r_2)}{ds_n} \bigg|_{s_n = \frac{1}{2}} = 0, \]

using (12) and (13) gives the ‘break point’

\[ \phi^B = \frac{4 + 6\mu - 4\mu \sigma + \gamma(4\sigma + \mu - 2) - 2\sqrt{2\sqrt{(\gamma - \mu)(\sigma - 1)(4 + \mu(4 + \gamma - 2\sigma) + 2\gamma\sigma)}}}{(2 + \gamma)(2 + \mu)} \]

B Sustain point

Solving (14) gives the sustain point

25
\[
\phi^S = \frac{(1+\mu)^\sigma (1 + \mu - \sqrt{(1 + \mu)^2 - (1 + \gamma)^2 (\frac{1+\mu}{1+\gamma})^{2\sigma}})}{1 + \gamma}.
\] (A.3)

Differentiating \( \phi^S \) gives:

\[
\frac{\partial \phi^S}{\partial \gamma} = -\frac{(1 + \mu)(\sigma - 1)}{(1 + \gamma)\sqrt{(1 + \mu)^2 - (1 + \gamma)^2 (\frac{1+\mu}{1+\gamma})^{2\sigma}}}, \quad \phi^S < 0 \quad (A.4)
\]

\[
\frac{\partial \phi^S}{\partial \mu} = \frac{\sigma - 1}{\sqrt{(1 + \mu)^2 - (1 + \gamma)^2 (\frac{1+\mu}{1+\gamma})^{2\sigma}}} \cdot \phi^S > 0 \quad (A.5)
\]

### C Locational Forces

The locational forces are obtained by evaluating the different forces at \( s_n = \frac{1}{2} \) for the symmetric region case, i.e. \( s_l = s_k = \frac{1}{2} \).

#### C.1 Intra-Industry Spillovers

To isolate the intra-industry spillover force we differentiate the capital reward gap with respect to \( s_n \), holding fixed the market crowding effect (the direct effect of the industry share on \( r_i \)) and inter-industry spillovers.

\[
\left. \frac{d(r_1 - r_2)}{ds_n} \right|_{s_n = \frac{1}{2}, \mu = 0} = \frac{\partial(r_1 - r_2)}{\partial \chi} \frac{\partial \chi}{\partial \gamma} \frac{\partial \gamma}{\partial s_n} = -\frac{32\alpha \gamma (1 - \sigma)}{(2 + \gamma)\sigma} \cdot \phi > 0.
\] (A.6)

This expression is positive for our parameter specifications and captures the agglomerative intra-industry spillover force.

#### C.2 Inter-Industry Spillovers

Holding fixed the market crowding effect and intra-industry spillovers yields the degglomerative inter-industry spillover force

\[
\left. \frac{d(r_1 - r_2)}{ds_n} \right|_{s_n = \frac{1}{2}, \gamma = 0} = \frac{\partial(r_1 - r_2)}{\partial \chi} \frac{\partial \chi}{\partial \mu} \frac{\partial \mu}{\partial s_n} = \frac{32\alpha \mu (1 - \sigma)}{(2 + \mu)\sigma} \cdot \phi \frac{\phi}{(1 + \phi)^2} < 0.
\] (A.7)
C.3 Market Crowding Effect

The second dispersion force denoted as the market crowding effect works through the direct effect of \( s_n \) on \( r_1 \) in (12). Holding fixed inter- and intra-industry spillovers yields

\[
\frac{\partial (r_1 - r_2)}{\partial s_n} \bigg|_{s_n = \frac{1}{2}, \mu = \gamma = 0} = -\frac{8\alpha (-1 + \phi)^2}{\sigma (1 + \phi)^2} \leq 0. \tag{A.8}
\]

which is unambiguously non-positive.

D Proofs

Proof of Proposition 1.

We show that irrespective of the welfare weight \( \lambda \), an industry allocation where all firms are located in the larger region is preferred by the social planner to an allocation with all firms in the smaller region. Comparing the sum of regional welfare for the case where the core is located in the larger region with the sum of regional welfare for the case where the small region hosts the core yields

\[
W \big|_{s_n = 1} - W \big|_{s_n = 0} = \left(2s_l - 1\right)\left[\frac{\mu (\sigma - 1) - \alpha \ln \phi}{\sigma - 1}\right] \lambda \quad \tag{A.9}
\]

which is unambiguously positive for \( s_l > \frac{1}{2} \). ■

Proof of Proposition 2.

Plugging in the respective subsidy levels \( z_2^d \) and \( z_1^{min} \) from (23) and (21) into region 2’s welfare function, respectively yields

\[
W^C_2(z_2^d) - W^P_2(z_1^{min}(z_2)) \big|_{\lambda = 1/2} = -\frac{\left(2s_l - 1\right)}{2} \left[\mu - \frac{\alpha}{\sigma - 1} \ln \phi\right], \quad \tag{A.10}
\]

which is negative for \( s_l > \frac{1}{2} \). It follows that Govt 2 sets \( z_2 = z_2^d - \varepsilon \) implying that \( W^C_1(z_1^{min}(z_2^d - \varepsilon)) > W^P_1(z_2) \). ■

Proof of Proposition 3.

(i) From Proposition 1, we know that without subsidies, welfare is higher if the core is in the larger region. Evaluating the effect of subsidies on welfare in the case where the core is in region 1 gives:

\[
(1 - \lambda)z_1^{min} - \frac{(1 - \lambda)s_k + \lambda s_l}{s_k + s_l}z_1^{min}. \tag{A.11}
\]
For $\lambda = \frac{1}{2}$, this is zero. The same holds for the welfare effect of subsidies if the core is in region 2. Hence, welfare with subsidies is still highest if the core is in the larger region.

(ii) From the proof above we know that $W_C^1(z_{min}^1(z_d^2 - \varepsilon)) > W_P^1(z_2)$ holds. Since $\frac{\partial W_P^1}{\partial z_2} > 0$ implies $W_P^1(z_2) > W_P^1(0)$ for $z_d^2 > 0$ it follows that $W_C^1(z_{min}^1(z_d^2 - \varepsilon)) > W_P^1(0)$, i.e. Govt 1 is better off after successfully snatching the core compared to the baseline welfare level.

Proof of Proposition 4.

The locus $\tilde{s}_l = s_l(\gamma)$ is implicitly defined by

$$\Delta(s_l, \gamma, \cdot) \equiv W_C^2(z_d^2, s_l, \gamma, \cdot) - W_P^2(z_{min}^1(z_d^2 - \varepsilon), s_l, \gamma, \cdot) = 0.$$ 

Part(i) follows immediately from setting $\lambda = \frac{1}{2}$ in (A.10). To prove (ii), differentiation of $\Delta$ gives the slope:

$$\frac{d\tilde{s}_l}{d\gamma} = \frac{d\Delta/d\gamma}{d\Delta/ds_l}; \quad (A.12)$$

where

$$\frac{d\Delta}{d\gamma} = \frac{\alpha(2\lambda - 1)(\sigma - 1)\phi(1 + \mu)^{\sigma - 1} \left(\lambda(1 - 2s_l)(1 - 2s_l) - 2s_l + \frac{2s_l + 1}{\sigma^2} - 2s_l + 3\right)}{2(1 + \gamma)\sigma(2s_l - 3)(\lambda(2s_l - 1) + 1)} > 0 \quad (A.13)$$

for $s_l, \lambda > \frac{1}{2}$.

The expression for $d\Delta/ds_l$ is rather messy and therefore omitted. However, we can show numerically that it is negative for the parameters used in the paper. Intuitively, when region 2 becomes smaller, it will be less willing to defend. Formally, we can show that differentiating $\frac{d\tilde{s}_l}{d\gamma}$ and evaluating at $\lambda = \frac{1}{2}$ gives

$$\frac{d(d\tilde{s}_l/d\gamma)}{d\lambda} \bigg|_{\lambda=1/2} = \frac{\alpha(\sigma - 1)^2 (4(s_l - 1)s_l - 1)(1 + \mu)^{\sigma - 1} \phi^{\phi^{\frac{\phi}{\sigma^2}} - \phi^{\phi^{\frac{\phi}{\sigma^2}}}} \left((2s_l + 1)\phi^{\frac{2s_l}{\sigma^2}} + (3 - 2s_l)\phi^{\frac{2s_l}{\sigma^2}}\right)}{(1 + \gamma)^{\sigma(2s_l - 3)(2s_l + 1)(\alpha \log(\phi) + \mu(1 - \sigma))}} > 0 \quad (A.14)$$

Hence, $\frac{d\tilde{s}_l}{d\gamma} > 0$ for $\lambda, s_l > \frac{1}{2}$.

Proof of Proposition 5.

(i) Since the industry allocation is not affected, we need to consider only the effect of subsidies on welfare. This is given by:

$$Z = (1 - \lambda)z_2 - \frac{(1 - \lambda)(1 - s_k) + \lambda(1 - s_l)}{1 - s_k + 1 - s_l}z_2 = \frac{(1 - 2\lambda)(1 - s_l)}{1 - s_k + 1 - s_l}z_2.$$
This expression is negative for $\lambda > \frac{1}{2}$, so subsidies decrease welfare.

(ii) and (iii). From $\frac{\partial W^C(z_2)}{\partial z_2} < 0$ and $\frac{\partial W^P(z_2)}{\partial z_2} > 0$ it follows that $W^C(z_2^d) < W^C(0)$ and $W^P(z_2) > W^P(0)$ for $z_2^d > 0$. Residents of region 2 will unambiguously experience a welfare decline whereas households in region 1 experience an unambiguous welfare gain. ■

References


